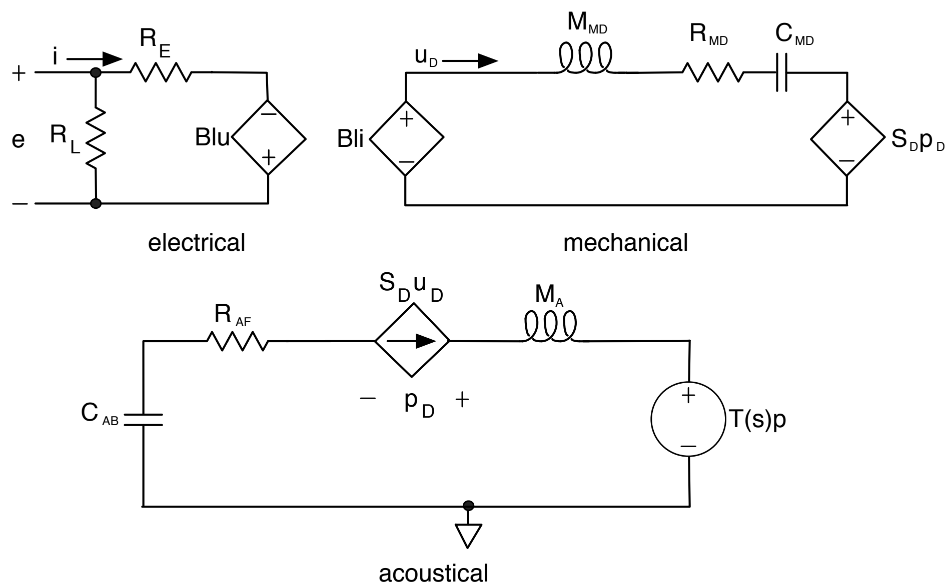
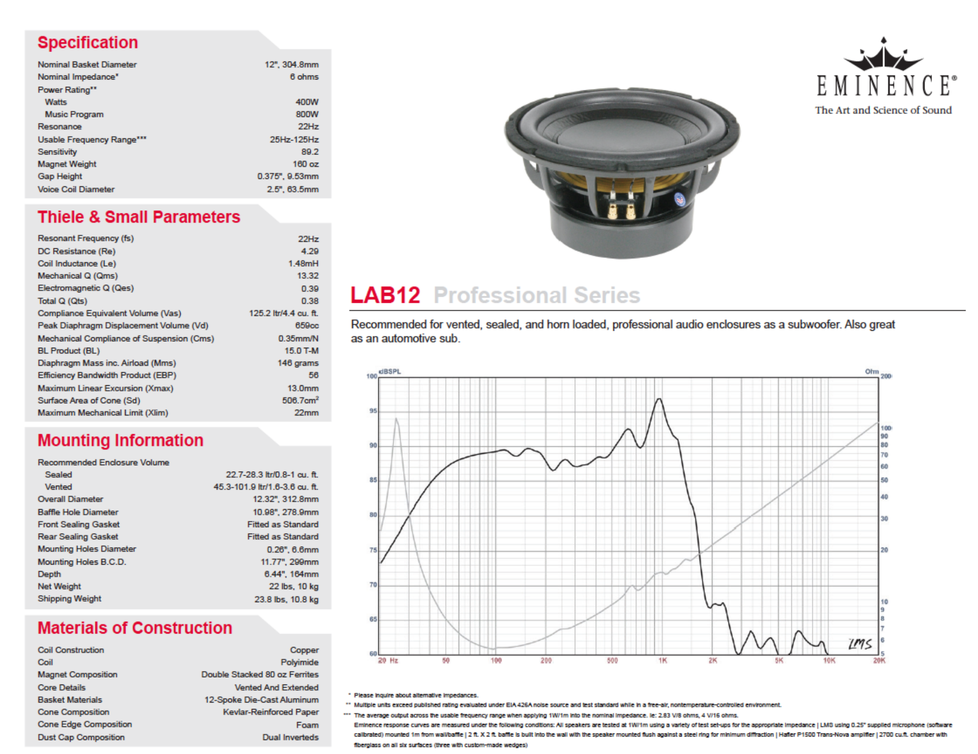
Transducer Theory

v2.0

Will Pirkle





Specification

Nominal Basket Diameter	12", 304.8mm
Nominal Impedance*	6 ohms
Power Rating**	
Watts	400W
Music Program	800W
Resonance	22Hz
Usable Frequency Range***	25Hz-125Hz
Sensitivity	89.2
Magnet Weight	160 oz
Gap Height	0.375", 9.53mm
Voice Coil Diameter	2.5", 63.5mm

Thiele & Small Parameters

Resonant Frequency (fs)	22Hz
DC Resistance (Re)	4.29
Coil Inductance (Le)	1.48mH
Mechanical Q (Qms)	13.32
Electromagnetic Q (Qes)	0.39
Total Q (Qts)	0.38
Compliance Equivalent Volume (Vas)	125.2 ltr/4.4 cu. ft.
Peak Diaphragm Displacement Volume (Vd)	659cc
Mechanical Compliance of Suspension (Cms)	0.35mm/N
BL Product (BL)	15.0 T-M
Diaphragm Mass inc. Airload (Mms)	146 grams
Efficiency Bandwidth Product (EBP)	56
Maximum Linear Excursion (Xmax)	13.0mm
Surface Area of Cone (Sd)	506.7cm²
Maximum Mechanical Limit (Xlim)	22mm

Mounting Information

Recommended Enclosure Volume	
Sealed	22.7-28.3 ltr/0.8-1 cu. ft.
Vented	45.3-101.9 ltr/1.6-3.6 cu. ft.
Overall Diameter	12.32", 312.8mm
Baffle Hole Diameter	10.98", 278.9mm
Front Sealing Gasket	Fitted as Standard
Rear Sealing Gasket	Fitted as Standard
Mounting Holes Diameter	0.26", 6.6mm
Mounting Holes B.C.D.	11.77", 299mm
Depth	6.44", 164mm
Net Weight	22 lbs, 10 kg
Shipping Weight	23.8 lbs, 10.8 kg

Materials of Construction

Coil Construction	Copper
Coil	Polyimide
Magnet Composition	Double Stacked 80 oz Ferrites
Core Details	Vented And Extended
Basket Materials	12-Spoke Die-Cast Aluminum
Cone Composition	Kevlar-Reinforced Paper
Cone Edge Composition	Foam
Dust Cap Composition	Dual Inverteds

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*** The average output across the usable frequency range when applying 1W/1m into the nominal impedance. Ie: 2.83 V/8 ohms, 4 V/16 ohms.

Eminence response curves are measured under the following conditions: All speakers are tested at 1W/1m using a variety of test set-ups for the appropriate impedance | LMS using 0.25" supplied microphone (software calibrated) mounted 1m from wall/baffle | 2 ft. X 2 ft. baffle is built into the wall with the speaker mounted flush against a steel ring for minimum diffraction | Hafler P1500 Trans-Nova amplifier | 2700 cu.ft. chamber with fiberglass on all six surfaces (three with custom-made wedges)

1 Basic Transducer Acoustics

We shall now turn our attention to acoustic **transducers**. A transducer is a device that converts energy one form into another. Loudspeakers and microphones are acoustic transducers. A loudspeaker converts electrical energy (electrons in motion) to acoustic energy (air particles in motion). An audio amplifier supplies the electrical energy to the loudspeaker. The loudspeaker converts this energy into a force that drives a diaphragm. The moving diaphragm pushes and pulls on the surrounding air to produce sound. A microphone is the electrical complement to a loudspeaker since it converts acoustic energy into electrical energy. A sound's vibrating air supplies the acoustic energy. This vibrating air provides a force that pushes on a diaphragm and the microphone converts this force into an electrical signal. In its most simplistic definition, *acoustics* is the study of vibrating air – how sound waves propagate in a medium. Up to this point, our focus has been the understanding and design of electrical circuits to manipulate an audio signal. We are now ready to discuss transducer theory, that is, the design and analysis of loudspeakers and microphones. Our general goals in transducer theory are to be able to analyze and predict:

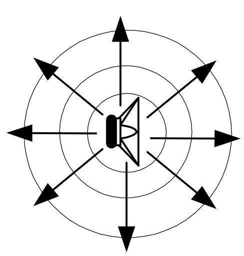
- Propagation – how sound radiates from a loudspeaker and travels through a medium
- Directivity – how the radiated sounds move in three dimensions through the medium
- Frequency Response – how the transducer and its enclosure alters the frequency components of the audio signal

In this chapter, we focus on the propagation and directivity aspects of sound – the fundamental theories of acoustics. In the remaining chapters, we will analyze, design, and predict loudspeaker and microphone frequency responses. We will find that speakers do not function well hanging in free-space – they need an enclosure. We will design loudspeaker enclosures for a given speaker to create desired frequency responses. We will also analyze or predict the frequency response of a loudspeaker based on the physical dimensions of the enclosure and the acoustic parameters of the loudspeaker itself. Gaining this understanding will require a combination of basic math, physics, and electrical engineering. As it turns out, the best way to analyze and design transducers is to model their behavior as if they were electrical circuits, so we are already well prepared in that respect.

In acoustics terminology there is a difference between a speaker and a speaker mounted in an enclosure (or box). We need to adjust our terminology to suit – unless otherwise specified, in this text we will use the following:

- A raw loudspeaker (not mounted in an enclosure) is called a **driver**.
- A driver mounted in an enclosure is called a **loudspeaker**.

1.1 The Ideal Loudspeaker



The ideal loudspeaker is known as a **point source** in transducer jargon. A point source is infinitely small and radiates all frequencies at equal power and in all directions. You could close your eyes and walk around the point source and the music would sound identical no matter where you stood – this is called **omni-directional radiation**. In general, we will try to design systems that behave like point sources, even though it is impossible to fully achieve.

Figure 1.1: The symbol for our ideal loudspeaker – a driver with concentric circles "radiating" from it.

1.2 Sound Pressure Level

As we discussed way back in Chapter 1, sound is vibrating air. Specifically, the vibrations consist of compression and rarefaction (or expansion) in which air particles are pressed closely together (compression) or pulled far apart (rarefaction). We use static air pressure (P_o) as our reference of how much the air is compressed or expanded. For our purposes, we will take P_o to be 15 lb/in^2 or 1.013x10^5 Pa (Pa = *pascals*, or N/m^2). Sound creates *pressure fluctuations* in its medium. Similarly, we've seen an AC source that creates voltage or current fluctuations in a circuit. In our audio circuits, we measured the relative strength of these fluctuations using either peak, peak-to-peak, or RMS measurements (see Figure 1.7). We might specify a "5 V_{peak}" signal or a "3 V_{RMS}" generator. We measure sound pressure fluctuations in the same manner: peak peak-to-peak, or RMS. There is a specific kind of measurement unit called **Sound Pressure Level**, or SPL. This measurement is made in decibels, and is sometimes referred to as **db_{SPL}**. The SPL measurement uses RMS pressure fluctuation as the variable. To be a decibel measurement, there must be a ratio. SPL is a comparison (ratio) of the measured RMS pressure fluctuations compared to the threshold of hearing in humans, 2x10^{-5} $Pa_{(RMS)}$. In this way, 0dB_{SPL} represents the pressure fluctuations for a sound that is just barely audible. Formally:

$$\begin{aligned} dB_{SPL} &= 20\log\left[\frac{p_{RMS}}{p_{ref}}\right] \\ &= 20\log\left[\frac{p_{RMS}}{2x10^{-5}Pa}\right] \end{aligned} \quad [1.1]$$

The pressure may not be measured in RMS units, but can easily be converted using the following formulae:

$$peak-to-peak = 2\sqrt{2}RMS \qquad RMS = \frac{peak-to-peak}{2\sqrt{2}} \qquad RMS = \frac{peak}{\sqrt{2}} \quad [1.2]$$

We can also convert db_{SPL} to pressure by solving the equation backwards, that is:

$$p_{RMS} = (2x10^{-5})(10^{dB/20}) \quad [1.3]$$

This value may then be converted to peak or peak-to-peak as needed.

Example 1.1 The threshold of pain in human ears in often considered to be 120dB_{SPL}. What is the peak-to-peak pressure ($p_{p\text{-}p}$) a listener experiences at this volume? How many times more than the threshold of hearing does this represent?

Answer: From [1.2], the RMS pressure is

$$p_{RMS} = (2x10^{-5})(10^{120/20}) = (2x10^{-5})(1000000) = 20Pa \quad [1.4]$$

20 Pa is one million times the pressure fluctuations of a just-audible sound. Our ears have a very high dynamic range!

1.3 Wave Propagation

Our next job is to understand the difference between the global motion of the sound wave, and the local motion of air particles. For example, suppose you witness someone far away from you clapping their hands – you first see the hands clap, then you hear the sound. The person who clapped vibrated the air next to their hands. Your ears (transducers in their own right) picked up the sound because the air particles next to your eardrums were vibrating. However, air particles vibrating next to your eardrum are not the same air particles that were initially set into motion by the handclap. The sound has propagated across a distance to reach your ears, yet the individual air particles have not moved across this distance. If the air had propagated too, then you would feel wind each time a sound occurs. This is clearly not the case. Another analogy might be a long line of dominoes falling – the first domino is set into motion, and this creates a 'wave' that propagates down the line of dominoes. Although each domino only moves a short distance, toppling over to hit its neighbor, the domino wave may travel a long way.

Since most of us have actually viewed a driver in motion, let's jump right in and analyze the physical behavior of the driver's cone as it pushes and pulls on the air. One of the most fundamental techniques in transducer theory is to use a circular, flat piston-head to model the behavior of the cone-shaped, and not necessarily circular, driver. Aside from the geometric simplifications, we can leverage on the work of the brilliant English physicist/ engineer Lord William Thomas Kelvin (1824 – 1907) who studied, among many other things, the behavior of vibrating piston heads. Additionally, we will find that drivers really do behave just like flat pistons under certain circumstances, and this piston behavior is actually highly desirable.

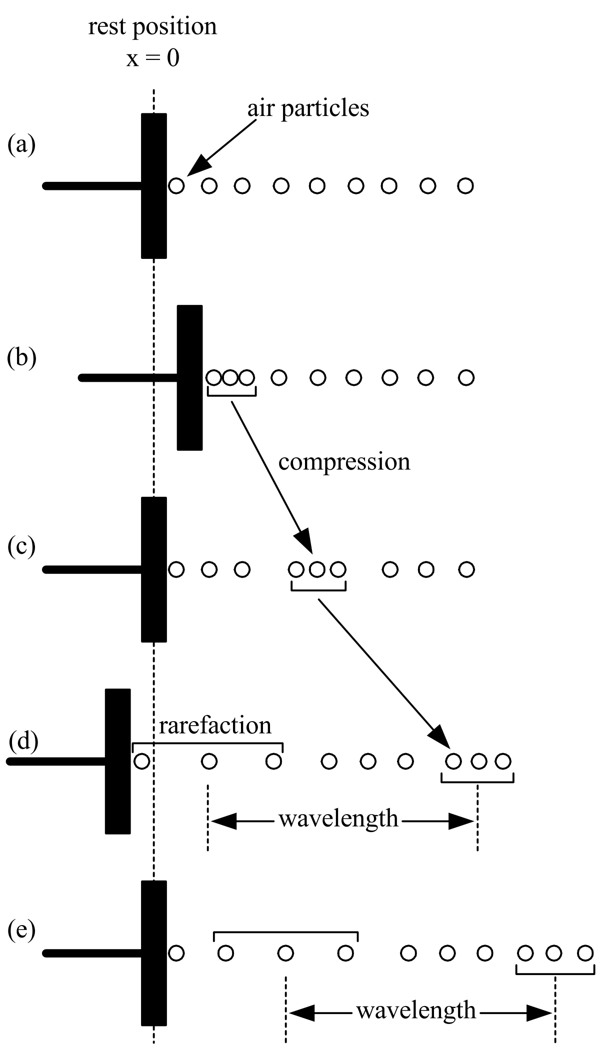


Figure 1.2: A breakdown of the motion of the piston head and air particles during sinusoidal oscillation. The piston is vibrating in the x dimension. In (a), the piston is at rest and the air particles are shown at rest (static air pressure position). For simplification, we will only observe a single line of air particles on the x-axis.

(b) As the piston pushes to the right, it compresses the adjacent air particles (simplified to three particles here). Note that the three particles are pressed tightly together.

(c) During the time it took for the piston to move back to rest position, the band of compressed particles moved to the right. Note that the *originally* compressed particles have essentially returned to their rest positions and the band of compression has moved to the *next* set of particles, via particle collision. Also, note that the new compressed particles are not pressed as tightly as before.

(d) The piston has moved backwards in the negative x-direction. This has expanded the distance between the first three adjacent air particles creating a band of rarefaction. At the same time, the band of compression has propagated further to the right, and the amount of compression has diminished again.

(e) The piston is back at rest position. Both the compression and rarefaction bands are not propagating to the right. The sound ***wave*** is now moving to the right. The distance from the center of one compression band to the center of the next compression band is called the **wavelength**. (NOTE: the diagram above has a typo; it should say "1/2 wavelength" instead). As the wave continues to propagate to the right, in the positive x direction, the wavelength remains the same, but the compression band becomes less compressed and the expansion band becomes less evacuated. As the wave propagates, it looses energy, so that at some distance far away from the piston, no particle vibration occurs – this is the point at which the sound wave dies.

NOTE: many of the transducer design equations require an "equivalent piston radius" in the calculation. Generally, the manufacturer specifies the diameter of the driver, usually in inches. A handy rule of thumb is that the piston radius in centimeters is approximately equal to the driver's diameter in inches, so a 12" driver would have a piston radius of 12cm (0.12 m). Additionally, the piston radius is denoted by the variable ***a*** in these calculations.

1.4 Driver Excursion and Air Particle Displacement

The "in and out" motion of a driver is called its **excursion**. Just as in sound level measurements, we can use peak, peak-to-peak, and RMS types of measurements for driver excursion. In this text, we will use the lower case italic ***x*** to represent driver excursion. One of the most fundamental driver specifications is called $x_{\max}$ and is the maximum distance away from the rest position that the driver can move. This is often called the *maximum excursion.* Note that this is inherently a peak measurement, since it only takes into account the peak excursion in one direction. The complete distance the driver can travel, both in and out, is actually double the value for $x_{\max}$.

The air particles along the wave front also move in and out (or back and forth) in a similar manner as the driver. We call this motion the **particle displacement** away from the particle's rest position. Again, we may choose peak, peak-to-peak, or RMS measurements. We will use the lower case Greek letter ξ to represent the local particle displacement. Closer examination of Figure 1.1 reveals that the air particle that is at a distance of x = 0 (i.e. the particle closest to the driver's surface) has essentially the same displacement as the driver's excursion. As we move further away from the driver, the air particle displacement diminishes. The physical chemistry of air determines how the particle displacement decreases with distance.

1.5 Wave Velocity

The global motion of a sound wave consists of bands of compression and rarefaction propagating moving outward and away from the source. The sound wave moves at a constant velocity, ***c***. The speed of sound in a gas can be calculated as

$$c = \sqrt{\frac{\gamma P_o}{\rho_o}} \qquad [1.5]$$

where

γ = specific heat ratio (equal to 1.4 for diatomic gasses)
P_o = static air pressure (1.013×10^5 Pa)
ρ_o = air density (1.1

The velocity of sound in air, c = 345 m/s (or about 1131 ft/sec)

You should go ahead and memorize the velocity of sound in air at room temperature:

1.6 Spherical vs. Plane Waves

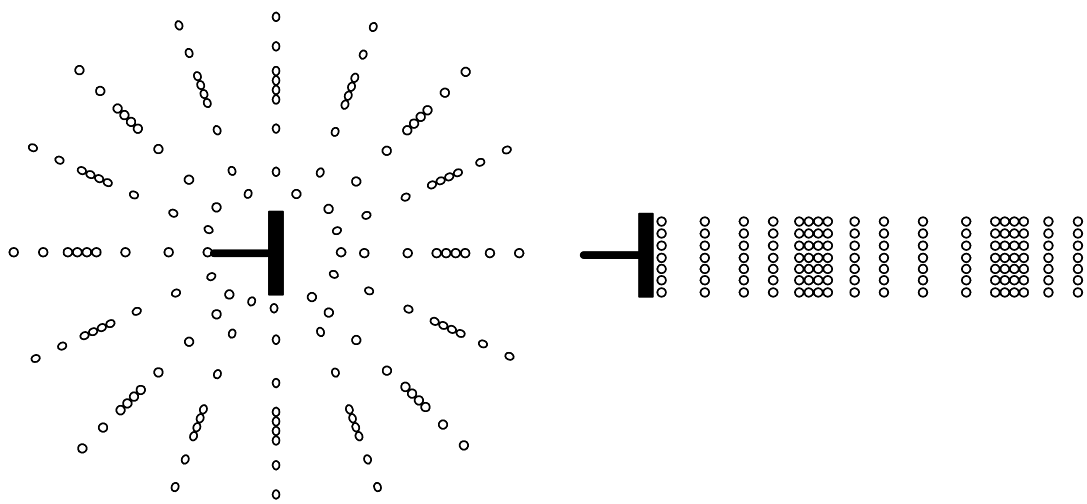


Figure 1.3: (a) spherical waves radiating outward from a piston; the bands of compression and rarefaction lie on spherical boundaries (note: only 2 dimensions shown). (b) Plane waves have bands aligned along planar surfaces (again, only 2 dimensions are shown)

There are two fundamental geometries of sound waves: **spherical waves** that radiate bands of compression and expansion omni-directionally and **plane waves** that radiate bands along a planar surface. All vibrating bodies attempt to radiate spherically. True plane waves can never be perfectly generated naturally.

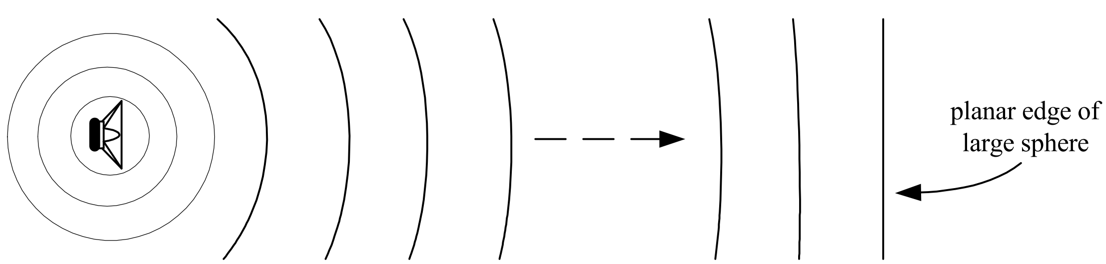


Figure 1.4: At a large distance from the source, the curvature of the spheres becomes so small that segments of the sphere appear to be planar.

However, plane waves can be set up as standing waves inside a tube.

Note that the curvature of the spherical waves becomes smaller as the sphere becomes larger, meaning that over a small segment of the sphere's surface, the geometry approaches being planar. If you are located sufficiently far away from a spherical source, you may experience these planar waves.
The distance (d_p) at which a spherical wave's curvature is small enough to create plane waves can be found as follows:

$$d_p = \frac{5c}{\pi f}$$
where
c = the velocity of sound [1.6]
f = frequency emitted by driver

1.7 Conditions for Omni-directional Radiation

There are two main conditions for omni-directional radiation. Lord Kelvin discovered that a piston radiates spherical waves if the wavelength of the emitted sound is greater than the circumference of the piston. Mathematically for omni-directional radiation,

$$\lambda \geq 2\pi a$$
λ = wavelength of frequency emitted [1.7]
a = piston-radius of driver

The frequency whose wavelength just satisfies [1.6] is called the **piston frequency**. Frequencies at or below the piston frequency will radiate omni-directionally from the piston-head surface. Equation [1.5] can be rewritten for the piston frequency as

$$f_{piston} = \frac{c}{2\pi a}$$
where [1.8]
c = the velocity of sound

For example, a 12" driver has a piston frequency of about 458 Hz. Below 458 Hz, the piston will radiate spherically. Above the piston frequency, the radiation pattern begins to deviate from spherical. We will cover this deviation in more detail in Section 1.8.

The second condition for omni-directional radiation involves the enclosure in which the driver is mounted. Bass frequencies are able to "bend" around obstacles. The physical size of the obstacle dictates the highest frequency of omni radiation. The larger the obstacle, the lower this frequency becomes. The enclosure that holds the driver acts as the obstacle. The rule of thumb is that for spherical radiation, the wavelength being emitted must be greater than the circumference of a circle calculated using the radius of the smallest sphere that just encloses the source. That is,

$$\lambda \geq 2\pi R$$
λ = wavelength of frequency emitted
R = radius of a sphere that completely encloses the source.
or
$$\mathrm{f} \leq \frac{c}{2\pi R}$$
f = frequency of sound emitted

Example 1.2

Suppose a 12" driver is mounted in a box measuring 0.5m x 0.75m x 2.0m. From trigonometry, we find that the radius of a sphere that encloses this box by

$$R = \frac{\sqrt{l^2 + w^2 + h^2}}{2} \qquad [1.9]$$

$l, w, h =$ length, width, height of enclosure

Thus R = 0.67m and the wavelength must be greater than 2 πR so the highest frequency of omni-directional radiation is about 82 Hz. Therefore, the loudspeaker enclosure places another limit on how close we can come to the ideal loudspeaker.

1.8 Volume Velocity

Ultimately, the sound pressure level that a driver can generate is related to how much air the driver can move. The volume of air a driver can displace over a given amount of time is called the **volume velocity** and is measured in m^3/sec. We use the uppercase **U** to denote volume velocity. A driver's volume velocity is dependent upon three factors:

1. the surface area of the driver (S_D)
2. the driver displacement (*x*)
3. the frequency of sound being emitted (f)

It is important to note that volume velocity is independent of distance away from the source. It is purely a function of the mechanical design and audio output frequency. Consider a 1 Hz tone applied to a driver. The driver will move back and forth once every second. Suppose the driver's surface area is 0.628 m^2 and the maximum peak excursion (x_{max}) of the driver is 1mm (0.001m). If the driver actually moves at its maximum excursion, what volume of air has been displaced during this one-second interval? It is the volume of air displaced during one waveform period times the number of cycles per second (frequency) of the audio signal:

$$\begin{aligned} U &= (S_D)(x)(f) \\ &= (0.628)(0.002)(1) \\ &= 0.012\, m^3/s \qquad [1.10] \\ & \textit{where} \\ x &= \text{the total excursion (not necessarily the maximum excursion)} \end{aligned}$$

The fundamental problem with this equation is in measuring the displacement, x. It can be very difficult to accurately measure this distance while the driver is operating. Fortunately, we can relate the SPL measured at some distance from the source to the volume velocity <u>and</u> driver excursion as follows:

For Pressure (dbSPL):

$$\begin{aligned} U &= \frac{4\pi r p}{\omega \rho_o} \\ & \textit{where} \\ U &= \text{volume velocity} \\ r &= \text{distance from source} \qquad [1.11] \\ p &= \text{pressure fluctuations at distance r} \\ \omega &= \text{audio frequency emitted by driver} \\ \rho_o &= \text{air density} = 1.18\ \text{kg/m}^3 \end{aligned}$$

$$p = \frac{U \omega \rho_o}{4\pi r}$$

For Excursion:

$$x = \frac{U}{\omega\pi a^2}$$

where

x = driver excursion [1.12]

U = volume velocity

ω = frequency emitted by driver = $2\pi f$

a = the equivalent piston radius

NOTE: Volume velocity, pressure, and excursion may be measured as peak, peak-to-peak, or RMS, but you must stick to one kind of measurement to produce the correct result.

Example 1.3

A driver has a piston radius of 10 cm (0.1m). Suppose a 100Hz sinusoid is applied to the driver. At a distance of one meter, we measure 92 dB_{SPL}. What is the driver's peak-to-peak volume velocity? What is the RMS excursion?

Solution:

At 92 dB_{SPL} the peak to peak pressure is:

$$p_{p\text{-}p} = 2\sqrt{2}\left(2x10^{-5}\right)\left(10^{92/20}\right) = 2.25 \text{ Pa}$$

therefore

$$U_{p-p} = \frac{4\pi(1)(2.25)}{2\pi(100)(1.18)} = 0.038 \text{ m}^3/\text{sec}$$

To calculate RMS excursion, we must first formulate the volume velocity as a RMS value:

$$U_{RMS} = \frac{U_{p-p}}{2\sqrt{2}} = 0.0134 \text{ m}^3/\text{sec}$$

therefore

$$x_{RMS} = \frac{0.0134}{(2\pi f)(\pi(0.10)^2)} = 0.006m = 6mm$$

Example 1.4

In Example 1.3, suppose we double the measurement distance from the source to 2 meters. What is the db_{SPL} at this distance?

Solution:

We already calculated the volume velocity to be U_{rms} = 0.013 m^3/sec. Using [1.4] we can solve backwards and find the pressure at a distance (r).

$$
\begin{aligned}
p_{rms} &= \frac{U_{rms}\omega\rho_o}{4\pi r} \\
&= \frac{(0.013)(2\pi(100))(1.18)}{4\pi(2)} \\
&0.3963Pa
\end{aligned}
$$

therefore

$$
\begin{aligned}
dB_{spl} &= 20\log\frac{0.3963}{2x10^{-5}} \\
&= 86dB
\end{aligned}
$$

This result shows that by doubling the distance from our spherical source, the dB_{SPL} drops by 6dB.

The Particle Displacement ξ is found by:

$$\xi = \frac{p}{\omega\rho_O c}$$

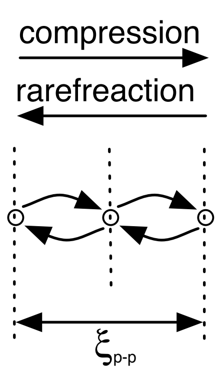
or

$$\xi_p = \frac{p_p}{\omega\rho_O c}$$

$$\xi_{p-p} = \frac{p_{p-p}}{\omega\rho_O c}$$

$$\xi_{RMS} = \frac{p_{RMS}}{\omega\rho_O c}$$

As the wavefront passes by the particle, it is displaced first forward, then backwards like this:



At the surface of the driver's cone, the particle displacement is the same as the driver excursion. As the wave propagates, it looses energy and the particle displacement falls off as the pressure drops.

1.9 Acoustic Intensity

The **acoustic intensity** of a sound wave at a distance (x) away from a source is defined as the amount of acoustic power flow per unit area of a sphere of radius = x. The acoustic power is spread over the surface of the sphere. The driver emits the spherical wave with a certain, finite amount of power. As the sphere expands, this power is spread ever more thinly over its surface. As you increase your distance away from the driver, the sound volume decreases, getting softer and softer. The acoustic intensity tells us how "thinly" the power is spread over the sphere. The average acoustic intensity (I_{AVE}) is found with:

$$I_{AVE} = \frac{p_{rms}^{\ 2}}{\rho_o c} \qquad [1.13]$$

where

p_{rms} = the rms pressure measured at some distance from the source

ρ_o = the density of air

Example 1.5

Calculate the average acoustic intensity in W/m^2 of the spherical wave at a distance of 1 meter in Example 1.3.

Answer:

The SPL was measured at 92 dB, therefore

$$p_{rms} = \left(2x10^{-5}\right)\left(10^{92/20}\right) = 0.796Pa$$

$$I_{AVE} = \frac{(0.796)^2}{(1.18)(345)} \approx 0.001556W/m^2$$

Example 1.6

Calculate the average acoustic intensity in W/m^2 of the spherical wave at a distance of 2 meters in Example 1.4.

The RMS pressure was found to be 0.3963 Pa. Therefore,

$$I_{AVE} = \frac{(0.3963)^2}{(1.18)(345)} \approx 0.000389W/m^2$$

We see that as the distance from the source doubles, the acoustic intensity drops by a factor of four. This result is often generalized as the **Inverse Square Law**, which includes the result we found in Example 1.3.

The **Inverse Square Law** states that the acoustic intensity of a spherical wave varies by the distance squared. A commonly used rule of thumb is that as the distance away from the source is doubled, the db_{SPL} drops by 6dB.

1.10 Acoustic Power

The **acoustic power** of a sound wave is found by multiplying the acoustic intensity by the surface area of the radiated sphere. The area of radiation will be either the area of a sphere or hemisphere whose size depends upon the distance from the source. If you are 3 meters away from an omni-directional source, you are standing at the edge of a sphere with a radius of 3 meters.

For Spherical Radiation :

$$P_{AR} = \frac{{U_{rms}}^2 \omega^2 \rho_o}{4\pi c} \qquad [1.14]$$

When a loudspeaker enclosure is mounted flush on a large surface (e.g. mounted in a wall), almost all the power will be radiated on the front of the loudspeaker. Sound radiation cannot be spherical since the wall prevents it. In this case, radiation will be hemispherical. The emitted power will be spread over the surface of a hemisphere, effectively doubling the power.

For Hemispherical Radiation :

$$P_{AR} = \frac{{U_{rms}}^2 \omega^2 \rho_o}{2\pi c} \qquad [1.15]$$

These can also be arranged as:

$$P_{AR} = U_{RMS}^2 R_{AR}$$

$$R_{AR} = \frac{\omega^2 \rho_o}{4\pi c} \quad \text{spherical}$$

$$R_{AR} = \frac{\omega^2 \rho_o}{2\pi c} \quad \text{hemispherical}$$

You should note that just as in the case of volume velocity, the acoustic power radiated into the spherical wave is independent of the distance away from the source. Similarly, you can think of the spherical wave being emitted from the driver with a fixed acoustic power. As you move away from the source, the total acoustic power does not change. However, the acoustic intensity does change, since the pressure fluctuations decrease as the distance increases. In general, when we describe the "loudness" of a sound, we are describing its acoustic intensity, not acoustic power.

Example 1.7

We would like to design a 10" driver to output one acoustic watt of power at across the entire audio spectrum of 20Hz – 20kHz, measured at a distance of 1 meter from the source. Assuming that radiation is spherical for all frequencies, this would mimic an ideal point source. What is the required peak-to-peak driver excursion at the two extreme frequencies, 20 Hz and 20 kHz? What is the particle displacement at 1 meter at the two extreme frequencies, 20 Hz and 20 kHz?

Answer:
Rearranging [1.14] yields the required RMS volume velocity.

$$U_{rms} = \sqrt{\frac{4\pi c P_{AR}}{\omega^2 \rho_o}}$$

$$U_{rms}\Big|_{20Hz} = \sqrt{\frac{4\pi(345)(1)}{(2\pi(20))^2(1.18)}} = 0.482\, m^3/\sec$$

$$U_{rms}\Big|_{20{,}000Hz} = \sqrt{\frac{4\pi(345)(1)}{(2\pi(20{,}000))^2(1.18)}} = 2.33x10^{-7}\, m^3/\sec$$

The peak-to-peak driver excursion is found with [1.5] (remembering to convert the volume velocity into peak-to-peak form):

$$x_{p-p} = \frac{U_{p-p}}{\omega\pi a^2}$$

$$x_{p-p}\Big|_{20Hz} = \frac{2\sqrt{2}(0.482)}{2\pi(20)\pi(1)^2} = 0.003m = 3mm$$

$$x_{p-p}\Big|_{20{,}000Hz} = \frac{2\sqrt{2}(2.33x10^{-7})}{2\pi(20{,}000)\pi(1)^2} = 1.7x10^{-12}\, m$$

$$p_{p-p} = \frac{U_{p-p}\omega\rho_o}{4\pi r}$$

$$p_{p-p}\Big|_{20Hz} = \frac{2\sqrt{2}(0.482)(2\pi)(20)(1.18)}{4\pi(1)} = 16.08Pa$$

$$p_{p-p}\Big|_{20{,}000Hz} = \frac{2\sqrt{2}(2.33x10^{-7})(2\pi)(20{,}000)(1.18)}{4\pi(1)} = 0.0078Pa$$

$$\xi_{p-p} = \frac{p_{p-p}}{\omega\rho_o c}$$

$$\xi_{p-p}\Big|_{20Hz} = \frac{16.08}{(2\pi)(20)(1.18)(345)} = 0.00031m = 0.31mm$$

$$\xi_{p-p}\Big|_{20kHz} = \frac{0.0078}{(2\pi)(20{,}000)(1.18)(345)} = 1.52x10^{-10} = 0.000000152mm$$

The 20 Hz excursion value is $1.9x10^9$ the 20 kHz value! It is very difficult to design a mechanical system to be sensitive enough to these specifications. At the upper frequency limit, the driver must move back and forth accurately across 1.7 pico-meters, 20,000 times per second. At the lower limit, the same material must (accurately) move $1.9x10^9$ times this distance. This, coupled with the fact that we desire spherical radiation leads to the design choice of multiple drivers mounted in a single enclosure. Typically, two or three drivers of various sizes are mounted in a box, creating *two-way* or *three-way* systems. The larger driver handles the lower frequencies with big excursions. The more rigid, higher mass *woofers* are able to move through the larger excursion range while pushing air without deforming. The more flexible lower mass *tweeters* are able to vibrate many more times per second accurately over small distances while retaining their shapes. Another option is the electrostatic loudspeaker that we will look at shortly.

Example 1.8

Suppose a loudspeaker outputs 0.75 W into a spherical wave. We cut a hole in a wall and mount the loudspeaker flush with the wall. What acoustic power is now radiated?

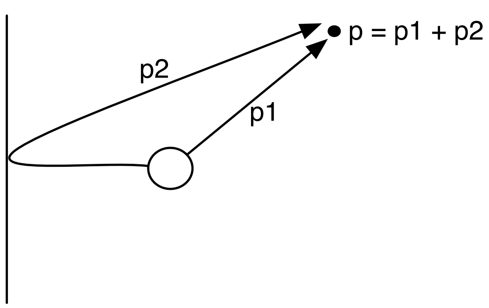
Solution:

Since we've mounted the enclosure in a wall, it will only be able to radiate to the front side in a hemispherical manner. Examination of [1.10] and [1.11] shows that the acoustic power is doubled when radiating into a hemispherical area, so the resulting output power will be 1.5 W.

At first glance, this result is astounding – mount the enclosure in a wall and double your acoustic power output! In fact, if you mount the enclosure at the intersection of 2 walls, the radiation will be into one quarter of a sphere, and the power will double again. Mount the enclosure at the intersection of 3 walls, and the radiation is into one eighth of a sphere and the power doubles yet again. It appears that we are multiplying our power by decreasing the size of the radiation area. The caveat is that this only works for spherical sources. We saw in example 1.2 that a 12" woofer mounted in a realistically sized enclosure only radiates spherically at and below 82 Hz. By mounting our loudspeaker flush with the wall, we only double the acoustic power for 82 Hz and below. While more bass may be what some listeners prefer, in many cases the result will be muddy and less intelligible bass. On the other hand, for a loudspeaker with poor bass response, this technique could help boost the bass a somewhat.

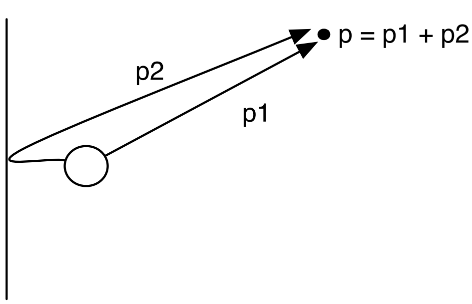
1.11 Acoustic Reflections

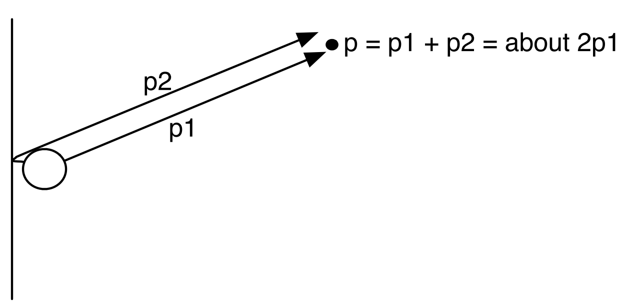
The power doubling you just saw when you mount the source flush with a wall can also be shown by using the Acoustic Reflection property of a wave. Acoustic waves reflect the same way as light waves, where the angle of incidence equals the angle of reflection. First consider the case where a source is located some distance from a large structure like a wall. Note that it must be radiating omni-directionally too.



In this case, the pressure p is the sum of the two components p1 (direct) and p2 (reflected).

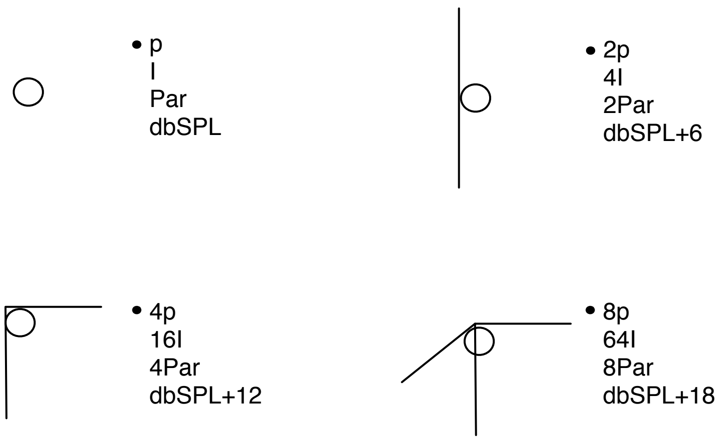
Now consider moving the source closer and closer to the wall until it is almost flush with it:





As the source gets closer the wall, the two paths (direct and reflected) become basically equal. With the source mounted flush to the surface of the wall, they become identical and the pressure doubles at point p. This doubling in pressure gives the same doubling of acoustic power that we saw previously by radiating into a half-sphere. This would also yield a +6dB increase in dB_{SPL} and a quadrupling of the acoustic power.

What happens if we place the source flush with the corner of 2 walls? How about 3 walls? The answer is that every new wall doubles the pressure again. So, for the 4 cases (free-air, then with boundaries) we can summarize as follows (adapted from Leach):



Four possible cases for acoustic reflections: note the doubling of power and the quadrupling of intensity with each boundary addition.

But, because the driver must radiate omni-directionally, these power and intensity boosts are generally going to be low frequency phenomena.

Frequency cancellation/ reinforcement from acoustic reflections

If a source is not mounted flush with a wall (or floor or ceiling) the reflected sound may arrive out of phase with the direct sound causing cancellation. The frequency of cancellation occurs at a wavelength that is half of the difference between the direct path and the reflected path. The reinforced frequencies will occur at twice this value.

The frequency that will experience cancellation occurs when the reflected path produces a time delay that puts the frequency 180 degrees out of phase with the direct path signal. Remembering that the angle of incidence equals the angle of reflection, you have to use a bit of geometry to find it. Considering just one surface, the frequency of cancellation is found with (Ballou):

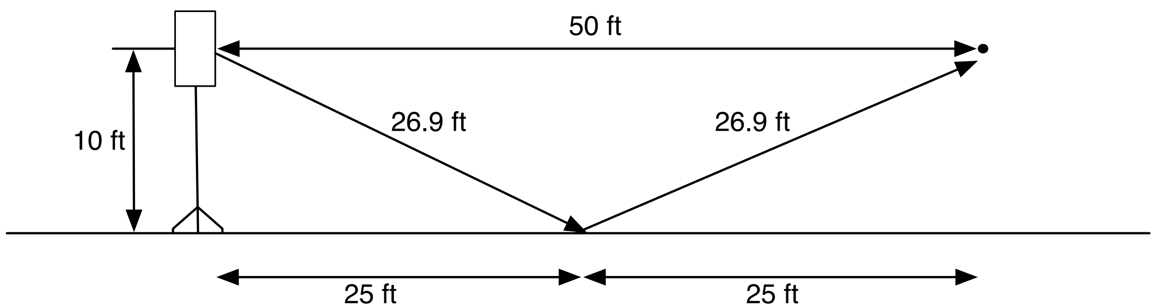
$$f_c = 0.5\frac{c}{d_{r1} + d_{r2} - d_{dir}}$$

d_{dir} = direct path

d_{r1} = reflection path from source to surface [1.17]

d_{r2} = reflection path from surface to listening point

For example, consider a loudspeaker mounted on a pole 10 feet tall with a listener 50 feet away and exactly on axis with the loudspeaker (10 feet from the ground). In this ***particular*** arrangement, the angle that produces the correct reflection creates an equilateral triangle so the distances are simple to calculate.



The total reflected path is 26.9 + 26.9 feet = 53.8 feet. Using [1.15] the frequency of cancellation is 148.6 Hz.

One simple fix is to move the speaker down to the ground. The closer it is to the ground, the higher the first frequency of cancellation will occur, assuming that frequency could actually be radiated omni-directionally from the source. For large subwoofers with large drivers, the highest frequency of omni-directional radiation is going to be very low.

1.12 Driver Directivity - Far Field

In Section 1.4.1 we observed that a piston-head would radiate omni directionally at and below the piston frequency, f_{piston} – the frequency whose wavelength equals the circumference of the piston. But what happens at frequencies above the f_{piston}? **Directivity plots** are used to describe the geometric shape of acoustic radiation. These directivity patterns are 2-dimensional geometric representations of wave propagation. Generally, the plots are made using polar coordinates such that 0º represents the position directly in front of the driver. This position is called "on-axis" or "incident" to the driver. The most widespread directivity plots are for a piston mounted in an infinite baffle. The ideal piston radiates hemi spherically on each side of the baffle (spherical when both halves are combined). For clarity, only one side of radiation is usually shown.

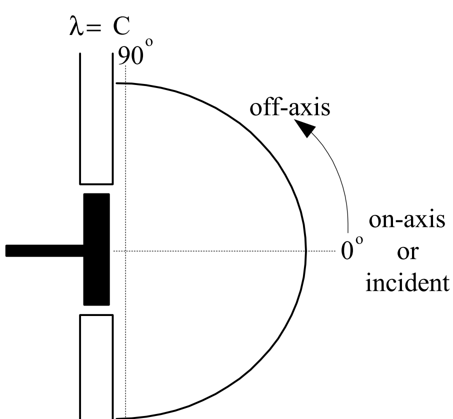


Figure 13.5

Figure 1.5 shows the directivity plot for an infinite baffle-mounted piston radiating at f_{piston}. As you move around the driver, deviating from the incident position, you are said to be "off-axis" from the driver. A listener standing anywhere within the hemisphere will hear the radiated sound. The boundary of the hemisphere marks a normalized contour of equal loudness. If you walked around the driver, staying on the contour, the apparent loudness would not change.

Near Field vs. Far Field:

$$dist_{FarField} \geq \frac{8a^2}{\lambda}$$

$$dist_{NearField} < \frac{8a^2}{\lambda} \qquad [1.18]$$

$a =$ piston radius

$\lambda =$ wavelength of emitted sound

These directivity patterns are considered to be valid at the far-field distances. The equation that relates the pressure to the on/off-axis angle is (M. Leach):

$$|p| \propto \left| \frac{2J_1(ka\sin\Theta)}{ka\sin\Theta} \right| \qquad [1.19]$$

$p =$ pressure

$J_1(ka\sin\Theta) =$ a Bessel Function

$ka\sin\Theta = \frac{2\pi a}{\lambda}\sin\Theta$ <- you can see the circumference vs. wavelength here

Figure 1.6 below is adapted from Leo Beranek's excellent text *Acoustics*, first published in 1954. It shows a plot of this equation, producing the directivity patterns for a piston mounted in an infinite baffle, radiating at six different frequencies, starting at the piston frequency, and increasing until the wavelength is 1/10th the circumference of the driver.

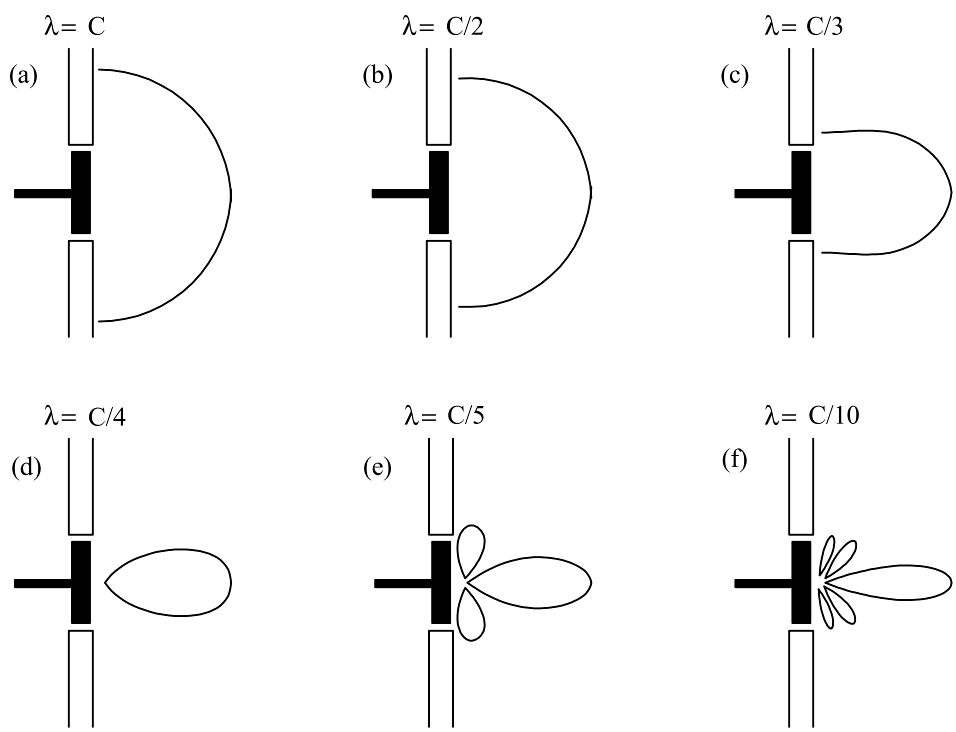
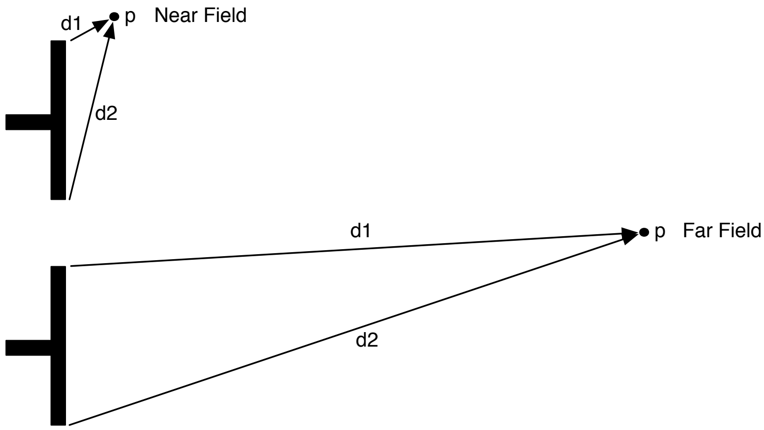


Figure 1.6: Six directivity plots adapted from Beranek, *op. cit*. p.102. In (a) the wavelength equals the circumference (f_{piston}) and we clearly see the perfect hemisphere shape emanating from the piston. In (b), the frequency has been increased such that the wavelength is now half the piston circumference. The hemispherical shape begins to shrink at the extremes. As the frequency increases to λ=C/3 (c) and λ=C/4 (d) the main lobe continues to pinch off at the base. In (d), the patterns is said to begin "beaming" (like headlight beams) as the base pinches completely off. As the frequency in further increased, "lobes" are seen forming, as the main lobe continues to shrink in girth. A person standing about 45^o off-axis in (e) would be in a dead spot, and would not hear anything at all!

Figure 1.6 graphically demonstrates the problem with trying to use a single driver to cover a broad range of frequencies. If the frequency continues to increase, the main lobe will continue to narrow, and multiple side lobes will form. Generally speaking, beaming and lobing are undesirable effects of piston radiation. Consider a large concert setting such as a stadium or arena. Because the audience is spread across a wide angle in front of the stage and is mostly in the far-field, chances are that some locations will be in the null spots in the directivity patterns. In order to get adequate coverage of a wide band of frequencies, multiple loudspeakers are often set up in arrays, fanning out and pointing in many different directions.

1.13 Driver Frequency Response - Near Field

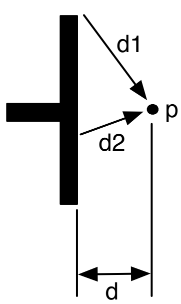
In the near-field, the Fresnel Diffraction Effect dominates the response. The diffraction is a combination of peaks and nulls in the frequency response caused by phase addition and cancellation. The peaks and nulls are called Maxima and Minima in the response. These peaks and nulls disappear outside the near-field.



Consider the two cases above where a random listening position p is shown. The surface of the piston is vibrating at some frequency. In the Near Field case, the distances d1 and d2 are very different so phase cancellation or addition will occur depending on the distances and the frequency. However, in the Far Field case, the distances d1 and d2 are essentially the same so the diffraction effect does not occur.

The **on-axis** Maxima and Minima distances (d) are given by (Leach):

On-Axis Fresnel Diffraction



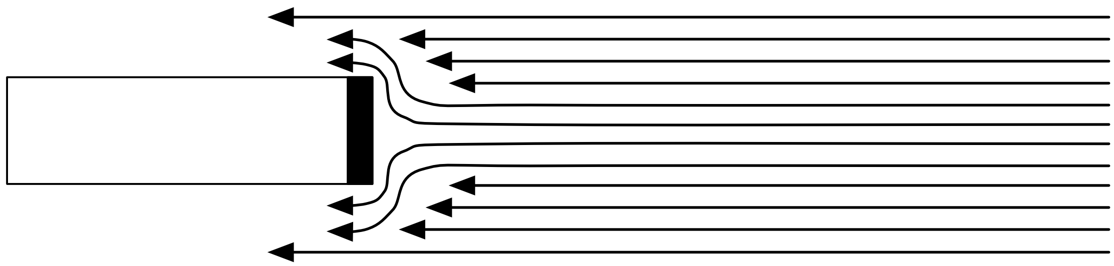
$$\text{Maxima:} \quad d = \frac{a^2 - \left(n\lambda/2\right)^2}{n\lambda} \qquad n = 1,3,5,7,... \qquad [1.20]$$

$$\text{Minima:} \quad d = \frac{a^2 - \left(n\lambda/2\right)^2}{2n\lambda} \qquad n = 2,4,6,8,...$$

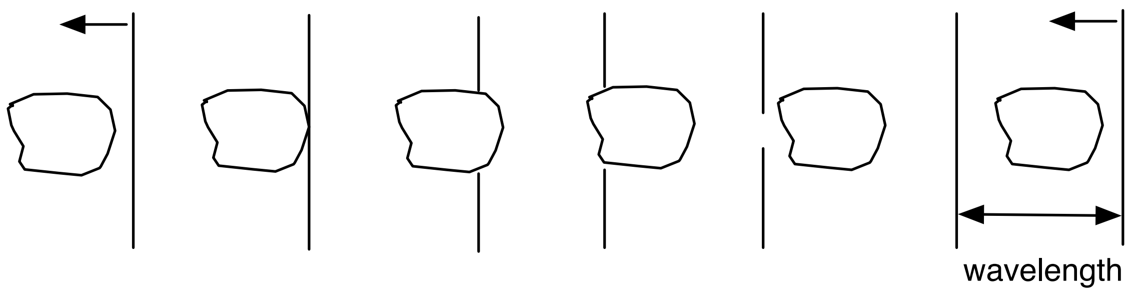
1.14 Microphone HF Response

The microphone's directivity pattern is determined by the enclosure it's mounted in, which we'll get to later. The loudspeaker's directivity pattern is a property of the driver (based on its circumference). Microphones also have a built-in consequence but it affects the frequency response more than directivity. Consider a microphone placed in a plane-wave field completely on-axis to the plane wave:

Low frequency waves strike the microphone's diaphragm and then "bend" around it.

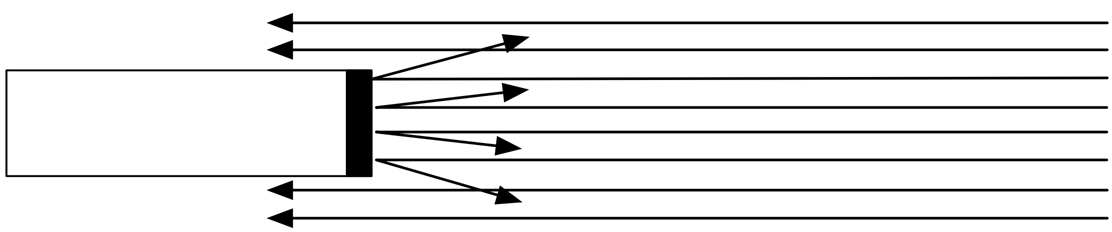


This is analogous to a large low frequency ocean wave hitting a rock (think of a large rolling swell-wave):

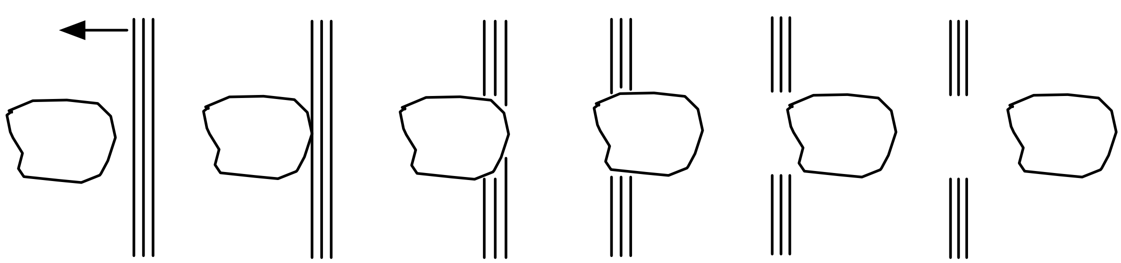


In this case, the rock breaks the wave, but it re-forms on the other side; it you've seen this, you've seen the water pour around the sides of the rock; that's the bending. The bending happens when the rock is small compared to the wave, specifically the wavelength (λ).

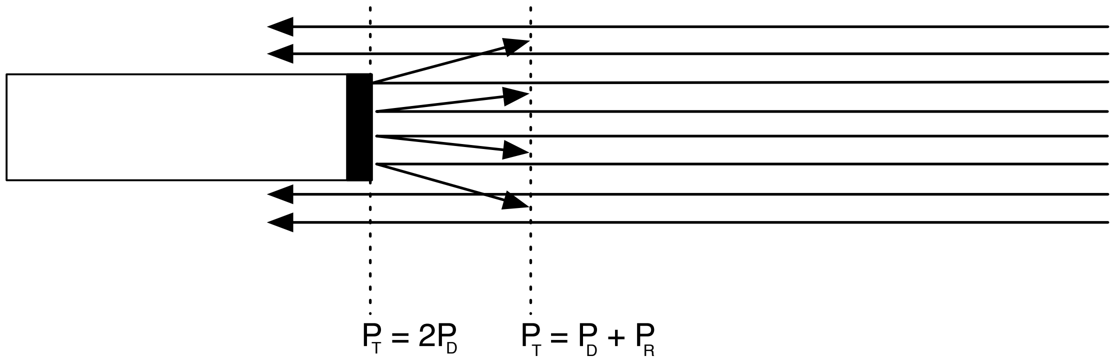
High Frequencies strike the microphone capsule and then reflect back off of it like this:



The same thing happens in the ocean waves - high frequency surface chop is stopped by the rock and a shadow area forms behind it:

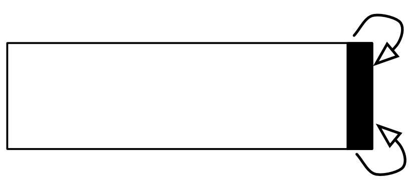


Here, the rock splits the waves and the shadow area forms behind it. In this case, the rock appears as a giant obstacle for the wave because its size is much larger than the wavelength of the surface chop. In the case of the microphone, the diaphragm looks like a large obstacle compared to the wavelength of the front.



What can we observe about the reflected wave? At some distance from the capsule, the total pressure is a combination of the direct + reflected wave. The closer you are to the capsule, the stronger the reflected component.

Right at the surface of the capsule, the reflected component has the same magnitude as the direct component, so the pressure is doubled. This effect is gradual, increasing with frequency until the wavelength is small enough for the doubling to occur. Just as before, this doubled pressure should result in a +6dB increase in output for the microphones; indeed all microphones have this built-in high frequency boost mechanism. But there's more: if the microphone's capsule has sharp corners (as shown in this example and in most microphones) there is a secondary reflected component from the perimeter.



The corner reflections are difficult to draw in a symbolic way but they do contribute to the total pressure at the surface of the capsule. At most, they add an additional +4dB for a total maximum of +10dB.

This maxima occurs at the frequency whose wavelength equals the ***diameter*** of the diaphragm:

$$D/\lambda = 1.0 \qquad [1.21]$$

Interestingly, instead of a continuous +10dB increase for frequencies above this, the response begins to decrease reaching a null point back at 0dB at the frequency whose wavelength equals the ***1/2 the diameter*** of the diaphragm:

$$D/\lambda = 2.0 \qquad [1.22]$$

As we increase the frequency further, the pressure increase forms again so that we get a set of maxima and minima at even and odd multiples of the first equation:

Maxima: $D/\lambda = 1,3,5,7...$

Minima: $D/\lambda = 2,4,6,8...$ [1.23]

These results are published in the Bruel & Kjaer Technical Manual No.1, 1959 and it includes the following diagram showing the phenomenon. The x axis is the normalized D/λ ratio.

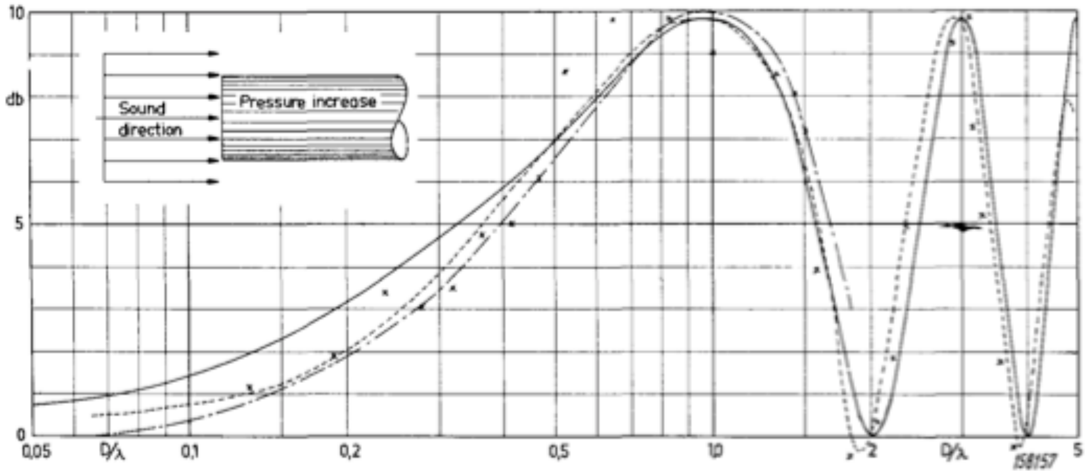


Fig. 3. Pressure increase at the axis of a cylinder the cylinder being placed in a free sound field. Curve drawn in full shows the theoretically calculated curve.

xxxxx Measured by Müller, Black, and Davis (1937)
—·—·— Measured by Danish Technical University (1948)
- - - - - - Measured by Brüel & Kjær (1958)

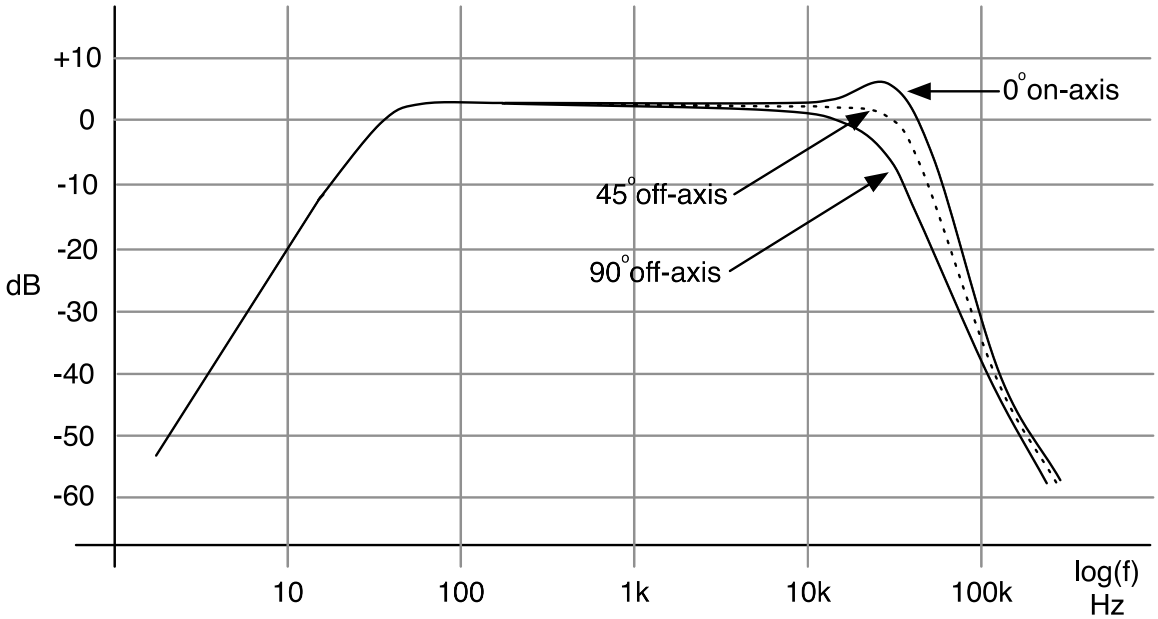
You can see that the +10dB peak is reached at the ratio of 1.0 and +6dB at a ratio of 0.4 so it is easy to calculate the responses of various diameter microphones. Here are a few:

Diaphragm	+3dB	+6dB	+10dB
1/2”	5.4kHz	10.8kHz	27.1kHz
3/4”	3.6kHz	7.2kHz	18.1kHz
1.0”			

As an exercise, fill in the row for the 1 inch diaphragm!

If you look at many frequency response plots for microphones you will often see a small hump in the high frequency portion, but not the kind of rise (starting at 3.6kHz for the 3/4” diaphragm) from the table. This is because many of the manufacturers are compensating for the increase with filtering (acoustically or electronically) to achieve the desired, flatter response. On the other hand, its good to note that microphones are generally already good at picking up high frequencies and that the larger the diaphragm, the more significant the potential effect.

Perhaps the most important ramification is in the way the frequency response will change with the orientation of the microphone: at 90 degrees off-axis, the waves are perpendicular to the diaphragm. So, there will be a reduction of high frequencies as you move off axis.


+10
0
-10
-20
-30
-40
-50
-60
dB
10
100
1k
10k
100k
log(f)
Hz
0° on-axis
45° off-axis
90° off-axis

5. A source of sound is located at the center of a hypothetical sphere of radius $r = 2\,\text{m}$. The source radiates an SPL of 86 dB over the surface of the sphere. *(a)* Solve for the acoustic intensity on the sphere. [$3.91 \times 10^{-4}\,\text{W/m}^2$]. *(b)* Solve for the total power radiated by the source. [19.7 mW]

6. Calculate the wavelength of a sound wave in meters and in feet at the frequencies 20 Hz, 200 Hz, 2 kHz, and 20 kHz. [at 20 Hz, $\lambda = 17.25\,\text{m} = 56.55\,\text{ft}$]

7. ~~Calculate the frequency of a sinusoidal sound wave in air at 22 °C which has the wavelength $\lambda = 1\,\text{m}$, $\lambda = 1\,\text{ft}$, and $\lambda = 1\,\text{in}$. [343.32 Hz, 1126.4 Hz, 13517 Hz]~~

8. The radiation from a loudspeaker driver in an infinite baffle is omnidirectional if the circumference of the diaphragm is less than the wavelength. *(a)* A woofer diaphragm has a piston radius of 12 cm. Calculate the highest frequency that it radiates an omni-directional wave. [458 Hz]. *(b)* Repeat the calculation for a midrange driver with a 4 cm piston radius. [1.37 kHz]

9. What is the rms, peak, and peak-to-peak particle displacement in a 100 dB 20 Hz tone? [0.039 mm, 0.055 mm, 0.11 mm]

15. The external measurements of a loudspeaker cabinet are 19 in by 12 in by 7 in. What is the highest frequency that the loudspeaker radiates as a simple source? [184 Hz] spherical

16. A circular piston of radius 6 cm vibrates sinusoidally in one wall of a sealed enclosure with a peak-to-peak displacement of 8 mm. Calculate the rms volume velocity it emits if the frequency is *(a)* 100 Hz [$0.0201\,\text{m}^3/\text{s}$], *(b)* 1000 Hz [$0.201\,\text{m}^3/\text{s}$], and *(c)* 10,000 Hz [$2.01\,\text{m}^3/\text{s}$]. ONLY

17. A midrange driver has a circular diaphragm with an effective piston diameter of 3.5 in. Calculate the required peak-to-peak diaphragm displacement if the loudspeaker is required to radiate 80 mW of acoustic power at 250 Hz into a 2π-steradian load. [2.24 mm]

18. A circular piston of radius 6 cm vibrates sinusoidally with a peak-to-peak displacement of 0.8 cm. Calculate the power it radiates into a 2π-steradian load at 100 Hz. [86.8 mW]

21. The SPL at 8 m from a 200 Hz source operated against a rigid, flat wall is 110 dB. The source radiates a simple spherical wave. *(a)* Calculate the rms volume velocity emitted by the source. [$0.214\,\text{m}^3/\text{s}$] *(b)* If the source is a piston of radius 12 cm, what must be its peak-to-peak displacement? [1.07 cm] *(c)* What is the total power radiated? [39.52 W]

22. A loudspeaker radiates a simple spherical wave into 4π steradians. The SPL is 70 dB at a distance of 100 ft from the loudspeaker. *(a)* Calculate the total power radiated. [0.1147 W] *(b)* Calculate the sound pressure level at a distance of 200 ft. [64 dB]

24. *(a)* Solve for the distance to the far field for a 12 in woofer at 100 Hz if the diaphragm can be modeled as a flat circular piston in an infinite baffle having a radius of 12 cm. [0.033 m] *(b)* Solve for the distance to the far field for a 1-inch tweeter at 10 kHz if the diaphragm can be modeled as a flat circular piston in an infinite baffle having a radius of 1 cm. [0.023 m]

CH1 HW

2 Modeling Acoustic Systems with Electrical Systems

	Components:	Variables
Acoustic Systems:	- boxes/enclosures/cavities	- pressure
	- tubes	- volume velocity
	- screen meshes/holes	- radiation impedance (resistance)
	- vents	
Electric Systems:	- resistors	- voltage
	- capacitors	- current
	- inductors	- impedance (resistance)
	- voltage sources	
	- current sources	

Volume Velocity

Notice that the equation for Acoustical Power Radiation is very similar to electrical power:

$$P_{AR} = U_{RMS}^2 R_{AR}$$
$$P_E = I^2 R$$

Could we equate Volume Velocity with Current? Current and Volume Velocity are defined as:

$$U = \frac{\text{volume flow}}{\text{time}}$$
$$I = \frac{\text{charge flow}}{\text{time}}$$

So there is an automatic connection; if charge were analogous to volume, this would work so let's go with it. So far we have:

I = volume velocity

Pressure

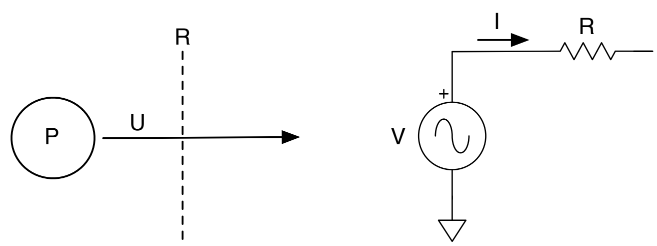
Acoustic Pressure is high when many air particles are crammed together and low when the particles are spread apart. In Electronics, Voltage is the amount of charge separation - the more electrons you have separated from their electron-deficient neighbors the higher the voltage. If we make air particle compaction analogous to charge separation, then we can model Pressure as Voltage. Now we have:

V = pressure
I = volume velocity

Resistance

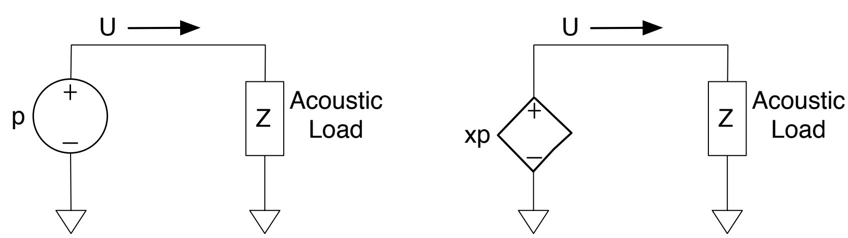
Air resistance is a non-frequency dependent opposition to air particle flow. Examples include fine mesh screens, cloths, and plates with very tiny holes drilled in them. Because the opposition is not frequency dependent, we can directly model the air resistance with an electrical resistor. But will this satisfy ohms law?

Consider a high air pressure on one side of a screen mesh. Air particles always want to move from an area of higher pressure to an area of lower pressure. So, there would be a flow of a volume of air through the screen which would resist it and limit the volume flow. This is identical to an electrical circuit with a potential (voltage) connected to a resistor. Current flows and the resistor limits the charge flow.

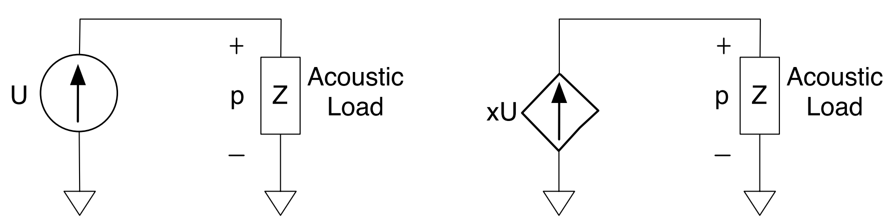


Sources

A pressure source becomes a voltage source, and there are 2 types: dependent and independent. The dependent version has an extra variable that links its instantaneous value to some other parameter - it is a source that is being controlled by something else.



Likewise, a volume velocity source becomes a current source that produces a pressure differential across the acoustic load impedance:



If the acoustic load was purely resistive, like the screen mesh or plate with tiny holes, the impedance block could be replaced with a resistor component.

Ohm's Law

Ohm's Law then becomes:

Electric :

$V = IZ$

Acoustic

$p = UZ$

Enclosures/Boxes/Cavities

An enclosed box of air is relatively uninteresting until you try to compress or expand the air within it. Consider a loudspeaker mounted in a sealed box. When the driver pushes backwards, it compresses the air in the box and a resistance builds up. When the driver pulls out, it forms a vacuum and an opposing resistance builds up. We call this a pneumatic air-spring. And it is immediately clear that the resistance can swap polarities - it is an AC resistance or impedance that is happening.

The force that the air spring generates is:

$f = kx$

where k is the *spring constant*. The value of k tells you how stiff or loose the spring is - high values indicate high stiffness. In acoustics, they use another variable called *compliance* which is exactly the opposite - high compliance = a loose spring. Mechanical Compliance is:

$$C_M = \frac{1}{k}$$

With a bit of calculus:

$$f = kx = \frac{1}{C_M}x$$

$$f = \frac{1}{C_M}\int u\,dt$$

$u =$ velocity

We also know that:

$$p = \frac{f}{S}$$

$p =$ pressure

$S =$ surface area

and

$$u = \frac{U}{S}$$

or

$$u = \frac{1}{S}U$$

Remember that dividing the Volume Velocity (m³/s) by the surface area (m²) gives you the velocity (m/s). Now we let force become pressure and convert velocity to volume velocity and mechanical compliance to acoustical compliance:

$$p = \frac{f}{S} = \frac{1}{S}\frac{1}{C_M}\int\frac{1}{S}Udt$$

$$= \frac{1}{S^2 C_M}\int U\,dt$$

If we assume the volume velocity is fluctuating sinusoidally, then we can write it like this:

$$U = U_0 e^{j\omega t}$$

then

$$\int U_0 e^{j\omega t}\,dt = \frac{1}{j\omega}U_0 e^{j\omega t}$$

so

$$\int U dt = \frac{1}{j\omega}U$$

then:

$$p = \frac{1}{S^2 C_M}\frac{1}{j\omega}U$$

let

$$C_A = S^2 C_M$$

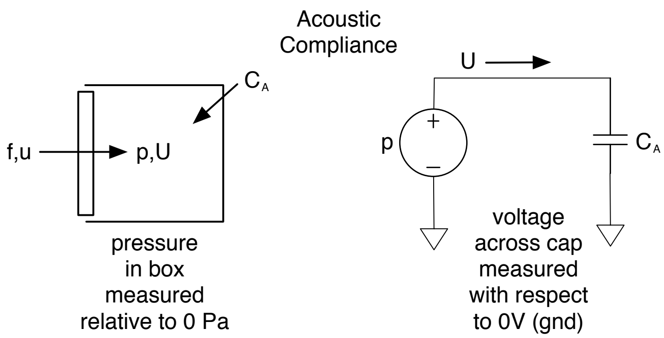
then

$$p = \frac{1}{j\omega C_A}U$$

Since the acoustic Ohm's Law is p = UZ, then we are looking for an electronic component that has an impedance that is inversely proportional to frequency = a **capacitor**.

$$Z_A = \frac{p}{U}$$

$$Z_{CA} = \frac{1}{j\omega C_A}$$



Whenever you model an acoustic compliance with a capacitor, <u>one side must always be connected to ground!</u>

The value of the cap depends on the volume of air in the enclosure:

$$C_A = \frac{V}{\rho_o c^2}$$

Tubes/Ports/Vents

An Acoustic Mass is a volume of air that can be accelerated without being compressed. The air in our box gets compressed so it has no acoustic mass. Air inside of a tube that is open at both ends can be moved without being compressed. There is an acoustic mass of air in a tube like this. If the tube has a length l and a cross sectional area S, then the volume in the tube is Sl. The mechanical mass of air is the density (mass/volume) times volume or

$$M_M = \rho_o Sl$$

If the air is moved with a velocity u, then the force is

$$\begin{aligned} f &= mA \\ &= M_m A \\ &= M_M \frac{du}{dt} \end{aligned}$$

since

$$p = \frac{f}{S}$$

then

$$f = M_M \frac{du}{dt}$$
$$p = \frac{1}{S} M_M \frac{du}{dt}$$

and

$$u = \frac{1}{S} U$$

so we can write

$$\begin{aligned} p &= \frac{1}{S} M_M \frac{du}{dt} \\ &= \frac{1}{S} M_M \frac{d(\frac{1}{S}U)}{dt} \\ &= \frac{1}{S^2} M_M \frac{dU}{dt} \end{aligned}$$

Now, we make the same assumption that the volume velocity is sinusoidal:

$$U = U_0 e^{j\omega t}$$
$$\frac{dU_0 e^{j\omega t}}{dt} = j\omega U_0 e^{j\omega t}$$
$$\frac{dU}{dt} = j\omega U$$

then

$$\begin{aligned} p &= \frac{1}{S^2} M_M \frac{dU}{dt} \\ &= \frac{1}{S^2} M_M j\omega U \end{aligned}$$

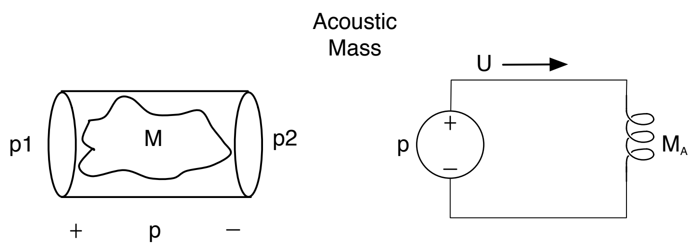
now define the Acoustic Mass like this:

$$M_A = \frac{1}{S^2} M_M$$

and

$$p = j\omega M_A U$$
$$so$$
$$Z_{MA} = j\omega M_A$$

We need a component whose impedance is directly proportional to frequency - an **inductor**.



For the air in the tube to be considered an acoustic mass, the wavelength must be:

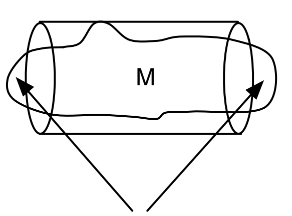
$$\lambda \geq 8l$$
$$\text{l = length of tube}$$

We are almost always going to make this assumption unless specifically noted.

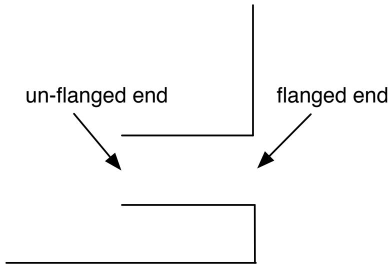
The simple equation to find the acoustic mass is:

$$M_A = \frac{\rho_o l}{S}$$

However a more accurate value can be obtained by taking into account that the air is kind of sticky and the actual mass sticks to pieces of air just outside the tube:



Correction factors are needed to accommodate the extra pieces of air mass sticking outside the tube ends. There are two equations, one for an un-flanged end the other for a flanged end. The diagram at the left shows a tube with 2 un-flanged ends. A flanged end occurs when you mount the tube so that one end is flush with a hole in an enclosure, like the vents you see on loudspeakers.



The equations are:

$$l_f = 0.8488\sqrt{\frac{S}{\pi}}$$

$$l_{uf} = 0.6132\sqrt{\frac{S}{\pi}}$$

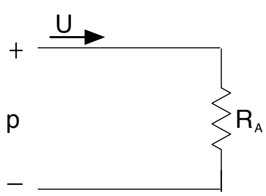
so the tube on the left would have one flanged and one un-flanged end - its total length would be:

$$l = l + l_{uf} + l_f$$

Tube Variations

Infinitely Long Tube

You saw that in order for the tube to act as an acoustic mass-trapper, the wavelength of the sound had to be at least 8 times the length of the tube. What happens if the tube is very, very long - infinitely long? The reason that the acoustic mass is frequency dependent is because a standing wave is set up in the tube - the air particles bounce off of the air mass at the end of the tube and reflect back and forth. With an infinitely long tube, there is no reverse or reflected wave and therefore no dependence on frequency, thus we can model it as an acoustic resistance instead:



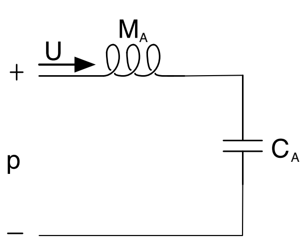
$$R_A = \frac{\rho_o c}{S}$$

$S =$ cross sectional area of tube/vent

Long Tube Closed on One End

A long tube closed on one end acts as both an acoustic mass trapper and a compliance if its length is greater then one tenth the wavelength of the sound of interest.

$$l > \frac{\lambda}{10}$$

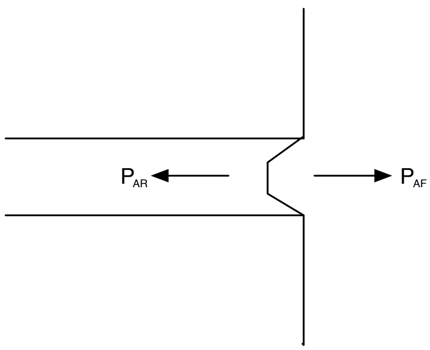


$$M_A = \frac{\rho_o l}{3S}$$

$$C_A = \frac{V}{\rho_o c^2}$$

Note that the equation for acoustic mass has the 3 in the denominator; it is slightly different than the regular equation. Also note that the other end of the cap isn't connected to ground - its not needed in this case because of the tube.

Example (from Leach):



A loudspeaker is mounted in a hole in a wall. One side of the speaker radiates out emitting a spherical wave into half-space. The back side is connected to an infinitely long tube. At 3 meters from the front side we measure 80dBSPL for a 200 Hz sinusoid. What is the acoustic power radiated to each side?

Solution: To find the power radiated to the front, first convert 80dBSPL to RMS pressure:

$$p_{RMS} = 0.2Pa$$

Next, find the average Intensity:

$$I_{AVE} = \frac{p_{RMS}^2}{\rho_o c} = 9.83x10^{-5} \, W/m^2$$

The power radiated to the front is the Average Intensity multiplied by the surface area of radiation - a hemisphere:

$$\begin{aligned} P_{AF} &= I_{AVE} A_{HEMI} \\ &= I_{AVE} 2\pi 3^2 \\ &= 5.56mW \end{aligned}$$

To find the power radiated to the rear, find the Volume Velocity square it, and multiply it by the Radiation Resistance - in this case a single resistor value. The Volume Velocity is found by re-arranging the equation that relates Power and Volume Velocity when driving a hemisphere:

$$U_{RMS} = \sqrt{\frac{2\pi c P_{AF}}{\omega^2 \rho_o}} = 2.54x10^{-3} \, m^3/s$$

The Radiation Resistance of the infinitely long tube is:

$$R_{AB} = \frac{\rho_o c}{S}$$

The Power delivered to the tube is:

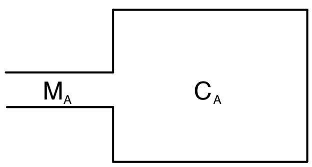
$$P_{AR} = U_{RMS}^2 R_{AB} = 0.335W$$

This is more than 60 times the amount radiated to the front. It is much more efficient to radiate into a tube, which is where horns (long tubes with flanged ends), stethoscopes, and submarine communications systems come from.

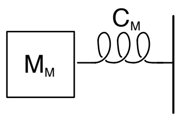
Helmholtz Resonator

A Helmholtz Resonator consists of a tube connected to an enclosure - the familiar soda-bottle trick where you blow air across the hole to create a tone uses this principle. The tube traps a mass of air in it. The enclosure (cavity) acts as an air-spring. The mass of air will vibrate back and forth against the spring under the correct conditions.

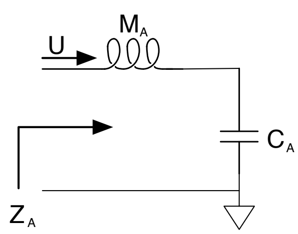
Symbolic diagram of the HH Resonator



Mechanical Equivalent



Electrical Equivalent



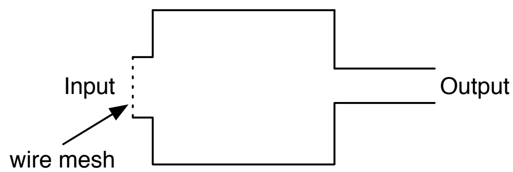
the Resonant Frequency is found with:

$$f_{HH} = \frac{1}{2\pi\sqrt{M_A C_A}}$$

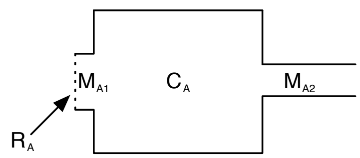
Challenge: prove this equation (hint use the impedance looking into the tube and some basic rules of electronics)

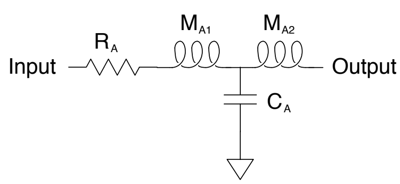
Acoustic Filters

Any number of electronic RLC filters can be fashioned into their acoustic counterparts. Various connections of tubes, chambers and screens can be used. For example,what kind of filter is this:



Can you see that the equivalent circuit is this:





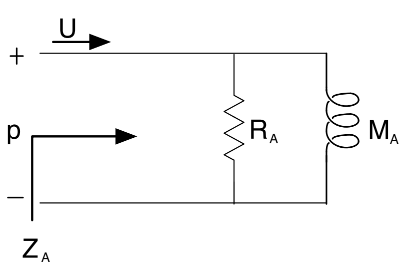
What kind of filter is it? It is a 3rd order Resonant Lowpass Filter; the resonant frequency is controlled by the 3 reactive components and the Q is controlled by the resistor in combination with the inductors.

The Impedance of Air

Air is invisible but it still has mass and because of its chemistry, its own way of compressing and expanding. Air is viscous like water, but we are designed not to notice the 1 atmosphere of air pressure present on our skin. The impedance of air changes depending on the source - what is radiating into the air load? The solution to find the circuits is very difficult in some cases and is outside the scope of this book. In all cases, the air-load that a vibrating entity feels has both real (resistive) and imaginary (reactive) parts at the same time.

Air Load on the Surface of a Spherical Wave

As the surface of a spherical wave pushes against the air load around it, it feels both a purely resistive opposition and a reactive one due to the acoustic mass of air it pushes. The circuit and equations are:



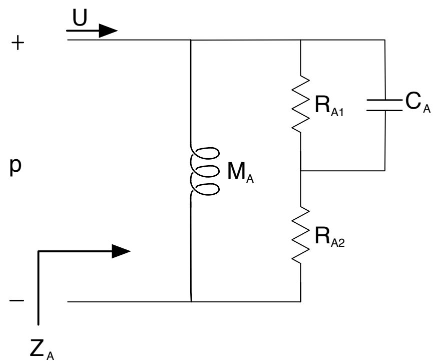
$$M_A = \frac{\rho_o}{4\pi r}$$

$$R_A = \frac{\rho_o c}{4\pi r^2}$$

$r =$ radius of sphere

Air Load on a Piston in a Baffle

Mounting a piston in a baffle (large surface like a wall) forces hemispherical radiation. This is what we care most about since most speakers are mounted flush with a surface. If the surface is large enough to block the lowest frequency of interest it could be consider a baffle. The solution for this circuit is extremely complicated - it has a high shelving filter portion and a HPF portion.



$$M_A = \frac{8\rho_o}{3\pi^2 a}$$

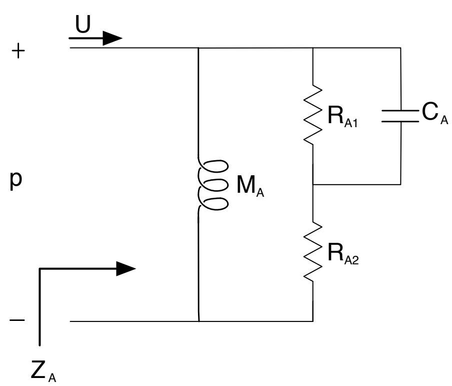
$$C_A = \frac{5.94a^3}{\rho_o c^2}$$

$$R_{A1} = \frac{0.4410\rho_o c}{\pi a^2}$$

$$R_{A2} = \frac{\rho_o c}{\pi a^2}$$

Air Load on a Piston at the end of a Tube

If the baffle isn't large enough (like the front of some small bookshelf speakers) then what do we do? It becomes very difficult to estimate the answer. But in the worst case scenario if the enclosure was folded all the way back on itself, it would become a tube. The circuit for the air load on a speaker at the end of a tube is identical to the one above in the baffle case, only the component values are changed:



$$M_A = \frac{0.6133\rho_o}{\pi a}$$

$$C_A = \frac{0.55\pi^2 a^3}{\rho_o c^2}$$

$$R_{A1} = \frac{0.5045\rho_o c}{\pi a^2}$$

$$R_{A2} = \frac{\rho_o c}{\pi a^2}$$

So the air-load a given loudspeaker in an enclosure feels on it will also have the same circuit, but the component values will be somewhere between the two above cases.

2. (a) An infinitely long tube has a diameter of 6 cm. Calculate the acoustic resistance seen looking into the tube. [1.44×10^5] (b) The tube is terminated with a rigid cap at a distance of 20 cm from the source. Calculate the acoustic compliance and acoustic mass seen looking into the tube. [4.03×10^{-9}, 27.8] (c) If the end cap is removed from the tube, calculate the acoustic mass seen looking into the tube. [83.5]

3. (a) A piston in the end of an infinitely long tube having a diameter of 6 cm radiates an average acoustic power of 1 mW into the tube. Calculate the SPL in the tube. [116 dB] (b) The tube is terminated with a rigid cap a distance of 20 cm from the piston. Calculate the SPL in the tube if the peak piston displacement is 1 mm and the frequency is 100 Hz. [148 dB] (c) Calculate the SPL in the 20 cm long tube if the end cap is removed. [130 dB]

4. A loudspeaker driver is located in one end of an infinitely long tube having a radius of 10 cm. At $f = 150$ Hz, the loudspeaker produces an SPL of 100 dB inside the tube. If the loudspeaker radiates a simple spherical wave into 4π steradians outside the tube, calculate the SPL at a distance of 1 m from the loudspeaker. [56.7 dB]

---

8. (a) A tube of air has a cross-sectional area S and is unflanged on both ends. What is its length ℓ if it is to have an acoustic mass M_A? [$M_A S/\rho_0 - 1.226\sqrt{S/\pi}$] (b) If the tube is flanged on both ends, what is the length? [$M_A S/\rho_0 - 1.698\sqrt{S/\pi}$]

9. (a) Fig. 3.18(a) shows a musical jug, with a diameter of 10 in and a height of 12 in. Solve for the Helmholtz resonance frequency of the jug. Model the neck as a tube that is flanged on one end and unflanged on the other. [53.2 Hz] (b) Draw the acoustical analogous circuit of the system shown in Fig. 3.18(b).

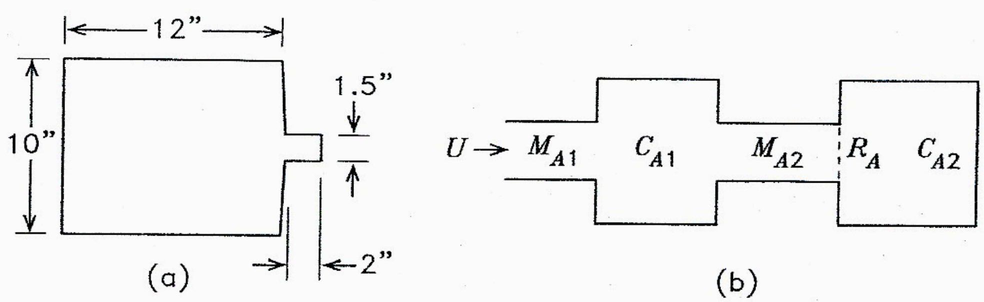


Figure 3.18: Figures for Problem 9.

CH. 2 HW

3 Modeling Mechanical Systems with Electrical Systems

	Components:	Variables
Mechanical Systems:	- masses	- force
	- springs	- velocity
	- surface friction	- friction
	- molecular (internal) friction	
Electric Systems:	- resistors	- voltage
	- capacitors	- current
	- inductors	- impedance (resistance)
	- voltage sources	
	- current sources	

Impedance and Admittance

The fundamental difference in approach for modeling mechanical systems is that we need to consider not only the impedance analogs (for example, the acoustic Impedance of a mass of air in a tube is modeled by an Inductor) but also the admittance analogs. Since the admittance Y = 1/Z, its very easy to swap components:

Impedance	Admittance
V	I
I	V
R	1/R
L	C
C	L
circuit mesh loops	circuit nodes
circuit nodes	circuit mesh loops

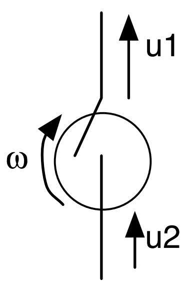
So in this section, each component will have 2 types: and Impedance analog and an Admittance analog. The Admittance analog is also called a “mobility analog.”

Velocity

Current and Velocity are defined as:

$$u = \frac{\text{distance}}{\text{time}}$$

$$I = \frac{\text{charge flow}}{\text{time}}$$

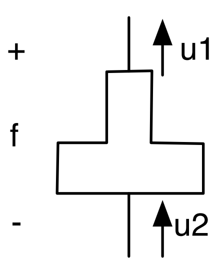


We could make Current the analog of Velocity in a mechanical system; if charge were analogous to distance, this would work so let's go with it. The velocity will need to be able to be sinusoidal so the mechanical symbol is a wheel on an axis with a rotor; it turns with an angular velocity with frequency ω. Generally, one side will be fixed but this does not have to be true and occasionally will not be.

Mechanical Component	Impedance	Admittance
Velocity (u)	Current (I)	Voltage (V)

Force

Force is pressure*area. In Electronics, Voltage is the amount of charge separation - the more electrons you have separated from their electron-deficient neighbors the higher the voltage. We already saw that pressure is analogous to voltage in acoustics, so we could make the case that since force is directly proportional to pressure, in mechanics we could model Force as a voltage.

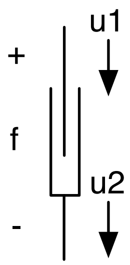


This is the mechanical symbol for a force generator (aka Force Source). It has 2 velocities u1 and u2 at each terminal. The force source pushes one end (u1) while pulling the other (u2). This is analogous to a Voltage source that pushes electrons from one terminal and and pulls electrons from the other.

Mechanical Component	Impedance	Admittance
Force (F)	Voltage (V)	Current (I)

Friction

Mechanical friction (surface friction or internal mechanical friction between molecules when a material is bent, twisted, flexed or otherwise distorted in shape) is considered to be non-frequency dependent under ideal conditions. The simplest mechanical model is a car's shock absorber. The shock absorber has two different velocities on each end (usually one end is fixed, but this doesn't have to be the case):



The mechanical symbol for Friction is the shock-absorber. A force across it produces two velocities at its two ends.

The equation that relates everything is:

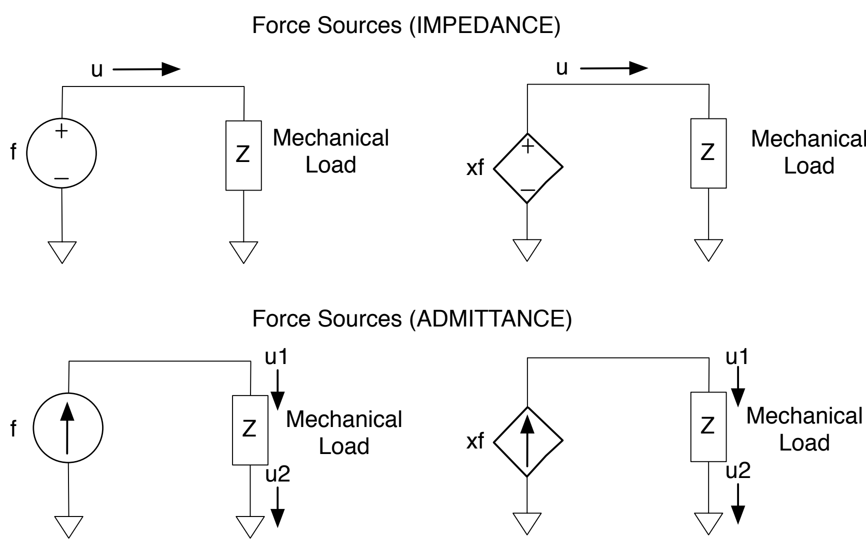
$$f = R_M \Delta(u)$$
$$= R_M(u_1 - u_2)$$

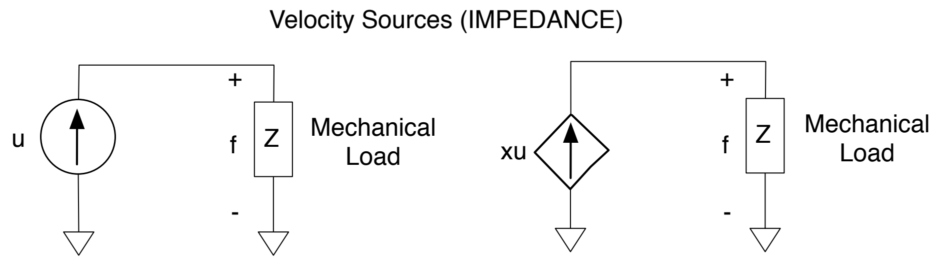
The impedance analog is a resistor and the admittance is a resistor of value 1/R

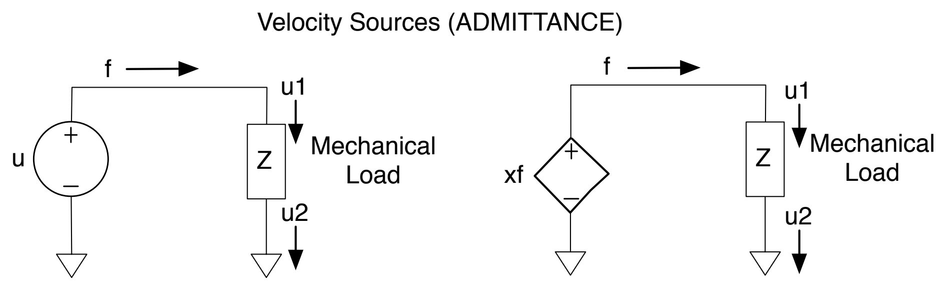
Mechanical Component	Impedance	Admittance
Resistance	Resistor (R)	Inverse Resistor (1/R)

Sources

The relationships between force, velocity and impedance follow Ohm's Law so the relationships between the various types (force, velocity in both impedance and admittance) are:







Ohm's Law

Ohm's Law then becomes:

Electric :

$V = IZ$

Mechanical

$f = uZ$

Springs

A force on a spring causes the spring to compress or expand depending on the direction of the force. The two ends of the spring move at different velocities. The spring exchanges energy with the force source; during compression it stores the energy and during expansion it returns it.

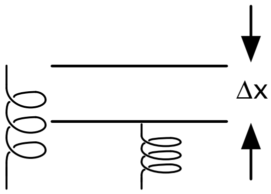
The force that the spring feels is:

$f = kx$

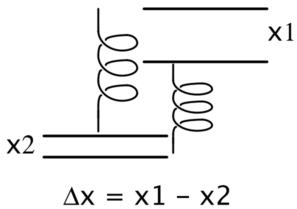
where k is the *spring constant*. The value of k tells you how stiff or loose the spring is - high values indicate high stiffness. In acoustics, they use another variable called *compliance* which is exactly the opposite - high compliance = a loose spring. Mechanical Compliance is:

$$C_M = \frac{1}{k}$$

If the spring is compressed with one end fixed (so that its velocity is 0), the displacement is Δx:



<u>However, the spring does not need to be fixed at one end.</u> If it is not fixed at one end, then there are two displacements and you still have a Δx.



With x = Δx and a bit of calculus:

$$f = kx = \frac{1}{C_M}x$$

$$f = \frac{1}{C_M}\int u\,dt$$

$$u = \text{velocity}$$

If we assume the velocity is fluctuating sinusoidally, then we can write it like this:

$$u = u_0 e^{j\omega t}$$

$$then$$

$$\int u_0 e^{j\omega t}dt = \frac{1}{j\omega}u_0 e^{j\omega t}$$

$$so$$

$$\int udt = \frac{1}{j\omega}u$$

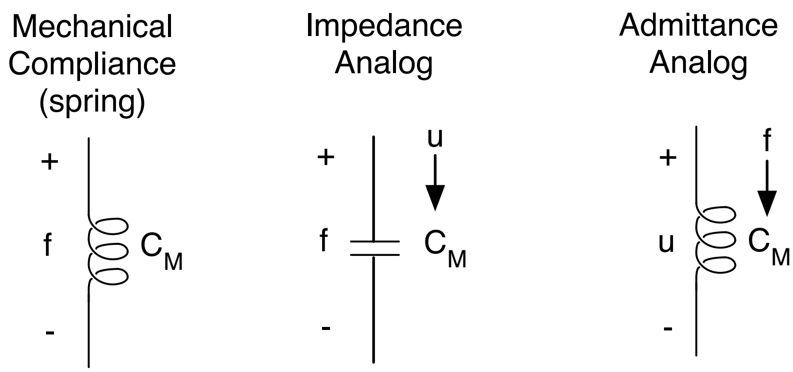
then:

$$f = \frac{1}{j\omega C_M}u$$

Since the acoustic Ohm's Law is f = uZ, then we are looking for an electronic component that has an impedance that is inversely proportional to frequency = a **capacitor**.

$$Z_M = \frac{f}{u}$$

$$Z_{CM} = \frac{1}{j\omega C_M}$$



Masses

Force and mass are related by Newton's Second Law:

$$\begin{aligned} f &= mA \\ &= M_m A \\ &= M_M \frac{du}{dt} \end{aligned}$$

Now, we make the same assumption that the velocity is sinusoidal:

$$u = u_0 e^{j\omega t}$$

$$\frac{du_0 e^{j\omega t}}{dt} = j\omega u_0 e^{j\omega t}$$

$$\frac{du}{dt} = j\omega u$$

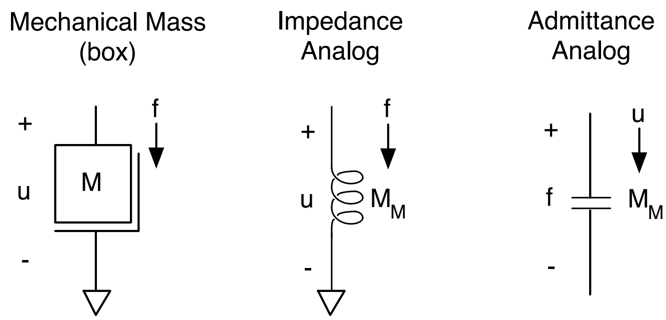
then

$$f = j\omega M_M u$$

so

$$Z_{MM} = j\omega M_M$$

We need a component whose impedance is directly proportional to frequency - an **inductor**.



Note that the mechanical and impedance analog are grounded on one side. This is because the velocity on the mass is measured with respect to 0 (rest or stand-still) so the velocity drop across the inductor must also be measured with respect to ground. The admittance version swaps grounded for non-grounded states.

Converting Mechanical Diagrams into Circuits

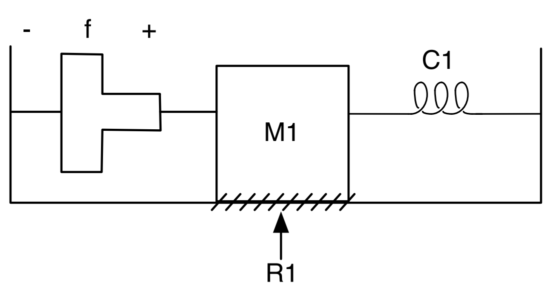
Converting the mechanical systems to electrical circuits is a bit more involved that in the Acoustics version. When converting Acoustic systems, you can simply follow the pressure and volume velocity through the tubes, cavities and resistances directly and write the circuit (see the Helmholtz Resonator and Acoustic Filter sections). However, in Mechanics, you must first create the Admittance circuit. Then, you convert it to an Impedance (normal) circuit using the same principles you learned in EE101 - you swap component types and turn loops into nodes and vica versa. The process for converting a mechanical diagram is in two general steps, each with sub-steps:

1) convert the mechanical drawing to an Admittance Circuit

2) convert the Admittance Circuit to an Impedance Circuit (this is a normal EE "trick")

The best way to show this is by doing examples.

Consider this diagram:



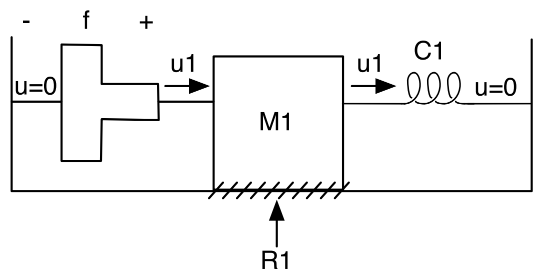
This diagram shows a sinusoidal force-source pushing and pulling on a mass M1 which is connected to the spring C1. The friction resitance R1 exists between the block and the floor.

Converting Mechanical Diagrams into Admittance Circuits

STEP 1: Identify all the different velocities in the system. The rules to use are:

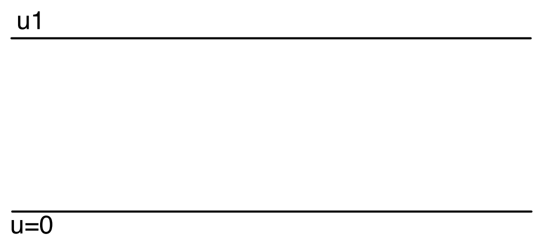
- The walls and floor have zero velocity
- A mass will have the same velocity on each side since it can't compress. All the other components and sources may have different velocities on each side but not necessarily. Often, they will be attached to a wall/floor.

This digram has 1 velocity (plus ground or zero-velocity):

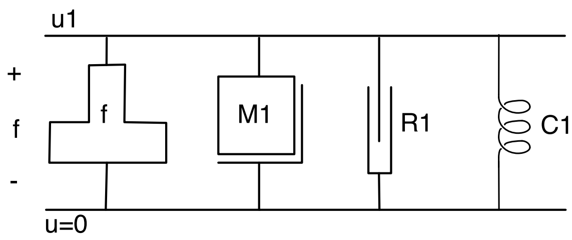


Aside from the u=0 parts, there is only one velocity. It must be the same on each side of M1.

STEP 2: Draw a horizontal bar for each velocity, plus another one at the bottom for ground (zero-velocity):

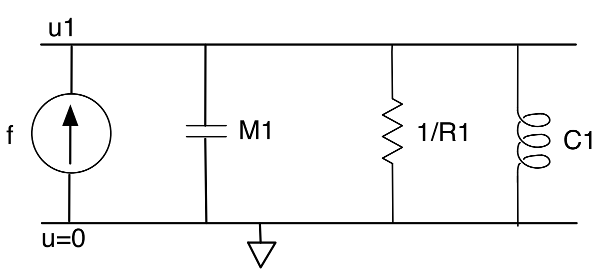


STEP 3: Attach each mechanical component to the velocities in question keeping the orientation of each the same as the mechanical drawing:



Notice the orientation of the force source, maintaining the polarity across the two velocities.

STEP 4: Convert each mechanical component into its **Admittance** component directly, preserving the relationships of polarity, etc. Also, add the ground node at u = 0:

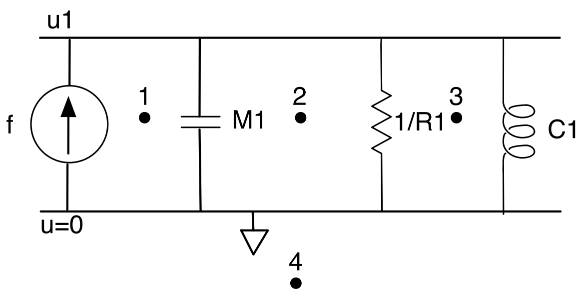


At this point, the Admittance circuit is complete. Note that the resistor value is 1/R1.

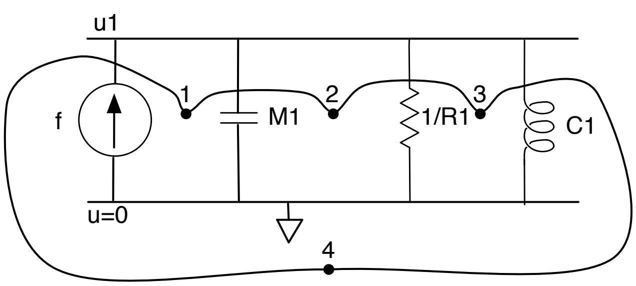
Converting Admittance Circuits into Impedance Circuits

Note - this is purely electronics - there's no physics involved and this step-by-step process is guaranteed to produce the Impedance analog (when followed properly)

STEP 1: Place a DOT **inside** of each mesh loop and number it. Also, place a dot outside the whole circuit (anywhere) that will represent the ground node.



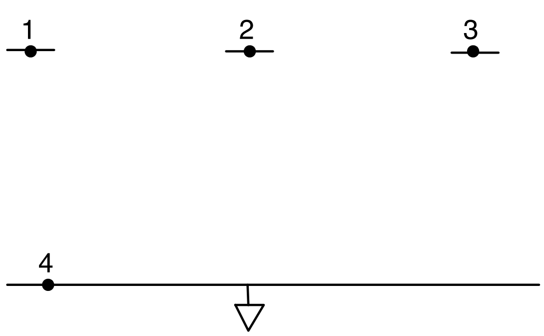
STEP 2: Draw a line to connect each dot so that you only draw one line through each component. A dot may be connected to more than one dot, but each element will have only one line through it:



Notice that each element has only one line through it.

STEP 3: Do the Admittance/Impedance Swap:

- the mesh loop numbers become nodes in a circuit - draw the skeleton:



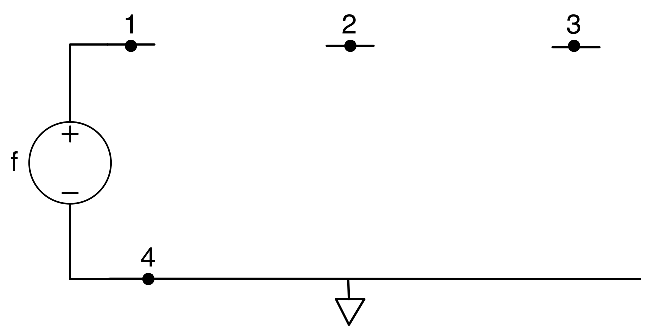
The gaps between the nodes are where the components will get slotted-in.

- connect the components between the nodes that were originally between the mesh loop dots AND swap the component types. The components are the ones with a line drawn through them:

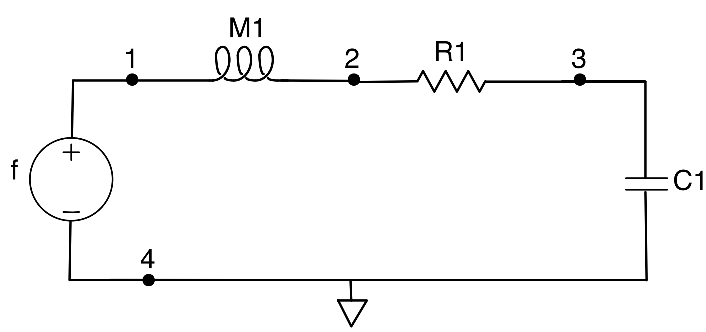
Admittance	Impedance
I	V
V	I
1/R	R
C	L
L	C

Example:

Between dots 1 and 4 we have a current source, so between nodes 1 and 4 we have a voltage source:



Conversion has started. Now do the rest:



Here's the final circuit.

STEP 4: Do a check: the old circuit had 3 loops plus ground, the new one has 3 notes plus ground. The old circuit had one giant node (u = u1) and the new circuit has one giant loop.

What kind of circuit is this? Its is a 2nd Order Band Pass Filter with a series RLC circuit. This mechanical system will resonate at the same frequency as the analogous electrical circuit and with the same damping factor (Q) as the electric one. For this circuit, those values are:

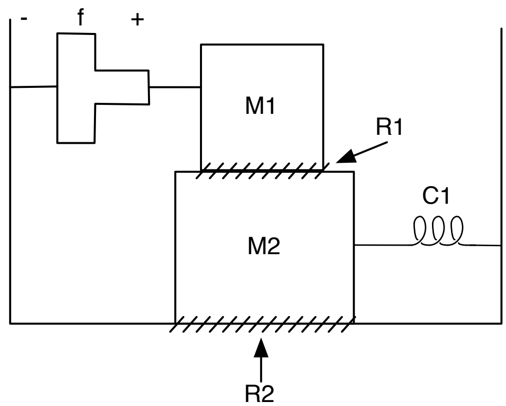
$$f_c = \frac{1}{2\pi\sqrt{M_1 C_1}}$$

$$Q = \frac{1}{R}\sqrt{\frac{M_1}{C}}$$

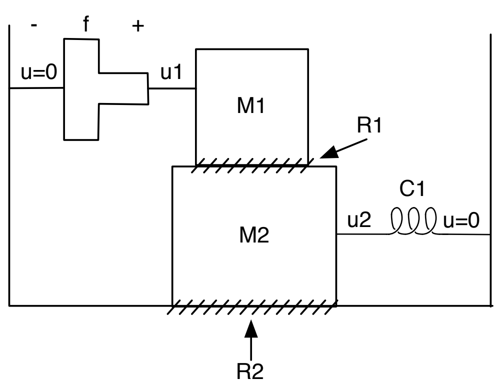
The mechanical system equations are the same as the electrical system equations.

More Examples:

Try this one:



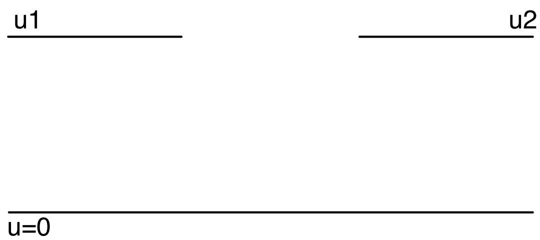
STEP 1: Identify all the different velocities in the system.



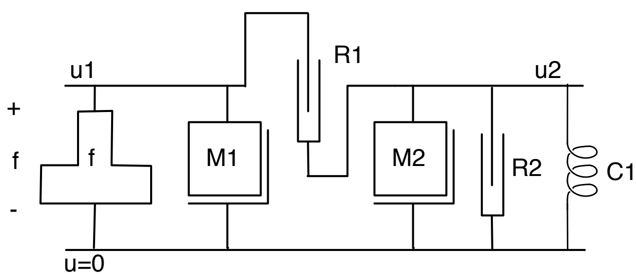
Notice there are 2 non-zero velocities, u1 and u2 which are on different masses; this is because the masses are not physically connected.

The resistance R1 is between u1 and u2 and R2 is between u2 and ground. This is because u1 is "stuck" to M1 and u2 is stuck to M2.

STEP 2: Draw a horizontal bar for each velocity, plus another one at the bottom for ground (zero-velocity):

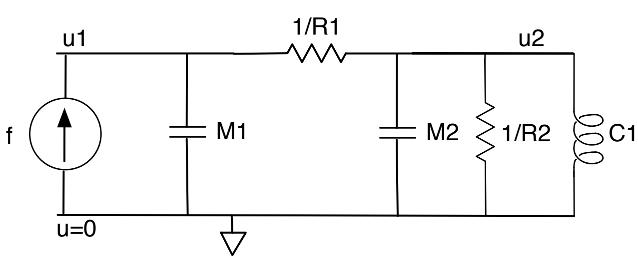


STEP 3: Attach each mechanical component to the velocities in question keeping the orientation of each the same as the mechanical drawing. Remember, <u>masses are always connected to ground at one end</u>!



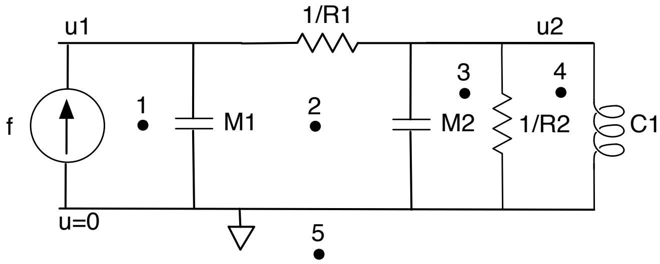
This might look funny at first but you can verify that the proper components connect to the proper velocities.

STEP 4: Convert each mechanical component into its **Admittance** component directly, preserving the relationships of polarity, etc. Also, add the ground node at u = 0:

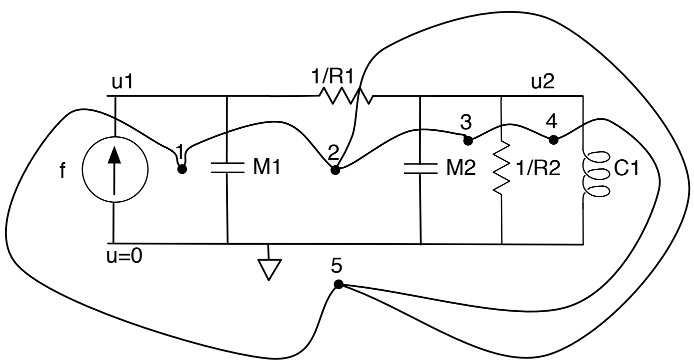


Converting Admittance Circuit into Impedance Circuit

STEP 1: Place a DOT **<u>inside</u>** of each mesh loop and number it. Also, place a dot outside the whole circuit (anywhere) that will represent the ground node.

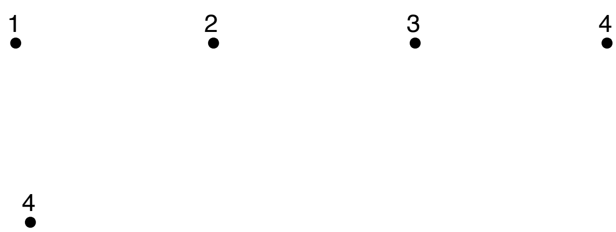


STEP 2: Draw a line to connect each dot so that you only draw one line through each component. A dot may be connected to more than one dot, but each element will have only one line through it:

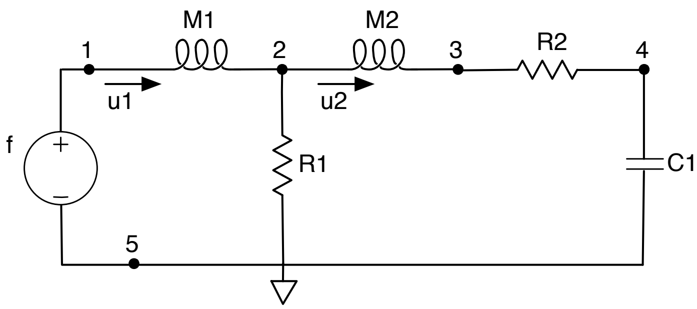


STEP 3: Do the Admittance/Impedance Swap:

- the mesh loop numbers become nodes in a circuit - draw the skeleton:



- connect the components between the nodes that were originally between the mesh loop dots AND swap the component types. The components are the ones with a line drawn through them:



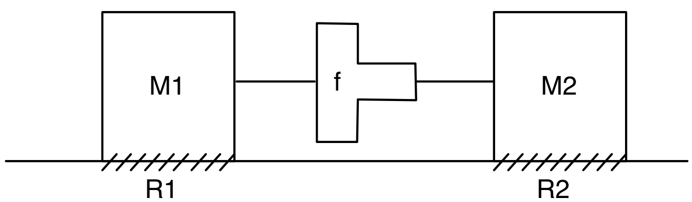
STEP 4: Do a check: the old circuit had 4 loops plus ground, the new one has 4 nodes plus ground. The old circuit had two nodes, the new one has two mesh loops.

Homework:

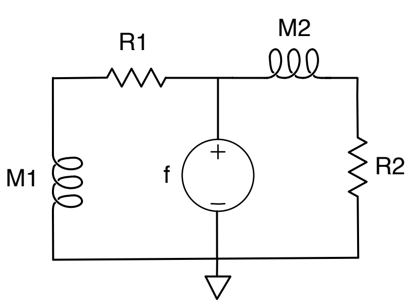
Verify the following Mechanical and Electrical Circuits are equivalent:

(1)

Mechanical

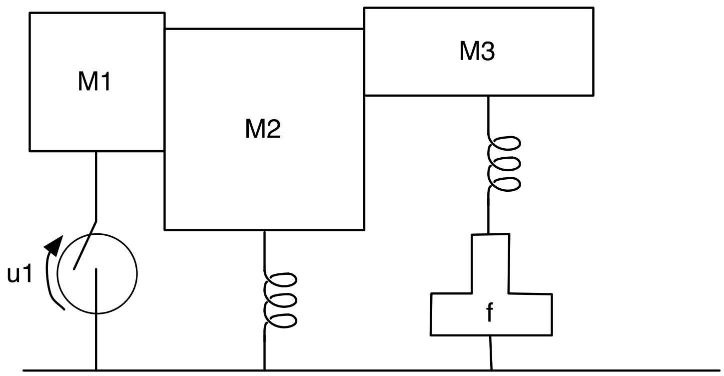


Electrical:

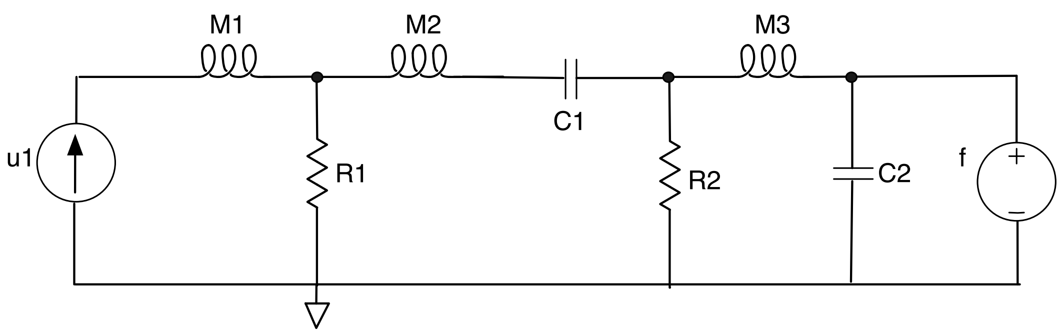


(2)

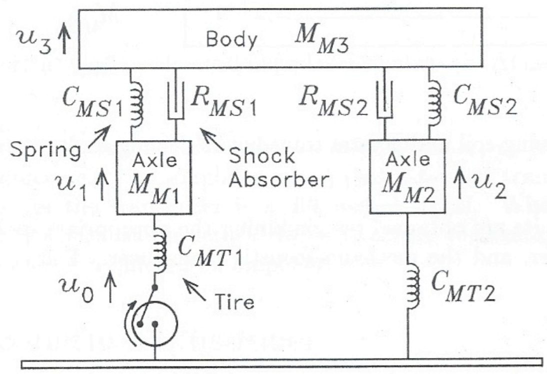
Mechanical:



Electrical:



2. An automobile and its suspension can be modeled by the mechanical diagram shown in Fig. 4.21. The velocity generator models bumps in the road. Form the mobility and the impedance analogous circuits for the system.



7. Fig. 4.23 shows a combination mechanical and acoustical system that is coupled by a mechano-acoustic transducer of area S_D. Form the analogous circuits for the system using an impedance analog for the mechanical part. [Mechanical part: a parallel force source f_0 and compliance C_M in series with a mass M_M, a resistance R_M, and a force source $f_a = S_D p_D$. Acoustical part: a volume velocity source $U_D = S_D u_D$ in series with two air load impedances. Back air load impedance: a compliance C_{A1} to ground. Front air load impedance: a mass M_A in series with the parallel combination of a compliance C_{A2} to ground and a resistance R_A to ground.]

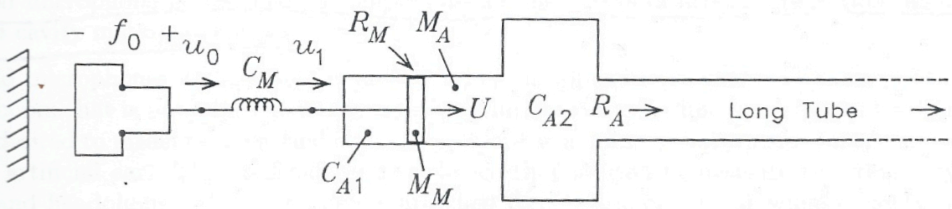


Figure 4.23: Figure for problem 7.

4 Transducer Modeling

In the last chapter, we examined the properties of sound waves that transducers convert either to or from electrical signals. Our end goal is to be able to predict the resulting frequency responses, acoustic output power, and efficiency of transducers in various enclosures. For a loudspeaker, the enclosure is the box that holds the driver. For microphones, the enclosure is sometimes called the capsule. We will find that our end goal is dependent upon the electrical and mechanical properties of the raw transducer itself, the mechanical and acoustic properties of the enclosure that holds it, and the physical and chemical properties of the air that acts as the wave propagation medium. Combining all these different disciplines together can be a daunting task. If we wish to predict the frequency response of a loudspeaker, then we need to be able to accurately describe its motion as a function of its frequency of oscillation. Traditionally, when faced with motional (dynamics) problems, we turn to traditional Newtonian physics. The Newtonian method involves first quantifying and qualifying all the forces acting on the system. Next, the sum of all the forces is set to zero (since all forces must have an equal and opposing force), and the dependent variable of choice (frequency here) is derived. In the field of transducers, the Newtonian method fails due to the complexity and interdependency of the forces and mechanics involved. Fortunately, there is a much easier way to generate frequency response and power predictions. This method uses electronic circuits to model the behavior of the complex electrical/mechanical/acoustical system. We've already seen how to calculate frequency response plots from circuits, so all we really need to do is define the circuit modeling technique.

Transducers have three distinct types of functional parts: electrical, mechanical and acoustical. When analyzing a transducer system, we look at each of these three parts separately, and then combine them together in a complete system. One or more of the three basic electrical elements – resistors, capacitors, and inductors, will be used to model each of these parts. We can break the parts down as follows:

Electrical

- resistance
- capacitance
- inductance

Mechanical

- friction
- springs
- mass

Acoustical

- air resistance
- pneumatic air springs
- air masses trapped in tubes or vents

In this chapter, we analyze the two most widely used transducer types, and identify the electrical and mechanical functional parts. We will finish the process by adding the acoustical parts when we design the enclosures.

Theile-Small Parameters

Richard Theile and Robert Small helped develop some of the theory of transducer enclosures. The electrical, mechanical, and acoustical parameters of a driver are called the *small signal* or more commonly, the **Theile-Small** parameters. Transducer manufacturers will test and specify the Theile-Small parameters for their products. The parameters are abbreviated such as R_E, the <u>E</u>lectrical <u>R</u>esistance, or M_{MD}, the <u>M</u>echanical <u>M</u>ass of the <u>D</u>iaphragm. We will identify several T-S parameters in this chapter, and more in the chapters to come.

4 Transducer Overview

Transducers must convert electric energy to acoustic energy. This ultimately involves converting an electrical signal to and from a force on a diaphragm. For a driver, the audio amp delivers the electrical signal to the transducer, which converts it to a force on a diaphragm (piston). The piston pushes and pulls the adjacent air particles. The air is a viscous, elastic medium and it opposes the piston's force. We call this opposition the "radiation resistance" or "impedance." We often say that the air is the "load" into which the piston delivers its power. The air load is more complex than a simple resistance, as in op-amps. It will be modeled with several electrical components.

Drivers

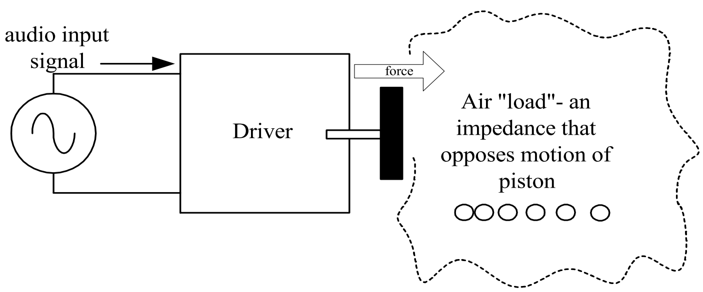


Figure 4.1: A conceptual diagram of an audio output transducer (driver). The electrical input signal is converted to a force on a piston that creates the sound waves. The air load on the piston provides a load impedance into which the piston delivers its acoustic power.

Microphones

For a microphone, we simply reverse the diagram. Vibrating air particles provide force on a piston. The transducer converts the force into an electric output signal. The input impedance of the connecting device (usually a high gain amplifier) provides the load that accepts the electrical output power.

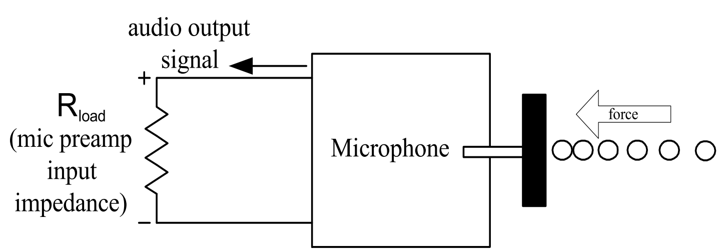
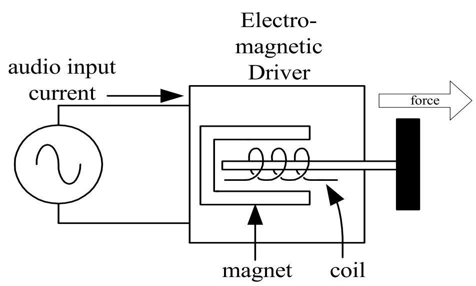


Figure 4.2: A conceptual diagram of a microphone transducer. The output signal is a voltage dropped across the connecting device's input impedance.

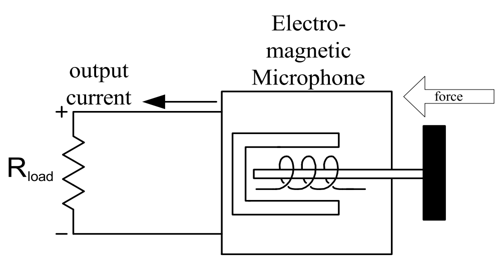
4.1 Fundamental Transducer Types: Moving Coil

There are two main types of transducers in wide use in audio called **electromagnetic** and **electrostatic** transducers. Electromagnetic transducers rely on Faraday's Law, which involves magnetic fields and induced currents.



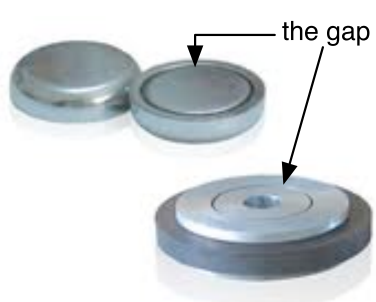
By far and away, the most popular is the electromagnetic or *moving coil* transducer. For loudspeakers, the audio amplifier injects AC current (the audio signal) into a coil of wire immersed in a magnetic field. Faraday's Law shows that the current will set up a magnetic field around the coil. This field attracts or opposes the immersion field causing the coil to move. The coil is glued to the speaker's diaphragm, or cone so that the cone moves in response to the current in the wire.

Figure 4.3: Conceptual diagram of a moving coil driver



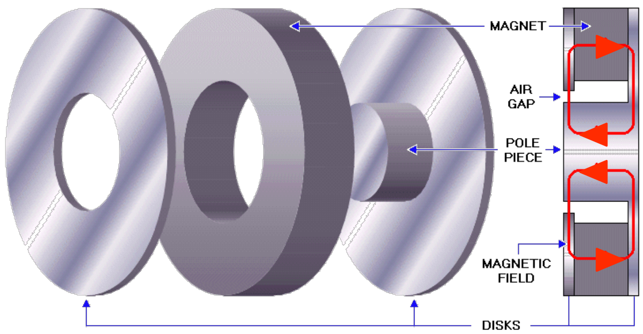
For microphones (commonly called *dynamic* mics), the system is simply reversed – air particle pressure and/or velocity causes a diaphragm to move. The diaphragm is connected to a coil of wire immersed in a magnetic field. The motion of the coil induces a current in the wire. This current is the audio output signal for the microphone.

Figure 4.4: Conceptual diagram of a moving coil (a.k.a. dynamic) microphone.



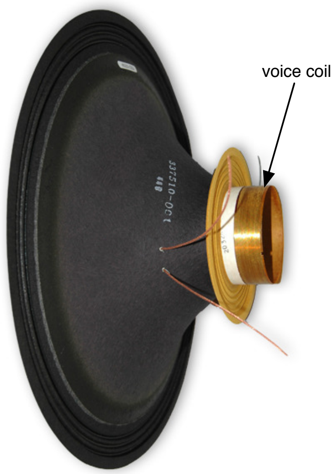
The magnet/coil combination is called the motor. The magnet is radial (circular) with the north and south engs located at the top and bottom of the ring.

At the left are some magnets for loudspeakers. Microphone motors are essentially the same but on a much smaller scale. The gap is the ring you can see which appears to be cut into the top plate. Actually, it is assembled that way as follows:



In this cut-away view you can see how the air gap is actully formed using a ring plate that is slightly smaller in diameter than the magnet's interior. The Pole Piece sticks up from the Back Plate.

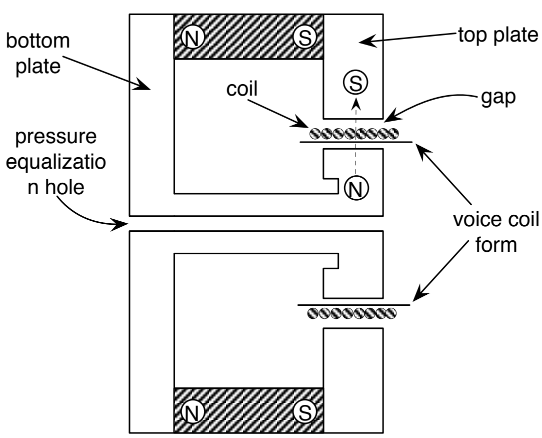
The magnetic lines of flux (B) shown here flow from North to South; the bottom plate is connected to the Pole Piece so flux flows from the center ourward radially.



The voice coil of the speaker is made from wire wrapped around a cardboard tube called the **voice coil form**.

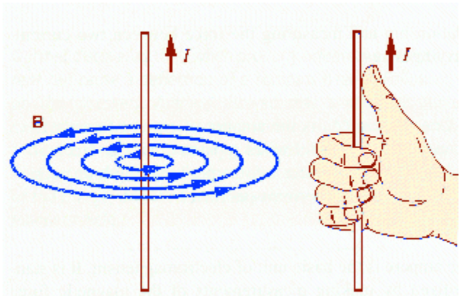
Leads from the coil run up along the base of the cone (or diaphragm) and then come out of holes on the side of it.

The coil sits inside the gap so that it is immersed in the magnetic field that the magnet produces.

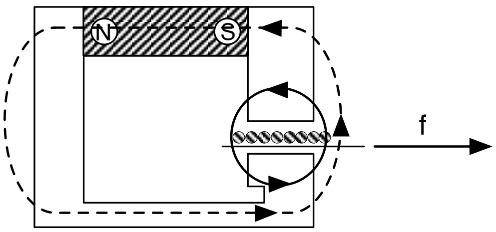


In this cut-away view you can see that the coils of wire sit perpendicular to the lines of flux. It is the perpinduclar arrangement that allows the motor to produce a force on the diapragm (for a speaker) or allows the diaphragm to induce a current in the coil (microphone).

Also shown is a pressure equalization hold drilled through the center of the assembly to prevent any kind of compliance being formed in the chambers between the plates.



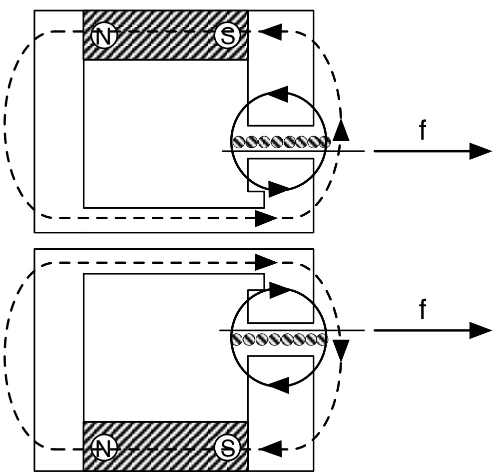
To understand how it works, consider the loudspeaker version: an AC current is injected into the coil. Following the right-hand-rule from physics (see picture to the left), a magnetic field is induced in the wire. The field is torroidal (circular) arround the wire and the polarity of the field dpends on the direction of the current. In this case the flux lines are flowing counterclockwise. Reverse the current direction and the flux lines also reverse as the polarity of the field switches.



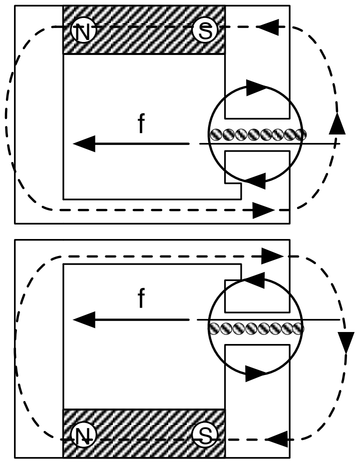
Consider just one half of the motor: suppose the current is coming out of the page and the flux lines are counter clockwise (solid) and at the same time the magnet's flux lines (dotted) are also counterclockwise. The two similar fields will repel each other producing a force **f** on the coil assembly:

$$f = Bli$$

where B is the flux-density of the magnet and l is the length of wire immersed in the field (the combined length of all the coils in the gap).



Now consider both halves: if the current is coming *out* of the page in the top half, it is going *into* the page in the bottom half so the flux lines are reversed, but so are the flux lines in the magnet (follow the route from north to south). The same opposition occurs to the like-polarity fields and the same force pushes outward.



When the current reverses direction, the fields switch direction so that they are opposite from before, but the speaker's flux direction is constant. So now the (opposite) fields attract each other and the voice coil gets pulled inward with the same, but opposite directional force.

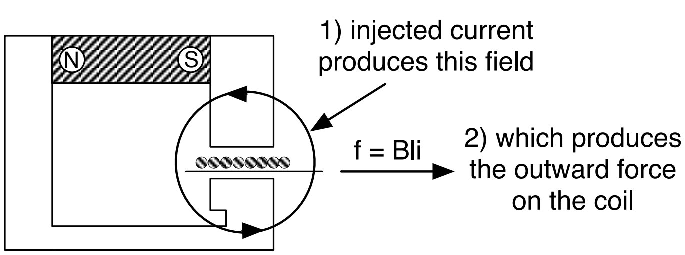
In the case of a microphone, the same situation exists but completely in reverse following the other half of Faraday's law. An inward force on the coil caused by pressure or velocity fluctuations of the air it's diaphragm is connected to will cause the coil to move with a velocity **u**. This velocity will induce a current **i** in the coil. Lenz's Law states that this current is set up in the direction that opposes the current that set up the field to begin with. When connected to a load impedance (the input impedance of the microphone preamp it is connected to), a voltage **e** will drop across the load. The voltage is:

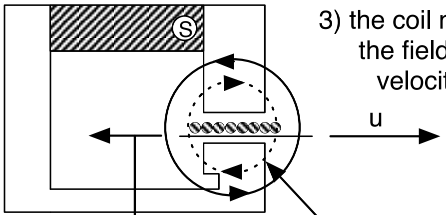
$$e = Blu$$

where B is the flux-density of the magnet and l is the length of wire immersed in the field and u is the velocity.

4.2 Back EMF

Moving coil transducers suffer from a condition known as Back EMF (Electro Motive Force) sometimes called *inductive kickback*. The problem is that Faraday's Law is working both for and against the driver. When an AC current is injected into the coil immersed in a magnetic field, another magnetic field is generated around the coil. As the coil moves in or out in response, Faraday's Law goes to work again – it says that a moving coil in a magnetic field will have a current induced in it. In other words, a current makes the coil move which induces a current that opposes the first one. Consider the first case above when the current in the top half of the magnet was coming out of the page (magnet flux lines are hidden for clarity):





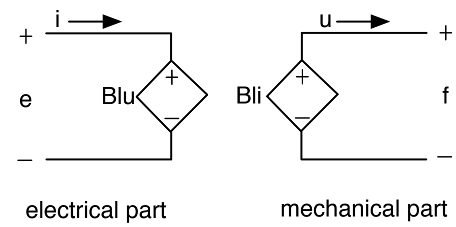
Note that the faster the change in current (think transient edges here), the higher the Back EMF. Back EMF produces a force on the diaphragm that is exactly opposite to the force we are trying to generate on the air particles. Fortunately, the Back EMF component is generally not as large as the forward force, so the driver will still be able to move. Back EMF causes a loss in efficiency, since the motion of the driver creates this opposition force. Even more problematic is the effect on the output section of audio power amplifiers, which do not like to have current spikes driven backwards *into* the output.

Back EMF is a force that opposes the force of the moving diaphragm.

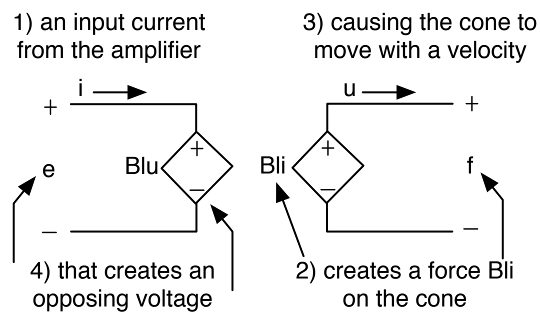
Back EMF is also a problem for microphones, only once again everything is reversed: the force and resulting velocity of the diaphragm induces a current in the wire. But the current in the wire immersed in the magmentic field produces an opposing field - the same problem.

4.3 Moving Coil Models

We need a model that takes all the forces, currents, velocities and voltages into account. This model is two back to back dependent voltage sources, one for the electrical part and one for the mechanical part:

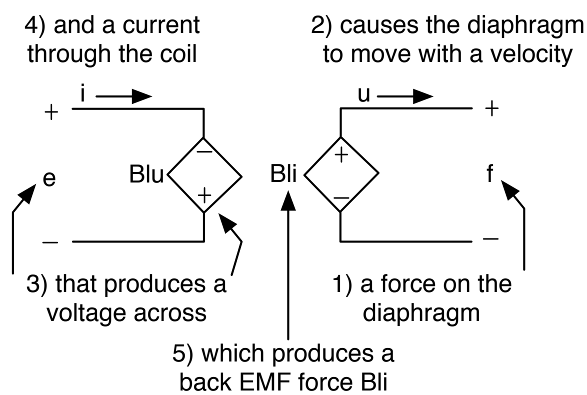


In the case of a loudspeaker, you can think of the sequence as follows:



The only tricky thing is that the voltage **e** has its polarity shown to reflect the normal operating conditions for forward current i. Don't let this bother you - you can think of the total voltage being the forward voltage from the amp minus the back EMF portion.

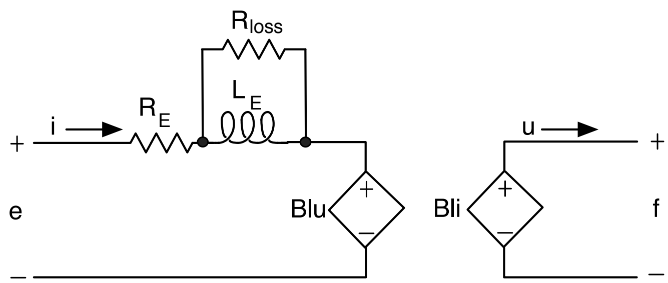
For a microphone, the sequence is reversed:



Loudspeaker Model:

Both the microphone and loudspeaker models need to account for the resistance of the wire in the voice coil. However, the considerably larger voice coil in the loudspeaker requires modeling the inductance of the coil along with magnetic losses in the motor system.

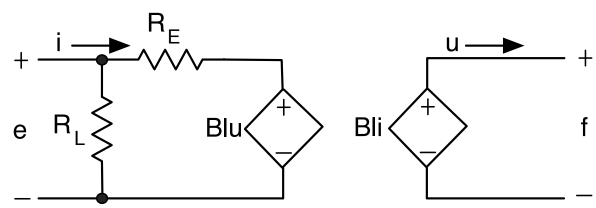
The completed Loudspeaker Model is:



Microphone Model:

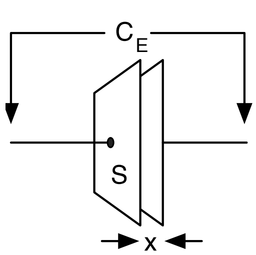
In the microphone model we can neglect the coil inductance and magnetic losses however we still need to keep the coil resistance. Additionally we need to take into account the input impedance of the pre-amp it is connected to.

The completed Microphone Model is:



4.4 Fundamental Transducer Types: Electrostatic

Electrostatic transducers convert forces applied to their surfaces directly into a voltage or current Electrostatic transducers are much more common in microphone form, where they are called *condenser*, *capacitor*, or *electret* mics. Much less common are the loudspeaker version, sometimes called *electrostats*. In all of these devices, a capacitance is created between two or more plates. At least one of the plates, called the **back-plate**, is electrically charged with a DC polarizing voltage and is fixed in position. The other plate is actually a very thin metal diaphragm, suspended so that it can flex easily. Capacitance (the ability to hold separated charges) is a function of the distance between the two plates.



$$C_E = \frac{\varepsilon_0 S}{x}$$

$\varepsilon_0 =$ dielectric constant (of air in this case)

$S =$ surface area of plates

$x =$ distance between plates

Microphone Model:

In condenser microphones, one of the plates is actually the diaphragm which must be made of a conducting type of material and is suspended in front of the second plate. The second plate (called the back plate) is stationary. Air particle pressure and/or velocity moves the diaphragm. As the distance between the two plates changes, the capacitance changes since the capacitance is inversely proportional to the distance between the plates.

$$C_E = \frac{\varepsilon_0 S}{x}$$

then

$$C_E - c(t) = \frac{\varepsilon_0 S}{x + x(t)}$$

C_E - c(t) represents the fluctuating capacitance caused by the fluctuating distance x + x(t). As the distance increases, the capacitance decreases. If a polarizing charge Q is placed on one of the plates (the back plate in microphones) then the total voltage across the capacitor is:

$$E = \frac{Q}{C_E}$$

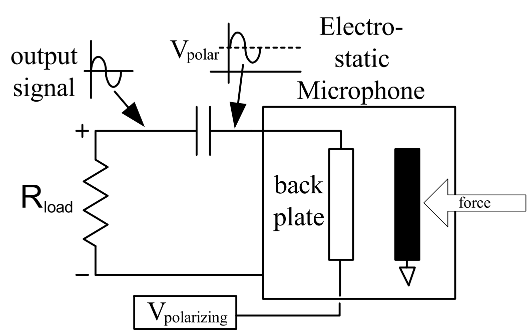
So the voltage is inversely proportional to the capacitance. If the capacitance varies in time, so will the voltage:

$$E + e(t) = \frac{Q}{C_E - c(t)}$$

or

$$E + e(t) \propto x + x(t)$$

The term E + e(t) represents the DC polarizing voltage E and the fluctuating voltage e(t) together. The DC component can be removed with an eternal capacitor to recover the audio signal e(t).



Here is a model of a condenser microphone. The polarizing voltage is called “Phantom Power” in studio lingo, typically +48V though most studio condensers can operate as low as a few volts. The higher the phantom power the higher the headroom.

The output signal is x(t).

Because distance and velocity are related, e(t) can be solved in terms of velocity **u** so the force on the diaphragm produces a velocity u and a voltage e(t) which is made up of two components: first the frequency dependent voltage drop across the cap itself:

$$e_1 = iZ$$
$$= \frac{1}{j\omega C_E} i$$

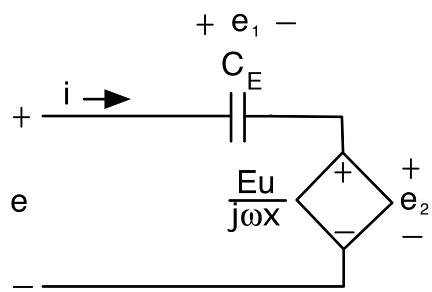
The second voltage is the one created by the change in distance (or velocity) and is also frequency dependent:

$$e_2 = \frac{Eu}{j\omega x}$$

so

$$e(t) = \frac{1}{j\omega C_E} i + \frac{Eu}{j\omega x}$$

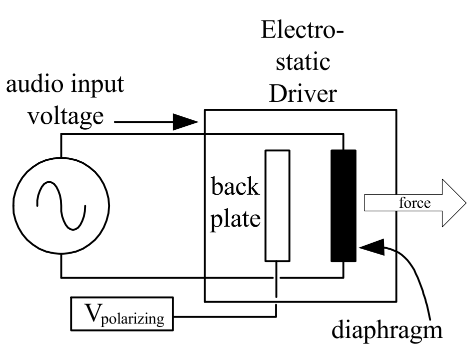
The model for this is:



In this model, you can see the two voltage components of the final signal e(t)

The dependent source shows that the diaphragm velocity produces an output voltage e_2 that is frequency dependent.

Loudspeaker Model:



In condenser loudspeakers, one of the plates is the flexible diaphragm while the other is the back plate. However, the signal e(t) is applied to the diaphragm side. Holding the back plate at a constant polarizing voltage causes the distance between the plates to change as all the above equations run backwards.

The force that is generated depends on the amount of current i(t) that is applied with the voltage e(t). The total force has two components.

The first component is the *force drop* across the mechanical compliance of the diaphragm - remember that f = u Z:

$$f_1 = -\frac{1}{j\omega C_M} u$$

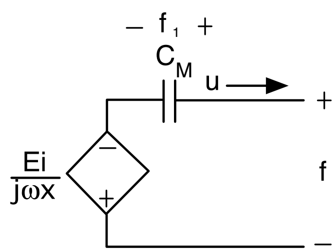
The negative sign indicates the force is in the opposite direction as the current which makes sense when you check the original equations showing an inverse proportional relationship between displacement and capacitance.

The second force component is due to the electrical component whereby the change in charge results in a forced change in distance of the plates; it too includes a negative sign for the same reason:

$$f_2 = -\frac{E}{j\omega x} i$$

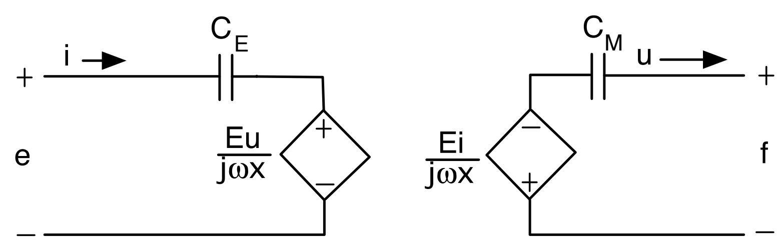
so

$$f = -\frac{E}{j\omega x} i - \frac{1}{j\omega C_M} u$$

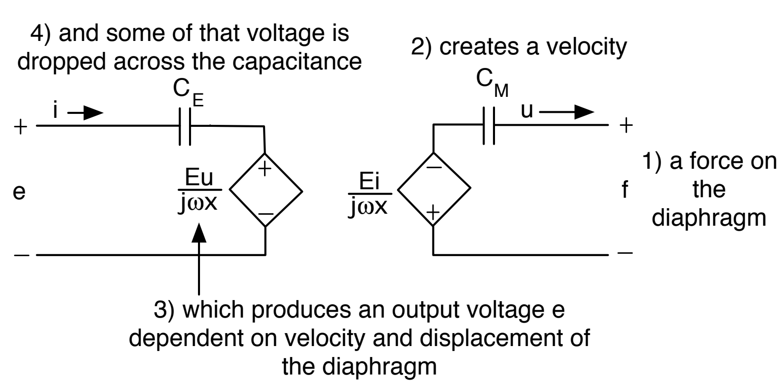


Here is the mechanical side, showing the total force delivered as a combination of two reverse force drops, one across the mechanical compliance and the other supplied by the dependent force.

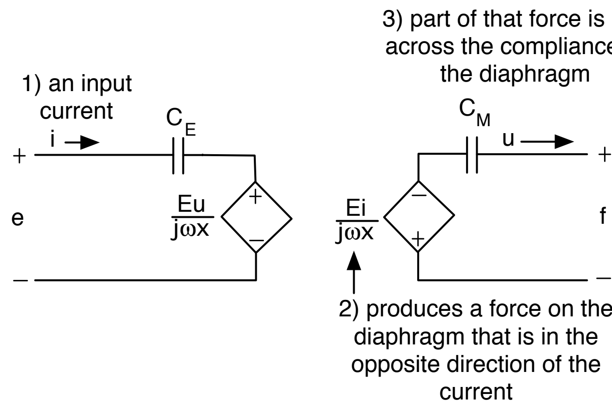
The complete model for the electrostatic transducer is:



So for the microphone version, you can think:

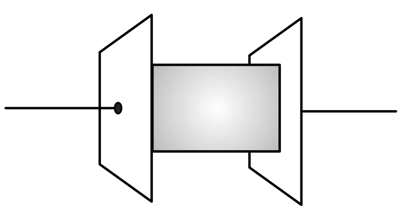


And for the loudspeaker version:



4.5 Fundamental Transducer Types: Piezoelectric Crystals

Piezoelectric crystals are crystalline structure materials which form a potential across them when deformed. Likewise, when a current is injected into the material, it deforms (twists, flexes or warps) in response. Therefore, these materials can be used as transducers. Piezo crystals have an upper and lower frequency limit; deformations outside their range do not result in a potential and vice versa. They have been relegated to buzzers and door-bells (loudspeaker version) and contact microphones for acoustic instruments or as MIDI triggers (microphone version). The familiar Quartz is a piezoelectric crystal but it has a very low output potential that requires a very large force to generate. Other materials are more efficient in that respect. They are usually disc shaped but sometimes are packaged in flexible rectangular strips.



The crystal is sandwiched between two metal plates that have a capacitance C_E :

When a current is applied across the crystal structure, a force is generated which is:

$$f = \frac{\tau}{j\omega} i - \frac{u}{j\omega C_M}$$

$\tau =$ crystal coupling coefficient

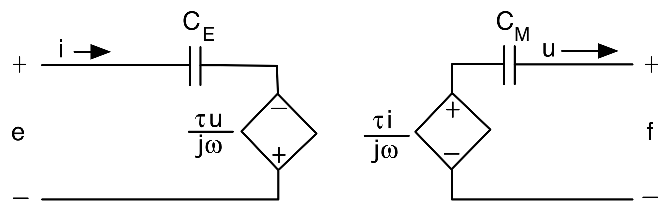
The first part is the force generated from the input current. It is in phase with the current and is directly proportional to the crystal coupling coefficient. The second component represents a force drop or loss across the mechanical compliance of the surface of the crystal.

When the crystal is deformed by a force f, it creates a velocity u in the material. The deformation velocity produces an output voltage due to the piezoelectric effect:

$$e = -\frac{\tau}{j\omega} u + \frac{1}{j\omega C_E} i$$

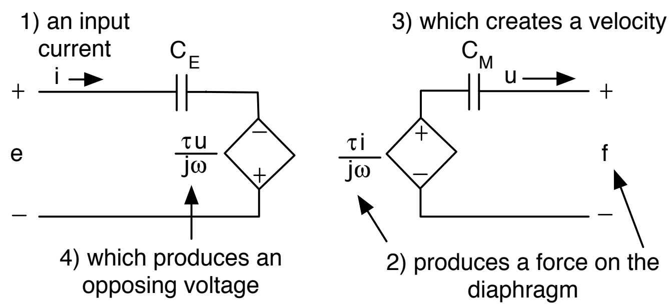
The first part is the voltage generated from velocity (u) of the crystal deformation. It is out of phase with the deformation and is directly proportional to the crystal coupling coefficient. The second component represents a voltage drop across the capacitance of the two plates.

The complete model is:



If you compare this to the moving coil mode. you will see that a similar back EMF effect is taking place (note the orientation of the two dependent sources).

You can think of it (in the loudspeaker version) like this:



So, of the three fundamental types, only the Electrostatic Transducer does not suffer from a backwards component, although its force direction is out of phase with the input current. On the other hand, only the moving coil transducer does not loose a significant amount of force across the compliance of the suspension system.

5 Microphone Models

Before we put the models together, we need to investigate a bit more exactly whats happening at the surface of the diaphragm.

The force across the diaphragm of a transducer is:

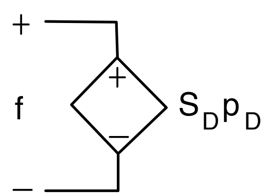
$$f_D = S_D p_D$$

where

$S_D =$ surface area of diaphragm

$p_D =$ pressure drop across diaphragm

$= p_{FRONT} - p_{REAR}$



This force is modeled as a voltage source:

The diaphragm itself is modeled as a Volume Velocity source where:

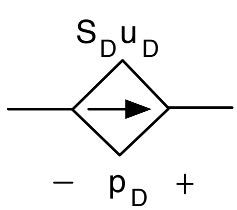
$$U_D = S_D u_D$$

where

$S_D =$ surface area of diaphragm

$u_D =$ velocity of diaphragm

The diaphragm can send Volume Velocity (driver) or receive it (microphone). The pressure drop is the voltage drop (pressure drop) across the diaphragm. In the models, the pressure across the diagram will be the sum of the voltages (pressures) in the circuit. The Volume Velocity source (diaphragm) will create some of the voltage drops as its current flows through various elements since p = UZ.



Modeling the Air Load

The air load on the outside of a microphone can be modeled with the same circuit as the piston at the end of a tube, given in Chapter 2. However, we still have the reflections off the diaphragm for high frequencies.

Looking at the plot of the frequency response of these reflections we see that the first rise looks just like a second order high shelving filter:

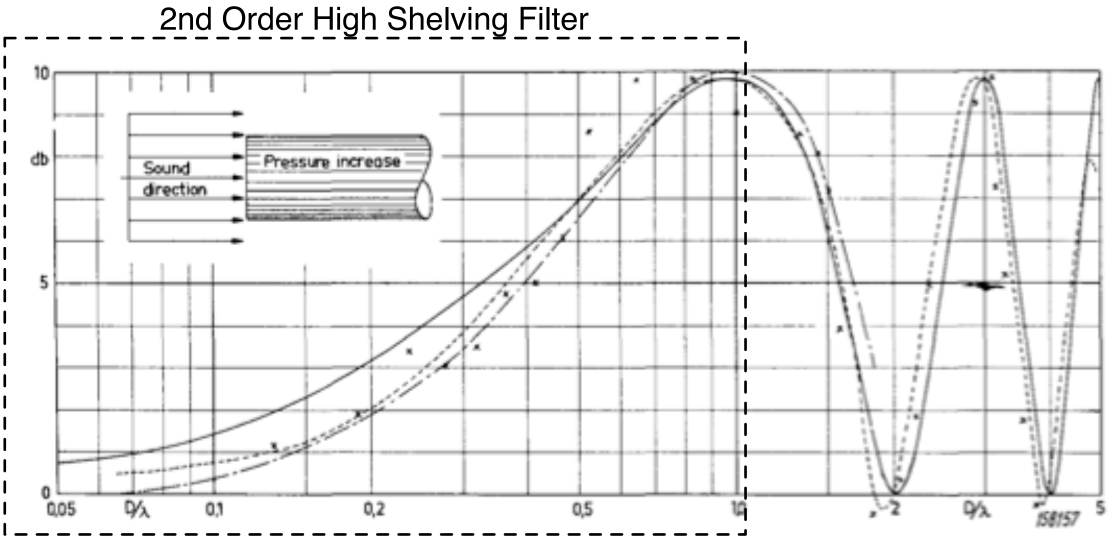
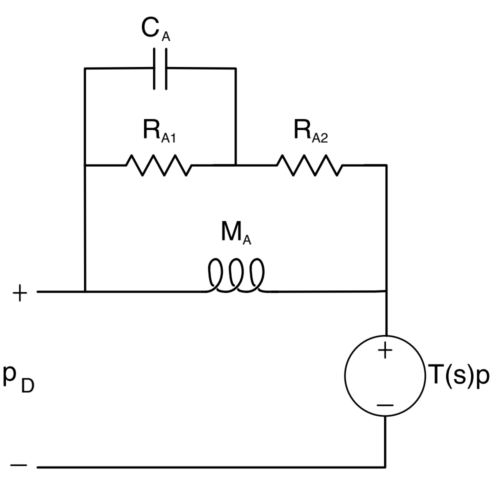


Fig. 3. Pressure increase at the axis of a cylinder the cylinder being placed in a free sound field. Curve drawn in full shows the theoretically calculated curve.

xxxxx Measured by Müller, Black, and Davis (1937)
—·—·— Measured by Danish Technical University (1948)
- - - - - - Measured by Brüel & Kjær (1958)

We can add a high shelving circuit to the original air load circuit by adding a pressure source (voltage source) that has a built-in 2nd order high shelving transfer function:



The circuit is turned on its side. The reflection source T(s)p is in phase with the pressure on the diaphragm so it adds or boosts the response. The transfer function T(s) is

$$T(s) = \frac{1 + b_1 s + b_2 s^2}{1 + c_1 s + c_2 s^2}$$

$$b_1 = (R_{A1} \parallel R_{A2})C_{A1} + \frac{M_{A1}}{(R_{A1} + R_{A2}) \parallel R_{A2}}$$

$$b_2 = \frac{2R_{A1}M_{A1}C_{A1}}{R_{A1} + R_{A2}}$$

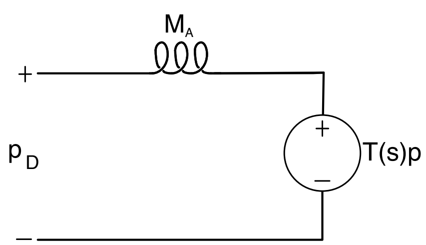
$$c_1 = (R_{A1} \parallel R_{A2})C_{A1} + \frac{M_{A1}}{(R_{A1} + R_{A2})}$$

$$c_2 = \frac{R_{A1}M_{A1}C_{A1}}{R_{A1} + R_{A2}}$$

The equations for the components are given in Chapter 2.

Low Frequency/High Frequency Approximation

At low frequencies, the dominant component is the acoustic mass, M_A whereas at high frequencies, the dominant component is the HF reflection source, T(s)p. An approximate circuit could therefore be constructed with just the two dominant components as:

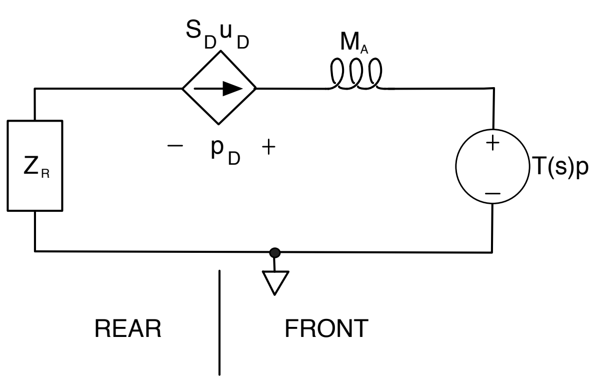


Basic Acoustic Model

So far, we've modeled the transducer's motor system and in this chapter we put it all together for complete models. The basic acoustic model is going to consist of:

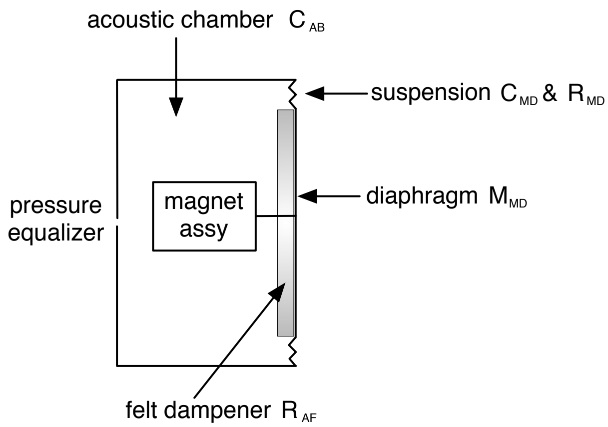
- the pressure drop across the diaphragm
- the air load on the front
- the air load on the rear (depends on enclosure)

So, the basic acoustic model will look like this:



5.1 Dynamic Microphone Model: Pressure

A Pressure microphone senses the instantaneous air pressure at the surface of the diaphragm. <u>All pressure microphones have an omni-directional pattern because they sense pressure equally from any direction.</u> This is done by mounting it in a sealed enclosure so that only the front side of the diaphragm is excited by the air pressure. A tiny hole is drilled in the enclosure to equalize pressure due to environmental location but it has no acoustic effect; its acoustic mass is so huge that it is an open circuit. The Pressure microphone senses pressure equally from all directions therefore its pickup pattern is Omnidirectional.

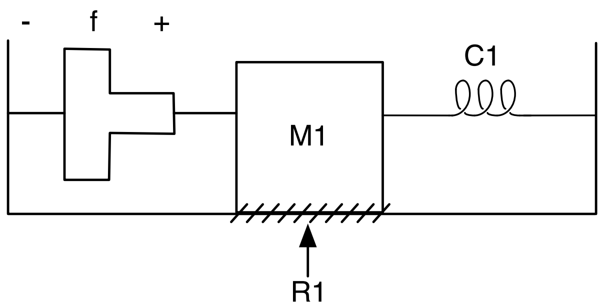


The Diaphragm has a mechanical mass M and the suspension provides both a compliance (springy restoring force) and a resistance within that spring.

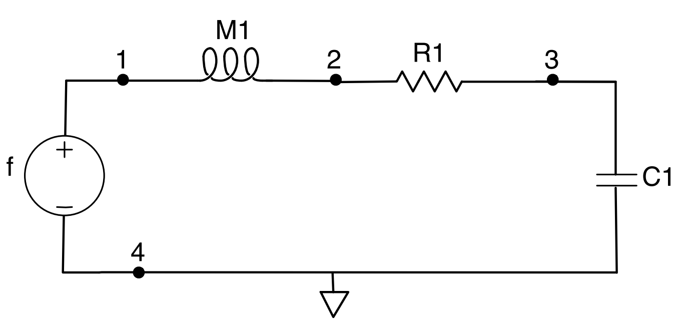
A felt dampener behind the diaphragm suppresses low frequency resonances and allows the designer to control the resonant hump (Q) in the low end response.

The acoustic chamber provides a compliance against which the mass of the diagram can resonate.

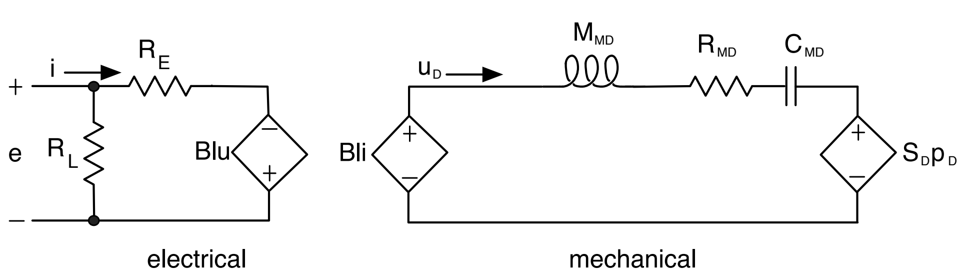
The Mechanical components consist of the diaphragm mass, compliance and resistance. The mechanical diagram is:



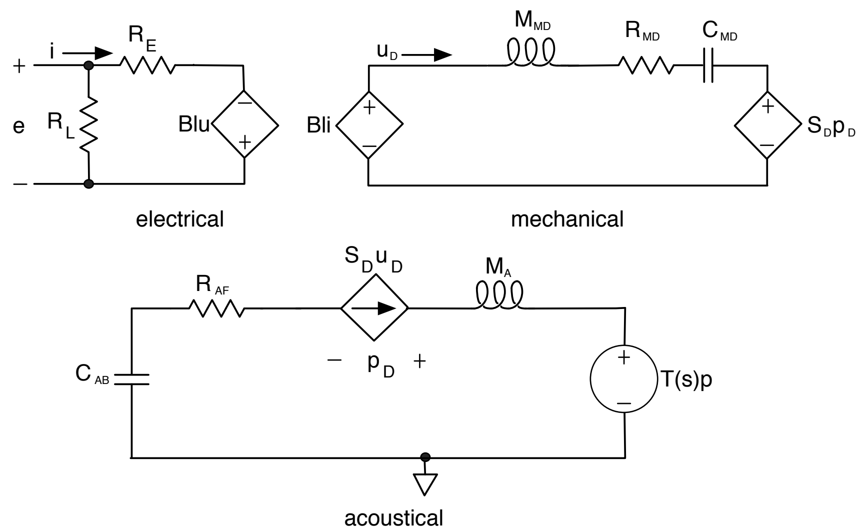
We have already modeled this system as an electric circuit. It turned out to be a Series LRC circuit.



So, we can put the first part of the model together like this:



We model the mechanical components as usual and the final force that appears at the surface of the diaphragm with its dependent source. We can add the acoustic circuit by observing that on the back side of the diaphragm we have a felt resistance and an acoustic compliance from the enclosure:



You can think of it like this: the pressure at the surface of the diaphragm produces a velocity in the mechanical circuit which controls a dependent voltage source in the electrical circuit which produces the final output voltage e(t) and current i(t).

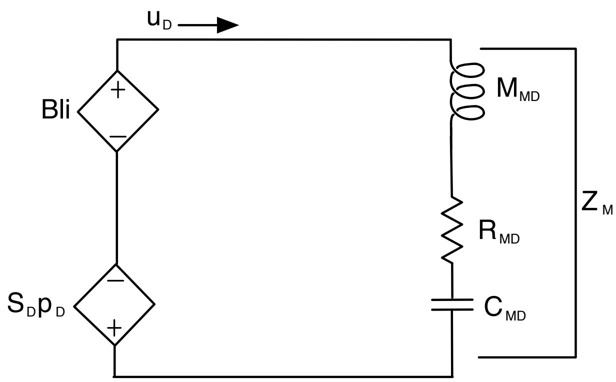
Solving for the final output voltage e(t) is fairly straight forward: it is going to be a voltage divider between the two resistances R_L and R_E. This requires solving 3 Ohm's Law equations for the 3 circuits. You need to find the impedances looking into the mechanical circuit (series RLC) and acoustic circuit (another series RLC).

For the electrical circuit, we can easily write two equations: one for the voltage divider which produces the output voltage e and the other for a current splitter:

$$e = \frac{-Blu_D R_L}{R_L + R_E}$$

$$i = \frac{-Blu_D}{R_L + R_E}$$

To fully solve the voltage divider equation, we also need to know the other two variables pressure and velocity. They come from a simple inspection of Ohm's Law for the other two.

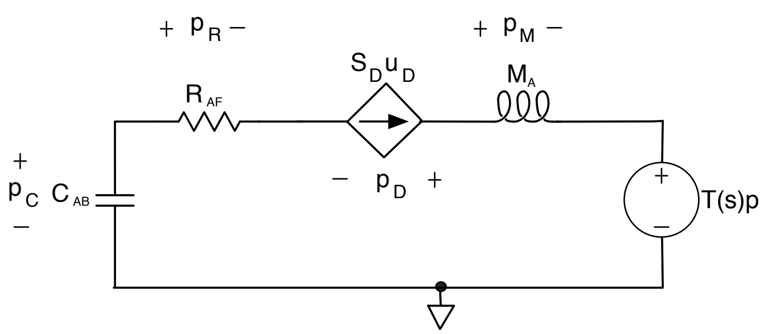


First, for velocity solve Ohm's Law for the circuit by rearranging and putting voltage sources on the left and the load on the right. The velocity will be V/Z.

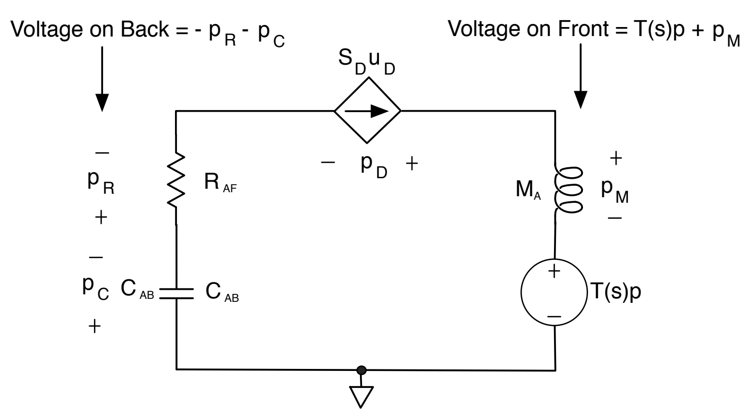
$$u_D = \frac{Bli - S_D p_D}{Z_M}$$

$$Z_M = j\omega M_{MD} + R_{MS} + \frac{1}{j\omega C_{MS}}$$

The pressure at the diaphragm is a sum of the pressure at the front and the pressure at the back. One way to look at this is that since the current flows in the same direction through each component, the voltage drop (pressure drop) will be the same polarity across each. Then, the total pressure is the sum of the incoming pressure (with reflection curve) and the pressure drops across each component, ZA:



Rearrange the circuit like this:



$$p_D = \text{ front pressure - rear pressure}$$
$$= T(s)p + S_D u_D (j\omega M_A) - \left[-S_D u_D R_{AF} - S_D u_D \frac{1}{jwC_{AB}} \right]$$
$$= T(s)p + S_D u_D (j\omega M_A) + S_D u_D R_{AF} + S_D u_D \frac{1}{jwC_{AB}}$$
$$= T(s)p + S_D u_D Z_A$$
$$Z_A = j\omega M_{A1} + R_{AF} + \frac{1}{j\omega C_{AB}}$$

With some some algebra, you can solve the original voltage divider equation to find the output voltage **e**:

$$e = \frac{-BlS_D R_L}{R_L + R_E} \frac{1}{Z_M + S_D^2 Z_A + \frac{(BL)^2}{R_L + R_E}} T(s)p$$

You can see the voltage divider equation in the first term with RL and RE. With more algebra you can fashion the equation into something more recognizable:

$$e = \frac{-BlS_D R_L}{R_L + R_E} \frac{1}{R_{MT}} \frac{(1/Q)(s/\omega_o)}{(s/\omega_o)^2 + (1/Q)(s/\omega_o) + 1} T(s)p$$

Examination of the third term reveals that this is a **2nd Order BPF transfer function**. (We did this in MMI401). The cutoff frequency and Q (bandwidth) are given by:

$$\omega_o = \frac{1}{2\pi\sqrt{M_{MT} C_{MT}}}$$
$$Q = \frac{1}{R_{MT}} \sqrt{\frac{M_{MT}}{C_{MT}}} = \frac{\omega_o}{bandwidth(\omega)} = \frac{f_o}{bandwidth(f)}$$
$$\omega_o = \sqrt{\omega_L \omega_H}$$
$$f_o = \sqrt{f_L f_H}$$

where

ω_L and f_L are the low frequency breakpoint of the BPF

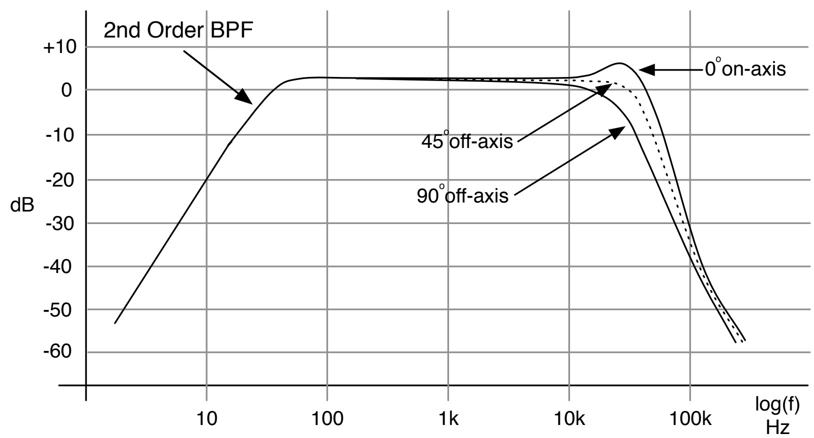
ω_H and f_H are the high frequency breakpoint of the BPF

These equations use combined component values that were created when the final algebra was done. They are:

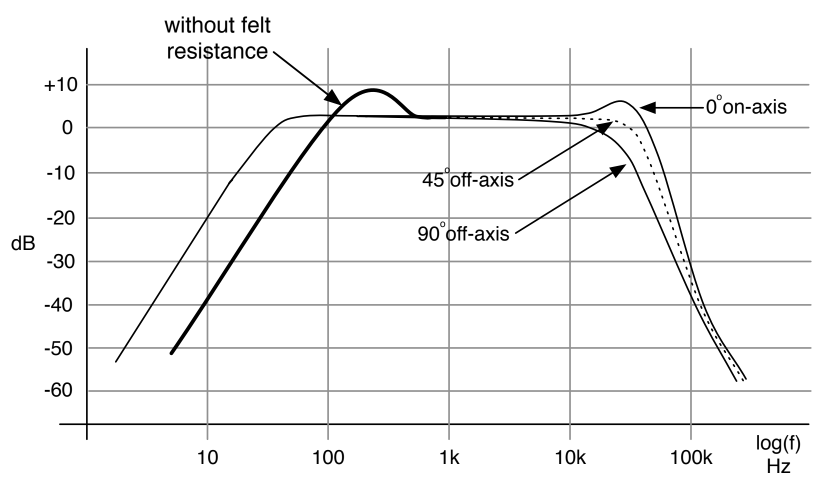
$$M_{MT} = M_{MD} + S_D^2 M_{A1}$$

$$R_{MT} = R_{MS} + S_D^2 R_{AF} + \frac{(Bl)^2}{R_L + R_E}$$

$$C_{MT} = \frac{1}{\frac{1}{C_{MD}} + \frac{S_D^2}{C_{AB}}}$$



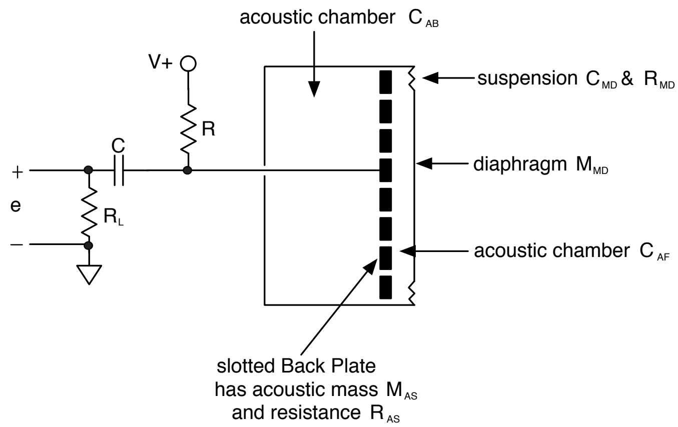
Therefore, the total predicted response is a bandpass filter shape (H(s)) multiplied by the HF Boost function (T(s)) which produces a bandpass plot with HF boost on axis. At 90 degrees off axis, the plot is theoretically a symmetrical BPF.



If the felt resistor is removed, a low frequency resonant hump appears. The felt resistance squashes the hump and flattens the response for more bass.

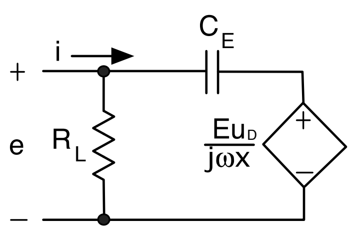
5.2 Condenser Microphone Model: Pressure

The Condenser Microphone uses the capacitor plates to produce the output voltage; as a pressure microphone it too will be housed in a sealed enclosure (again, we neglect the mass in the pressure equalization hole which also doubles as a wiring hole). The basic construction looks like this:

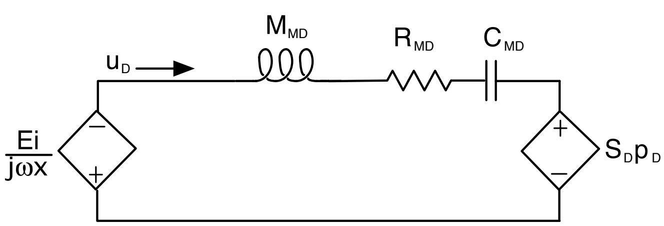


The slotted backplate is polarized with V+ through R; the voltage is removed by coupling cap C. The output voltage of the microphone is dropped across the input impedance of the microphone preamp, R_L. The diaphragm mass is very small compared to the dynamic so the felt resistor is not required. However, the back of the diaphragm feels the acoustic mass and resistance of the air in the slots along with two compliances from the air chambers.

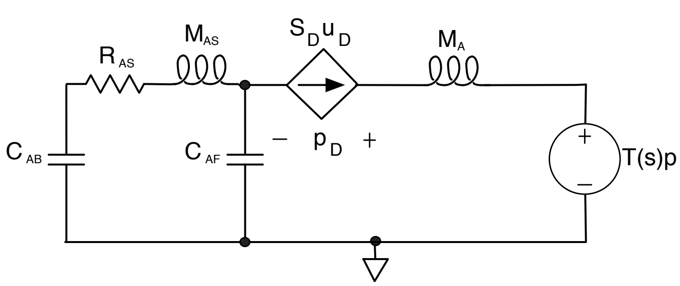
By combining the models from the last 2 chapters, we can fashion the complete condenser model as follows:



Electrical: consists of the velocity-dependent voltage source, the capacitance of the capsule, and the load impedance.



Mechanical: essentially identical to the moving coil version because of same mass, compliance and resistance in the diaphragm although the values are much different.



Acoustical: the front side is the same as moving coil with the low frequency approximation of the air load (Ma) and the reflected pressure. To figure out the back side, use the acoustic circuit equivalents tracing your way from the back surface of the diaphragm, through the first capacitance, then through the slots into the second capacitance.

You can see that the mechanical circuit produces the same BPF shape for the volume, u. The acoustical circuit has a resonant filter built into the back-side where the resistance of air in the slots tunes the Q. By forming the similar Ohm's Law equations for these three circuits, we can also obtain the output voltage as a transfer function equation. It is:

$$e = -\frac{EC_{MT}S_D}{x}\frac{1}{(s/\omega_o)^2 + 1/Q(s/\omega_o) + 1}T(s)p$$

where E is the polarizing voltage and the other variables are given by:

$$C_{MT} = \frac{1}{1/C_{MD} + S_D^2/C_{AB}}$$
$$M_{MT} = M_{MD} + S_D^2(M_A + M_{AS})$$
$$R_{MT} = R_{MD} + S_D^2 R_{AS}$$

Observation of the second term yields a 2nd Order Lowpass Filter. The manufacturer tunes the Q with slot size and the cutoff frequency with both the total mass and compliance factors. The equations for the cutoff and Q are:

$$\omega_o = \frac{1}{2\pi\sqrt{M_{MT}C_{MT}}}$$
$$Q = \frac{1}{R_{MT}}\sqrt{\frac{M_{MT}}{C_{MT}}}$$

The resonant hump in the output response has the following properties:

$$\omega_{peak} = \frac{\omega_o}{Q}\sqrt{Q^2 - 0.5}$$
$$\omega_{-3} = \left(1 + \left[\beta + \sqrt{\beta^2 + 1}\right]^{1/2}\right)\omega_o$$
$$\beta = \frac{1}{2Q^2} - 1$$
$$|PEAK| = \frac{Q^2}{\sqrt{Q^2 - 0.25}}$$
$$PEAK(dB) = 20\log(PEAK)$$

The microphone’s component values can be found with:

$$M_{MD} = \frac{4}{3}\pi a^2 t_D \rho_D$$
$t_D =$ diaphragm thickness

$p_D =$ diaphragm density

$$C_{MD} = \frac{1}{8\pi T}$$

$T =$ tension on diaphragm in N/m

$$C_E = \frac{\varepsilon_o \pi a^2}{x}$$

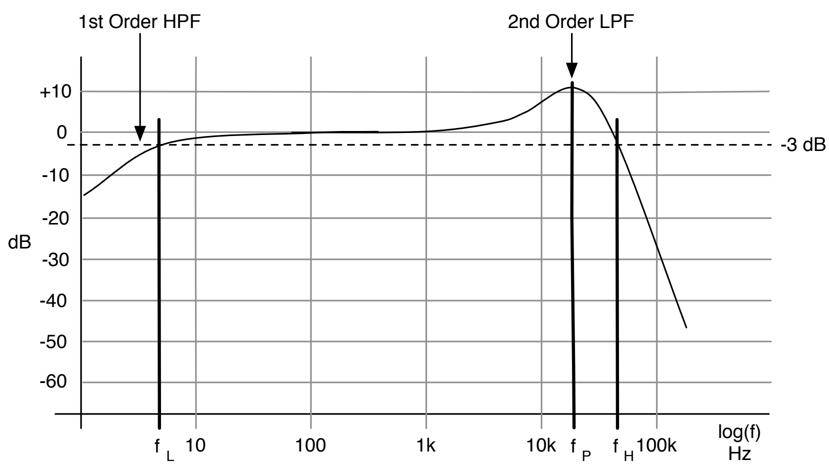
The capacitance is in the range of 40-80pF or so.

The microphone's capacitance combines with the load impedance to produce a 1st Order High-pass Filter which ultimately controls the low frequency rolloff point at:

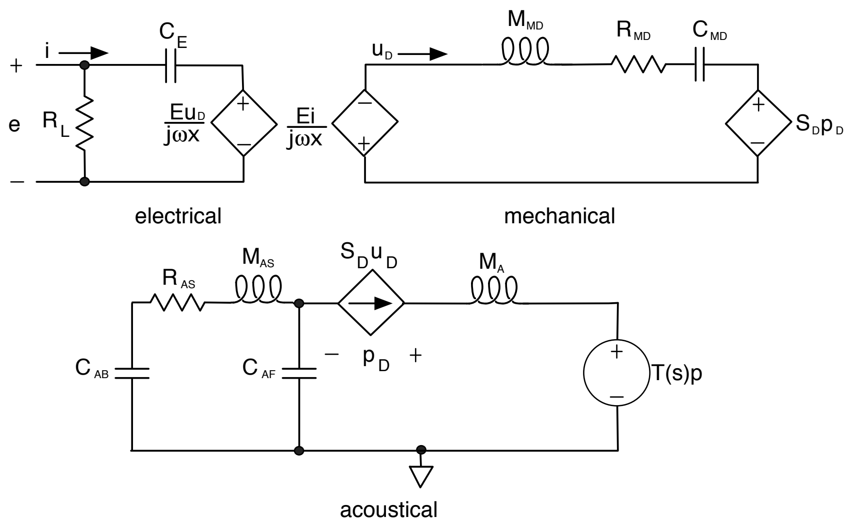
$$f_L = \frac{1}{2\pi R_L C_E}$$

Because of the value of the capacitance, the load resistance must be very high for good LF performance, but most low impedance microphone preamps only have about 1.5k to 2k input impedance. Usually, these mics have a built-in high-impedance buffer (powered off the phantom power supply, E) to produce the impedance necessary.

A typical Omni-condenser microphone might have a frequency response like this:



Complete Circuit:



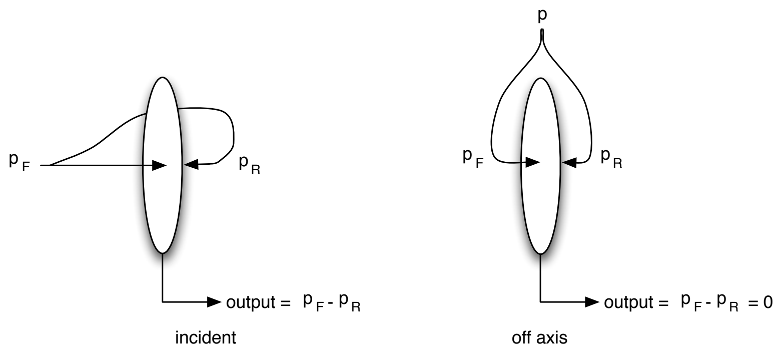
Critical Polarizing Voltage

You can see that the output e(t) is proportional to the polarizing voltage E (in the second term). It seems that the larger we make this voltage, the larger our output could be. However, if the voltage is too large, it will pull the diaphragm all the way back to the back plate causing a short circuit with arcing which will destroy the device. The voltage where this occurs is called the Critical Polarizing Voltage and is given by:

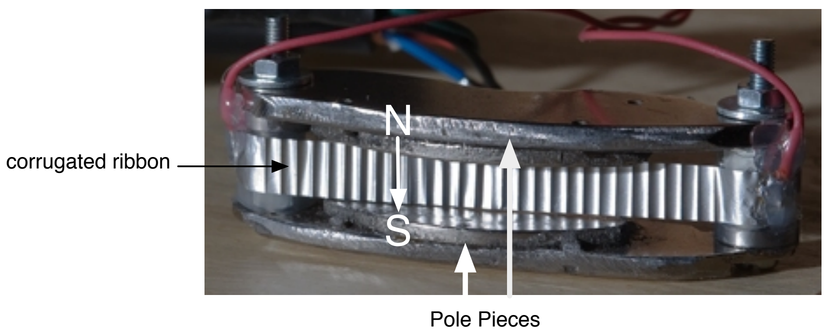
$$E_{CRIT} = \frac{x}{\sqrt{C_E C_{MT}}}$$

5.3 Ribbon Microphone Model: Velocity or Pressure Gradient

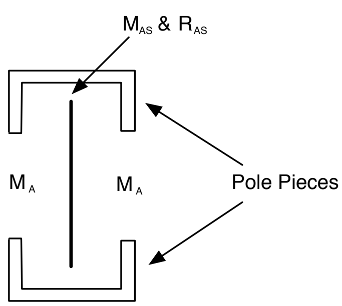
The Ribbon Microphone lends itself easily as a Velocity (aka Pressure Gradient) sensing microphone. A Velocity microphone senses air particle velocity by having both the front and back sides of the diaphragm fully exposed to the oncoming pressure. The velocity is proportional to the pressure difference, Δp between the two sides. <u>All velocity microphones have a figure 8 pattern because the pressure difference at 90 degrees off-axis is 0 - the pressure is the same on each side.</u>



The ribbon is a piece of corrugated metal suspended in a magnetic field. A ribbon physically acts like a single coil (or a few coils) electronically, so it is actually a type of Moving Coil microphone and has the same electric and mechanical models; only the acoustic model is different.

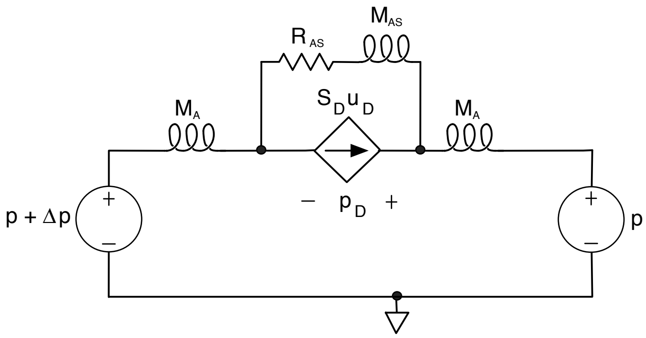


The Ribbon Mic consists of the Ribbon and two pole pieces. The ribbon is suspended between the pole pieces with End-Clamps.



A Top-View of the system reveals that the ribbon is sitting in a slot in the pole pieces. The slot provides an acoustic mass and a resistance too.

The acoustic circuit has to take this into account along with the fact that both sides of the diaphragm feel the normal air load - there are no acoustic cavities or tubes.



In the Acoustic model for the Ribbon Mic, you can see the two air loads (simplified down to just the acoustic inductance) with the combined air masses and resistances that couple the front and rear of the diaphragm.

Because the ribbon microphone senses velocity, it does not include the reflected pressure-sources that the pressure-microphones have in their models. The pressure on the back side is equal to the front pressure plus the change in pressure, front to back.

The directivity of the microphone can be found to be related to the angle of incidence and the distance between the front and back of the diaphragm:

$$e_{OUT} \propto \cos\Theta\Delta l$$
$$\Theta = \text{ angle of incidence}$$
$$\Delta l = \text{ distance from front to back (thickness of diaphragm)}$$

You can solve for the output voltage **e** in exactly the same way as the moving coil microphone using the Ohm's Law equations from the three circuits. After some algebra, you can find that:

$$e = \frac{BlS_D R_L}{cM_{MT}}(R_L + R_E)\frac{\left(s/\omega_o\right)^2}{\left(s/\omega_o\right)^2 + \left(1/Q\right)\left(s/\omega_o\right) + 1}\,p\cos\Theta\Delta l$$

Examination of the second term reveals that this is a 2nd Order High Pass filter. The term at the end is the directivity portion. The 2nd Order HPF will have a cutoff and Q of:

$$\omega_o = \frac{1}{2\pi\sqrt{M_{MT}C_{MR}}}$$
$$Q = \frac{1}{R_{MT}}\sqrt{\frac{M_{MT}}{C_{MR}}}$$
$$M_{MT} = M_{MD} + 2S_D M_A$$
$$C_{MT} = \text{ varies due to ribbon geometry}$$
$$\mathrm{R}_{MT} = R_{MD} + \frac{\left(Bl\right)^2}{R_L + R_E}$$

The equations that relate the 3dB and peak frequency for this HPF are:

$$\omega_{peak} = \frac{Q\omega_o}{\sqrt{Q^2 - 0.5}}$$

$$\omega_{-3} = \left[\beta + \sqrt{\beta^2 + 1}\right]^{1/2} \omega_o$$

$$\beta = \frac{1}{2Q^2} - 1$$

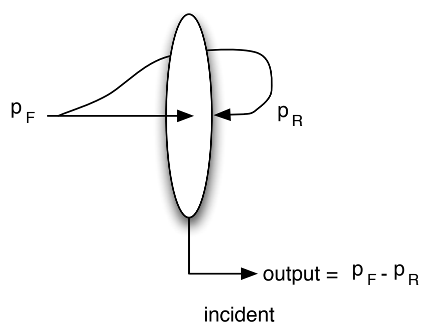
$$|PEAK| = \frac{Q^2}{\sqrt{Q^2 - 0.25}}$$

$$PEAK(dB) = 20\log(PEAK)$$

Note these are similar but not identical to the equations for a 2nd Order LPF from the condenser version.

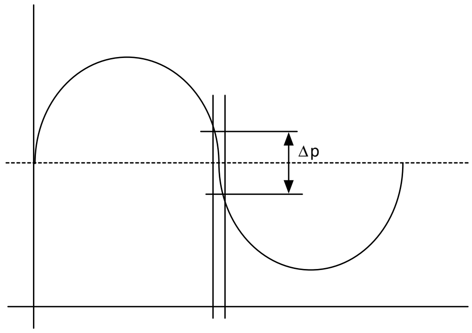
5.4 Proximity Effect

Velocity microphones suffer from a bass-boosting problem called the Proximity Effect. It should be noted that many do not see this as a problem as it can be used to deepen the sound of a vocalist or instrument, however from a system design standpoint (trying to get a microphone with a flat response) it is an issue. The Proximity Effect is a bass boost that occurs when the microphone is very close to the source. Ironically, it is actually a treble boost that the manufacturer has accommodated for to make the response as flat as possible.

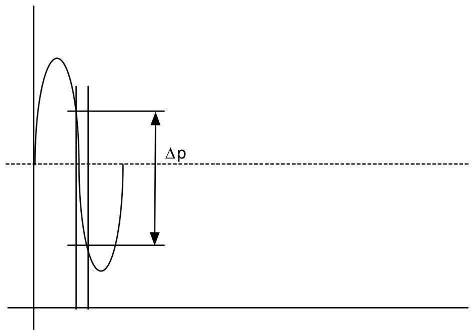


To understand where it comes from remember the plot showing the two paths to the front and back of the diaphragm.

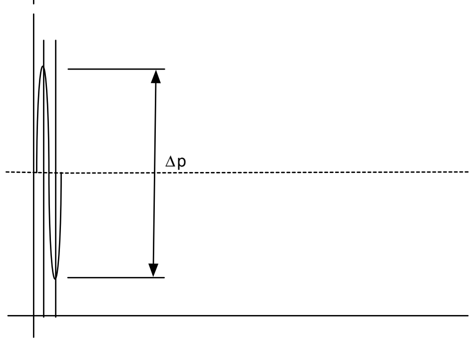
Suppose the longer path around the back added an extra 10 mm of travel distance. That path would be delayed an extra 10 mm's worth of wavelength. What does that 10 mm distance look like for sinusoids?



For any frequency, the worst case scenario is if the delay crosses the transition point. For the low frequency at the left, this results in some difference in pressure, Δp.



However if the frequency is increased, the same distance interval results in a larger Δp.



When the delay time equals the wavelength, you get the maximum Δp.

Above this frequency, comb filtering will occur as the pressure difference begins to fall out of phase with the signal. The comb filtering affects the HF portion of the output only.

These plots show that the pressure gradient is going to rise as the frequency rises - we can't escape the geometry. These plots are also from the far field where the distance creates a delay effect only. This means the amplitude of the signal on the back and front is about the same because the path distances are about the same. But the phase is delayed by the path distance giving the treble boost. Manufacturers build a bass boost into the system in the output transformer circuit. [1] http://artsites.ucsc.edu This flattens the response out to the beginning of comb filtering.

However, as the microphone is moved very close to the source the distances from the front and back become much larger with respect to one another. Because of the inverse square law, the pressure on the front will be much larger than the pressure on the back, thus the Δp will be large. This will put a boost on the bass frequencies. As the source halves the distance in the near field, the dBSPL will go up by 12dB. Now the flat response in the far field has an added boost in the near field. Many microphones have built in circuits to try to remove the proximity effect altogether; the circuits might be electronic or acoustic.

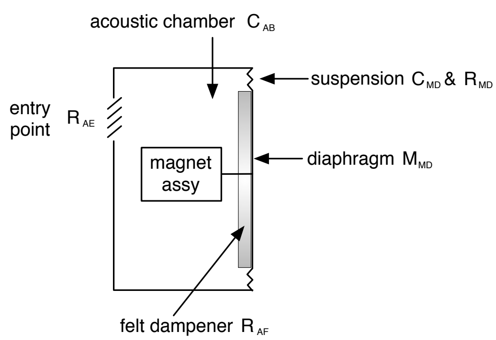
5.5 Combination Pressure & Velocity Microphones

The two types can be combined resulting in "Combination" microphones. This can be done one of several ways:

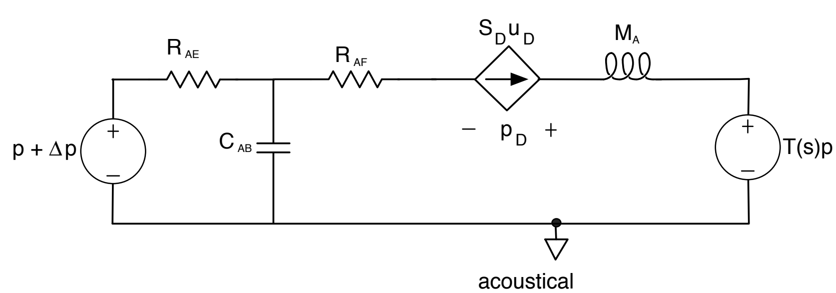
- mathematically & electronically combine the outputs of a pressure capsule and a velocity capsule
- mathematically & electronically combine the outputs of back-to-back capsules (either type)
- build an enclosure that combines features of both types

The first method is not popular. The second one is popular and results in multi-pattern microphones. The details of these types are in the circuits that combine the outputs.

The third method might be the most common because of the numerous vocal and instrument mics that employ it. This involves mounting a diaphragm in an enclosure but allowing entry points along the sides and/or back to couple air pressure from these areas to the back side of the diaphragm. The distance from the entry points to the back of the diaphragm will determine the frequency band it operates over (rising up until comb filtering starts to occur). The enclosure itself will determine how much of the back-side pressure is allowed to reach the diaphragm in addition to providing mass and compliance - and therefore filtering - operation. The number and location of the entry points varies with model and manufacturer. Typically, the entry points are along a ring on the outside of the tube. Sometimes, they are along straight lines too. The entry points that all result in the same distance to the diaphragm (aligned in a ring around the mic tube) can be combined together into one. The entry point will be covered with an acoustical resistance material - foam. If the capsule (tube) is thick enough, it might also be an acoustic mass trap. A simple combination moving coil microphone might look like this:



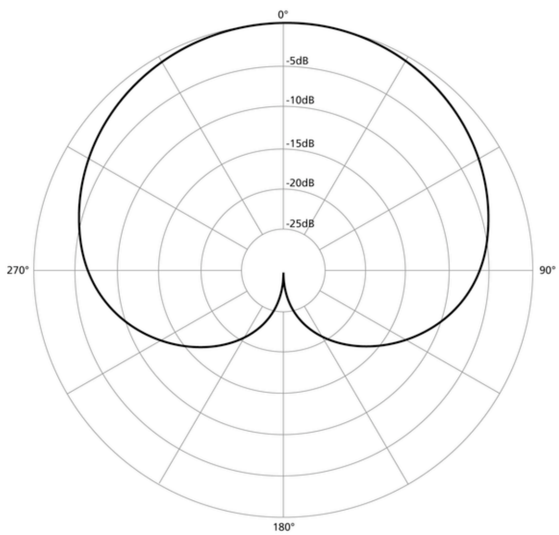
This will add a resistor to the backside of the enclosure's model and will couple to a pressure source of value p + Δ p due to the path difference to the back of the diaphragm.



The other models remain unchanged. However, the derivations of transfer functions for both moving coil and condenser microphones is difficult to factor into a form we can recognize like the 1st and 2nd Order LPF, HPF and BPF circuit from the previous work. Adding more entry points laterally down the tube will add more components and sources so the math can be difficult. However, computer simulations are always an option so it is important that you could use the skills you have to model other types of capsules.
The combination microphones also suffer from the Proximity Effect, though to a lesser extent, and like their velocity counterparts often have circuits to compensate (e.g. MD421).

If you only look to solve the pressure drop across the diaphragm, an interesting component occurs that explains the polar patterns of these microphones:

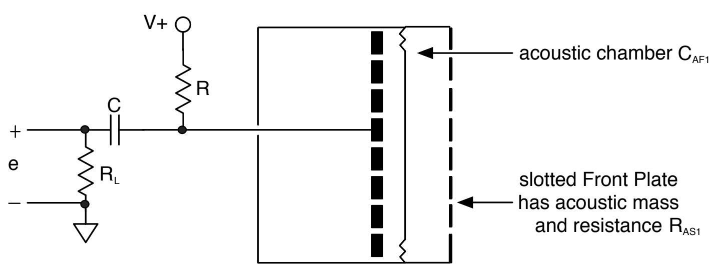
$$p_D = p\frac{j\omega R_A C_{AB}}{1+j\omega R_A C_{AB}}\left(1+B\cos\Theta\right)+S_D u_D\left(j\omega M_{A1}+\frac{R_A}{1+j\omega R_A C_{AB}}\right)$$



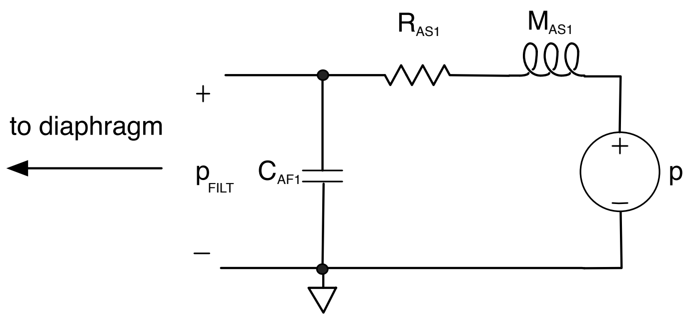
The interesting part is the 1+Bcos(Θ) which is the directivity portion arising from the partial velocity nature of the capsule. A polar plot of this little function reveals the Cardiod shape often associated with these microphones. In fact, placement of the entry points and other acoustical circuits allow for all kinds of variations from cardiod to hyper-cardiod to super-cardiod.

5.6 Acoustic Capsule Filters

One last enhancement comes in the form of a snap-on or screw-on acoustic filter that connects to the front of the capsule. Some microphones may have an assortment of them with different shape and number of holes or slots cut into the front and of varying lengths. These acoustic filters can be added to further shape the frequency response (check the documentation for your microphones). Often they will flatten the HF response or emphasize it or perform some other HF filtering.



Consider the Condenser microphone model with one of these filters connected to it. Slots or holes in the front plate along with the compliance of the chamber form an acoustic filter. Tracing the filter from the incoming pressure source to the front of the diaphragm results in the following circuit:



You should be able to tell by now that this is a 2nd Order LPF. It has the familiar equations:

$$\omega_o = \frac{1}{2\pi\sqrt{M_{AS1}C_{AF1}}}$$

$$Q = \frac{1}{R_{AS1}}\sqrt{\frac{M_{AS1}}{C_{AF1}}}$$

By adjusting the slots and tube length, the manufacturer can flatten or enhance resonant peaks or even perform a non-resonant LPF (Q < 0.707) to further tailor the HF response.

Homework:

An example application of the circuits for the condenser microphone is given in this section. The assumed specifications are: aluminum diaphragm material of density $\rho_1 = 2700\,\text{kg/m}^3$, diaphragm, electrode, and front air cavity radii $a = 1\,\text{cm}$, diaphragm thickness $t = 40\,\mu\text{m}$, diaphragm to back plate spacing $x_0 = 40\,\mu\text{m}$, diaphragm tension $T = 2 \times 10^4\,\text{N/m}$, back air cavity volume equal to 100 times front air cavity volume, polarizing voltage $E = 300\,\text{V}$, and total quality factor $Q = 1$ at the fundamental resonance frequency.

1. If we assume that the diaphragm vibrates in its fundamental mode, its mechanical mass M_{MD} and compliance C_{MD} are given by

$$M_{MD} = \frac{4}{3}\pi a^2 t\rho_1 = 4.52 \times 10^{-5}\,\text{kg} \qquad C_{MD} = \frac{1}{8\pi T} = 1.99 \times 10^{-6}\,\text{m/N}$$

The acoustic compliance of the front cavity is given by

$$C_{AB1} = \frac{V_{AF}}{\rho_0 c^2} = 8.95 \times 10^{-14}\,\text{m}^5/\text{N}$$

The compliance of the back cavity is $C_{AB2} = 100C_{AB1} = 8.95 \times 10^{-12}\,\text{m}^5/\text{N}$. A diaphragm mechanical damping resistance $R_{MD} = 0.178\,\text{N s/m}$ is assumed. To set the quality factor at $Q = 1$, the screen perforations in the back plate must have an acoustic resistance $R_{AS} = 5.22 \times 10^7\,\text{N s/m}^5$. We will take the acoustic mass of the screen perforations to be $M_{AS} = 132\,\text{kg/m}^4$. The electrical capacitance is given by

$$C_{E0} = \frac{\epsilon_0 \pi a^2}{x_0} = 69.5\,\text{pF}$$

An effective load resistance of $R'_L = 20\,\text{M}\Omega$ is assumed for the simulation.

1) for the Condenser mic above, (a) find the critical polarizing voltage [3,440V] and (b) calculate the fundamental resonant frequency [14.7kHz] HINT: use the equation for $M_A = M_{Aq}$ for impedance of air on a piston at the end of a long tube).

2) A pressure microphone has a circular diaphragm. At 10kHz, use the equation on Page 2 T(s) to determine the theoretical increase in response due to reflections for a diaphragm diameter of (a) 1/2 inch [3.37dB] and (b) 1 inch [5.54dB]. HINT: you need to find the magnitude of T(s) at f = 10kHz. This requires complex algebra similar to what we did in MMI401.

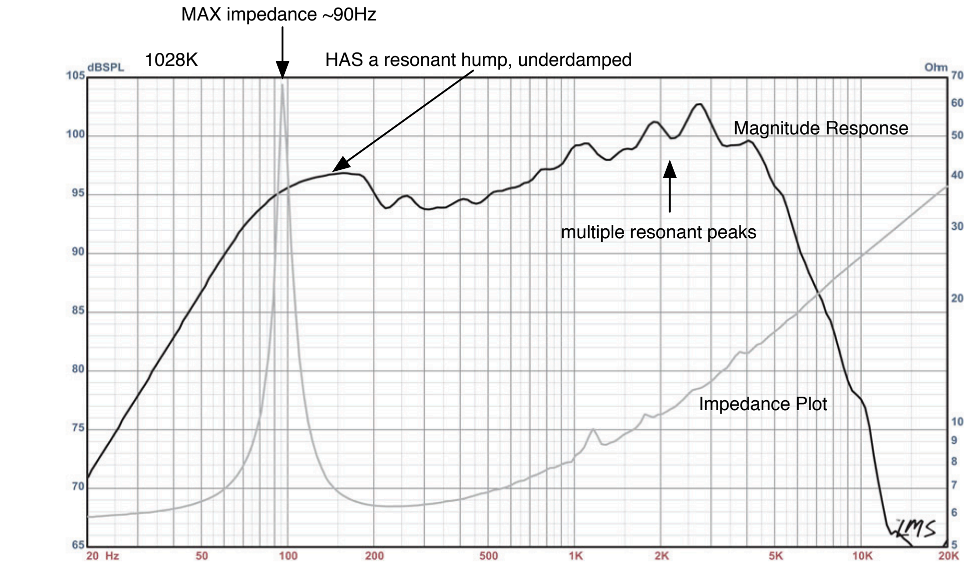
3) A dynamic microphone is to be designed for these specifications: diaphragm diameter = 1/2 inch, lower cutoff f_L = 30Hz, upper cutoff f_H = 8kHz. (a) calculate the fundamental resonant frequency and Q [490Hz, 0.0615] and (b) sketch the frequency response of the microphone for both normal and parallel (90 deg) incidence.

6 Moving Coil Driver Modeling & The Infinite Baffle Enclosure

First have a look at the two different driver data-sheets on the next two pages. The first is an Eminence Legend 1028K 10" driver. The second is an Eminence LAB12 12" driver. You should be able to recognize the following Thiele-Small Parameters:

- BL Product (BL)
- Surface Area of Cone (Sd)
- Maximum Linear Excursion (Xmax)

Now it is time to learn the rest of these parameters. Before we start, let's make some observations about the different graphs we see on the two specification sheets:



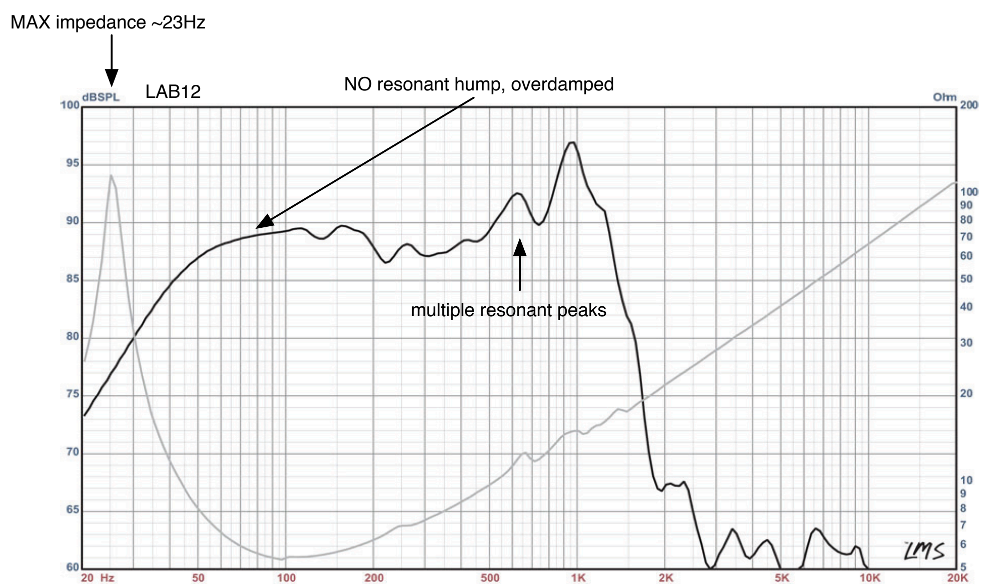


Figure 6.1: The Magnitude (dark line) and Impedance (light line) Responses plotted against Frequency. The Magnitude Response is also called the Frequency Response. The Impedance values (ohms) are on the right Y-Axis while the Magnitude values (dB) are on the left

EMINENCE®

The Art and Science of Sound

Specification

Nominal Basket Diameter	10", 254mm
Nominal Impedance*	8 ohms
Power Rating**	
Watts	35W
Music Program	N/A
Resonance	95Hz
Usable Frequency Range***	100Hz-5.5kHz
Sensitivity	97.4
Magnet Weight	6 oz
Gap Height	0.25", 6.2mm
Voice Coil Diameter	1", 25.4mm

Thiele & Small Parameters

Resonant Frequency (fs)	95Hz
DC Resistance (Re)	5.8
Coil Inductance (Le)	0.51mH
Mechanical Q (Qms)	17.96
Electromagnetic Q (Qes)	1.61
Total Q (Qts)	1.47
Compliance Equivalent Volume (Vas)	31.5 ltr/1.1 cu. ft.
Peak Diaphragm Displacement Volume (Vd)	72cc
Mechanical Compliance of Suspension (Cms)	0.19mm/N
BL Product (BL)	5.7 T-M
Diaphragm Mass inc. Airload (Mms)	15 grams
Efficiency Bandwidth Product (EBP)	59
Maximum Linear Excursion (Xmax)	2.1mm
Surface Area of Cone (Sd)	344.9cm²
Maximum Mechanical Limit (Xlim)	

Mounting Information

Recommended Enclosure Volume	
Sealed	Acceptable
Vented	Acceptable
Overall Diameter	10.11", 256.8mm
Baffle Hole Diameter	9.05", 229.9mm
Front Sealing Gasket	Fitted as Standard
Rear Sealing Gasket	
Mounting Holes Diameter	0.22", 5.6mm
Mounting Holes B.C.D.	9.6", 243.8mm
Depth	4.8", 122mm
Net Weight	2.9 lbs 1.3 kg
Shipping Weight	4 lbs 1.8 kg

Materials of Construction

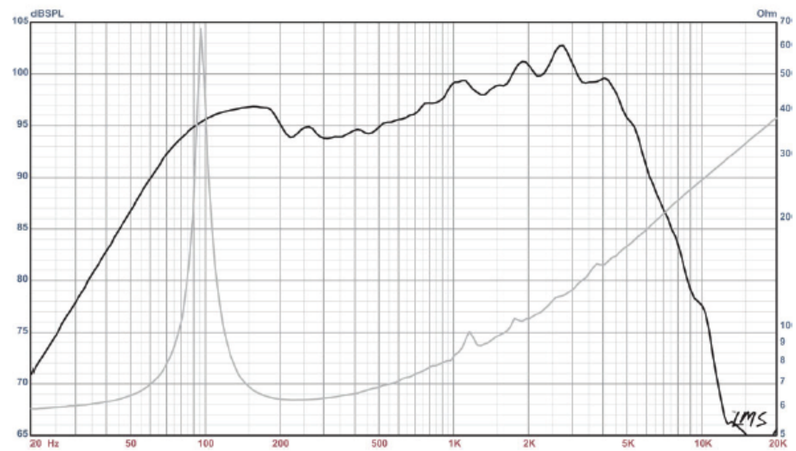
Coil Construction	Copper
Coil	Polyimide
Magnet Composition	Alnico
Core Details	Non-Vented
Basket Materials	Pressed Steel
Cone Composition	Paper
Cone Edge Composition	Paper
Dust Cap Composition	Solid Composition Felt

LEGEND 1028K

Vintage alnico and seamed cone tonality for guitar. Ideal Vintage alnico Jensen replacement.

Coloration: A very bluesy speaker with lots of sparkle, definition, and percussive characteristics.

Genre: Blues, Country, Rock.



* Please inquire about alternative impedances.

** Multiple units exceed published rating evaluated under EIA 426A noise source and test standard while in a free-air, nontemperature-controlled environment.

*** The average output across the usable frequency range when applying 1W/1m into the nominal impedance. Ie: 2.83 V/8 ohms, 4 V/16 ohms.

Eminence response curves are measured under the following conditions: All speakers are tested at 1W/1m using a variety of test set-ups for the appropriate impedance | LMS using 0.25" supplied microphone (software calibrated) mounted 1m from wall/baffle | 2 ft. X 2 ft. baffle is built into the wall with the speaker mounted flush against a steel ring for minimum diffraction | Hafler P1500 Trans-Nova amplifier | 2700 cu.ft. chamber with fiberglass on all six surfaces (three with custom-made wedges)

Specification

Nominal Basket Diameter	12", 304.8mm
Nominal Impedance*	6 ohms
Power Rating**	
Watts	400W
Music Program	800W
Resonance	22Hz
Usable Frequency Range***	25Hz-125Hz
Sensitivity	89.2
Magnet Weight	160 oz
Gap Height	0.375", 9.53mm
Voice Coil Diameter	2.5", 63.5mm

Thiele & Small Parameters

Resonant Frequency (fs)	22Hz
DC Resistance (Re)	4.29
Coil Inductance (Le)	1.48mH
Mechanical Q (Qms)	13.32
Electromagnetic Q (Qes)	0.39
Total Q (Qts)	0.38
Compliance Equivalent Volume (Vas)	125.2 ltr/4.4 cu. ft.
Peak Diaphragm Displacement Volume (Vd)	659cc
Mechanical Compliance of Suspension (Cms)	0.35mm/N
BL Product (BL)	15.0 T-M
Diaphragm Mass inc. Airload (Mms)	146 grams
Efficiency Bandwidth Product (EBP)	56
Maximum Linear Excursion (Xmax)	13.0mm
Surface Area of Cone (Sd)	506.7cm²
Maximum Mechanical Limit (Xlim)	22mm

Mounting Information

Recommended Enclosure Volume	
Sealed	22.7-28.3 ltr/0.8-1 cu. ft.
Vented	45.3-101.9 ltr/1.6-3.6 cu. ft.
Overall Diameter	12.32", 312.8mm
Baffle Hole Diameter	10.98", 278.9mm
Front Sealing Gasket	Fitted as Standard
Rear Sealing Gasket	Fitted as Standard
Mounting Holes Diameter	0.26", 6.6mm
Mounting Holes B.C.D.	11.77", 299mm
Depth	6.44", 164mm
Net Weight	22 lbs, 10 kg
Shipping Weight	23.8 lbs, 10.8 kg

Materials of Construction

Coil Construction	Copper
Coil	Polyimide
Magnet Composition	Double Stacked 80 oz Ferrites
Core Details	Vented And Extended
Basket Materials	12-Spoke Die-Cast Aluminum
Cone Composition	Kevlar-Reinforced Paper
Cone Edge Composition	Foam
Dust Cap Composition	Dual Inverteds

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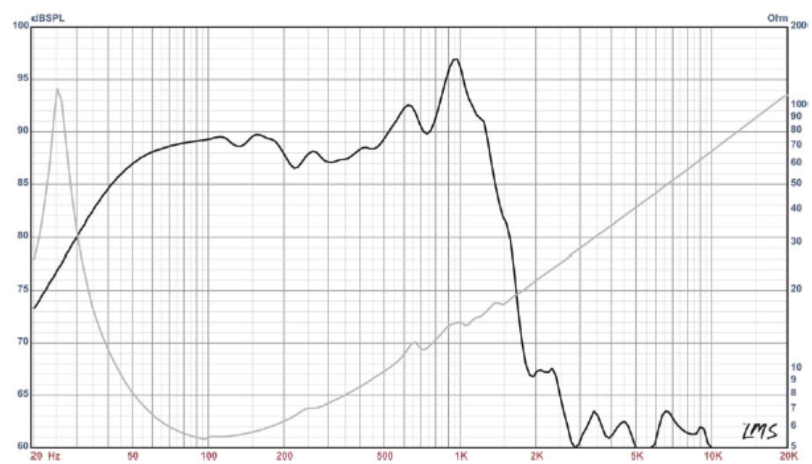


* Please inquire about alternative impedances.

** Multiple units exceed published rating evaluated under EIA 426A noise source and test standard while in a free-air, nontemperature-controlled environment.

*** The average output across the usable frequency range when applying 1W/1m into the nominal impedance. ie: 2.83 V/8 ohms, 4 V/16 ohms.

Eminence response curves are measured under the following conditions: All speakers are tested at 1W/1m using a variety of test set-ups for the appropriate impedance | LMS using 0.25" supplied microphone (software calibrated) mounted 1m from wall/baffle | 2 ft. X 2 ft. baffle is built into the wall with the speaker mounted flush against a steel ring for minimum diffraction | Hafler P1500 Trans-Nova amplifier | 2700 cu.ft. chamber with fiberglass on all six surfaces (three with custom-made wedges)

In the 1028K, we can see a low frequency resonant hump whereas there is no LF resonance in the LAB12. Both responses show multiple resonant peaks and valleys that occur before the upper band edge rolls off significantly. In the Impedance responses, we observe spikes at different frequencies; the 1028K around 95Hz and the LAB12 around 23Hz. NOTE: you may also see pronounced anti-resonances (dips). In this chapter, you will find out where these traits come from.

6.1 Electromagnetic Drivers

Let's investigate electromagnetic transducers in more detail by close inspection of a typical moving coil driver.

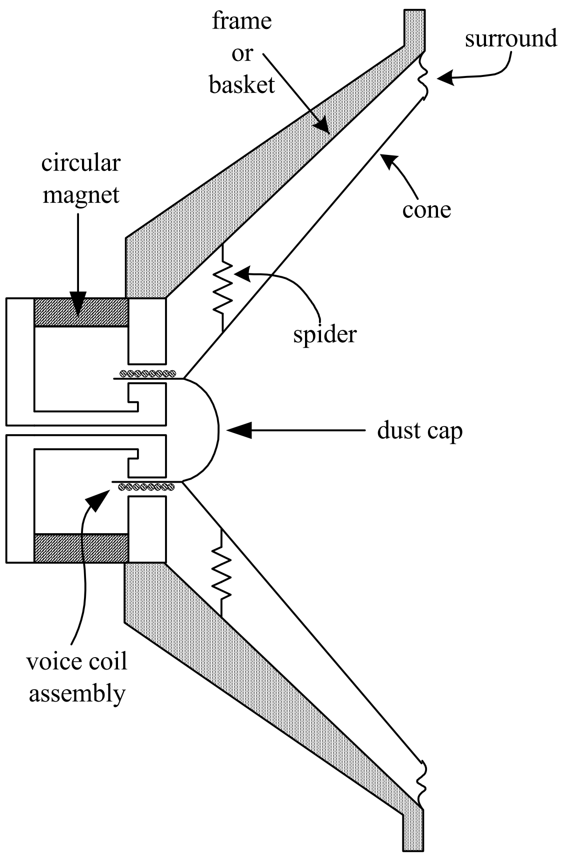


Figure 16.2: The outer parts of a moving coil driver include:

magnet: the heart of the driver is a circular magnet. It looks like a section cut out of a metal pipe. The north pole is on one end of the ring and the south pole on the other.

frame: also called the basket, this metal frame supports the outer rim of the cone at one end, and the magnet assembly at the other

surround: this flexible rubber ring connects the outer edge of the cone to the basket. The surround is connected using glue. The surround is fundamental to the analysis of a transducer. It supplies a major part of the restoring force for the cone. Together with the spider, the surround forms the **suspension** component of the driver.

cone: usually made of paper, paper/felt, or plastic, the cone transfers the acoustic power into the air load. It may have straight edges and be conical (as shown) or have flared edges that are horn-like.

spider: an accordion looking cloth material whose main purpose is to keep the voice coil assembly centered perfectly in the gap. It also proves the rest of the restoring force for the cone. Together with the surround, it forms the **suspension** component of the driver.

dust cap: protects the voice coil assembly and gap from foreign debris. In some drivers, the dust cap may be inverted (concave), or even flat like a disc. The dust cap may also be porous to air, connecting to the pressure equalization hole drilled through the center of the magnet assembly.

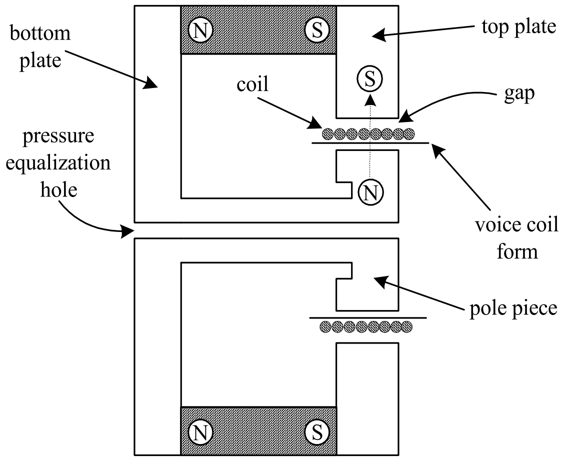


Figure 16.3 below shows the voice coil and magnet assemblies. The parts consist of:

top and bottom plates: these metal plates create a magnetic gap that the coil assembly sit in. Note the way the north and south poles are transferred to the gap area with the north pole in the center, and the south pole at the outer edge.

gap: a small (few millimeters) gap is formed between the plates. Magnetic lines of flux radiate from the center (N) outward through the coil to the edge (S). Note that the lines of flux are perpendicular to the voice coil.

coil: the coil (or voice coil) is formed by wrapping turns of wire around a cardboard **voice coil form**. There may be multiple layers of turns of wire, but only a single layer is shown here for simplicity.

pole piece: the pole piece connects to the bottom plate and forms the center plug that sits inside the voice coil form.

pressure equalization hole: because the gap may be very small, and the voice coil moves rapidly, pressure may build up inside the chambers. This hole prevents pressure build-up. In some systems, there are two holes, connecting the two chambers formed by the plates and pole piece.

6.2 The Voice Coil

The voice coil can be overhung or underhung as shown in Figure 6.4. The motor magnet strength is given by the BL Product where L is the length of wire immersed in the gap. In Figure 6.4, both drivers would have the same BL Product (assuming their magnetic flux B are identical) because they both have 6 turns of voice coil wire inside the gap width.

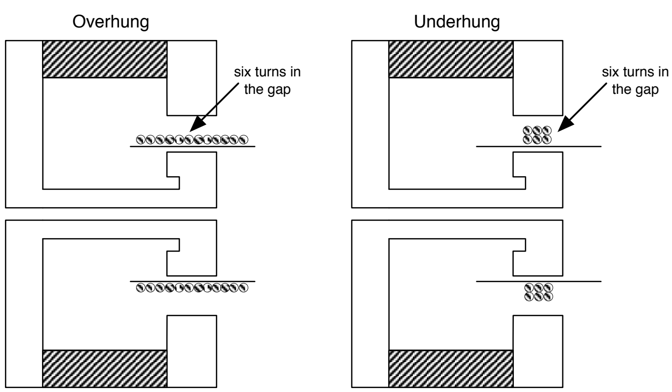


Figure 6.2: The overhung voice coil has wraps that go outside the gap width while the underhung coil's wraps are all inside the gap width.

6.3 Voice Coil Non-Linearities

As the driver moves back and forth, it is crucial that the number of turns of wire in the gap remains constant. When the driver exceeds a maximum distance (not necessarily Xmax) and the number of turns is no longer constant, non-linearities in the cone motion will occur. This will result in harmonic distortion in the audio. Another source of distortion occurs when the driver's suspension "bottoms out" or reaches its limit, which occurs at Xmax. Both the surround and the spider contribute to the overall compliance of the driver so whichever one bottoms out first sets the Xmax. The final (and most obvious source of distortion in the frequency response plots) non-linearity occurs when mechanical resonances are setup on the *surface* of the cone. These modal frequencies cause the surface to warp, flex and twist creating harmonic distortion in the output. The modal frequencies usually occur above the piston frequency.

Figure 6.5 shows the first two Axial Modes of resonance. In the first nodal resonance, the center portion of the cone is moving backwards (-) while the outer portion is moving forwards (+) thus producing a non-linear flex in the surface. In the second nodal resonance, two separate bands of resonance occur on the surface.

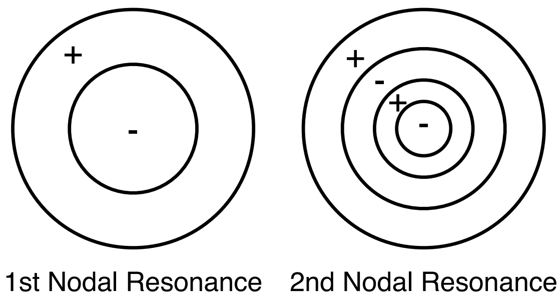


Figure 6.3: Axial Resonances form concentric circles of the surface moving in opposite directions

Figure 6.4 shows Radial or Bell Resonances. In the first order resonance, two portions of the cone are moving out (+) while the other two are moving in (-). In the second order case, all three portions are moving out and three are moving in, alternating directions.

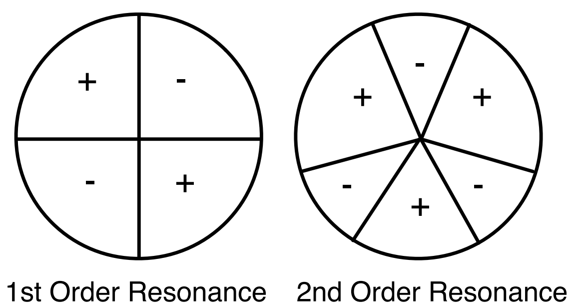


Figure 6.4: Radial Modes slice the cone into wedges, each moving in the opposite direction.

Manufacturers can try to offset these resonances by adding strengthening concentric rings (for radial modes) or radial bands (for axial modes) impregnated into the surface. As the modal resonances (standing waves) propagate outwards, the eventually hit and terminate at the rim where the surround is glued into place. The vibrations travel though the surround and can bounce backwards again. If the surround is mechanically terminated properly (by choosing the geometry of the glued flat-section where the surround connects to the cone) the reflections can be minimized.

6.4 First mode of breakup

The driver will operate linearly until the resonances on the cone surface occur. The first mode of resonance (first mode of breakup) is the first frequency where we see a resonant peak. Sometimes this mode is the most offensive of the bunch in its magnitude. One way to find it is to look for glitches in the impedance plot. This represents a sudden change in impedance often attributed to breakup or modal frequencies.

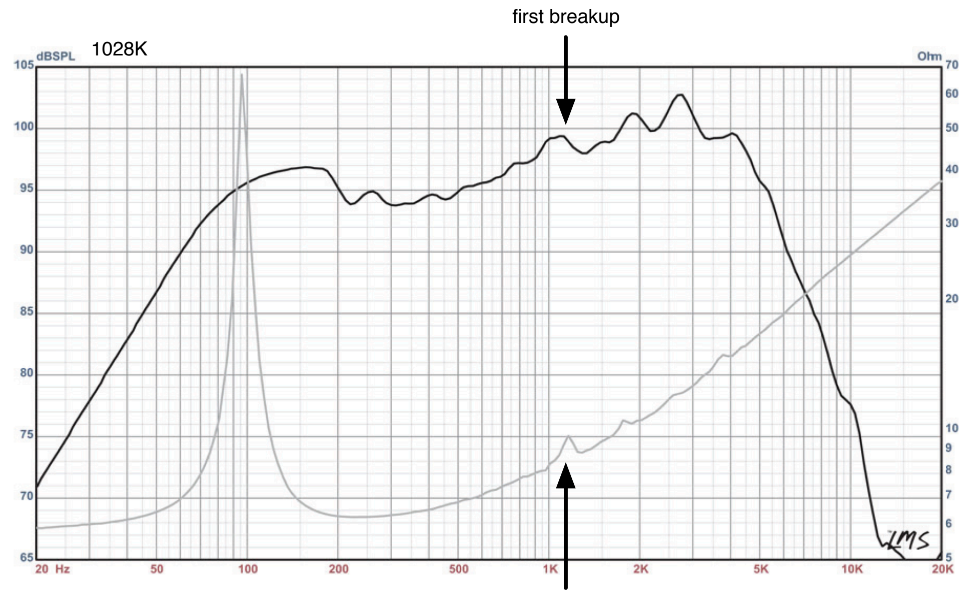


Figure 6.5: the first frequency of breakup can sometimes be found by examining the impedance plot for humps/notches.

The first mode of breakup can be calculated as follows:

$$f_{breakup} = 0.523\frac{t}{a}\sqrt{\frac{E}{\rho}}$$

$t =$ thickness of driver

$a =$ piston radius

$E =$ Young's Modulus of driver

$\rho =$ density of driver

Table 6.1 lists the Young's Modulus and densities of several popular cone materials.

Material	Young's modulus E, x10^{10}N/m^2	Density ϱ, x10^{3}kg/m^3	Specific modulus E/ϱ, x10^7(m/s)^2	Sound velocity $\sqrt{E/\varrho}$, x10^{3}m/s	Internal loss tanδ, –	Melting point °C
Beryllium	28	1.85	15	12	0.002	1284
Boron	40	2.34	17	13	0.002	2225
Aluminum	7.0	2.7	2.6	5.1	0.002	660
Titanium	10	4.5	2.2	4.7	0.002	1668
Boronized titanium	25	4.5	5.6	7.5	0.002	
Carbon fiber	23	1.74	13	11.5		
CFR-olefin	0.37	0.45	0.82	2.9	0.025	
Polymer-graphite	7.0	1.8	3.9	6.2	0.05	
Cone paper	0.1~0.2	0.5	0.2~0.4	1.4~2.0	0.02~0.05	
Graphite Glass	35	1.4	25	5.0	.005	

Table 6.1: Properties of some cone materials. (source: http://www.pearl-hifi.com/06_Lit_Archive/07_Misc_Downloads/Cone_Diaphragm_Mtls.pdf)

6.5 Functional Components

Electrical

- The voice coil has an inductance, $\mathbf{L_E}$ (the subscript E is for "electrical")
- The coil also has a DC resistance, $\mathbf{R_E}$ since the wire is usually very long (with many turns in the coil)
- At audio frequencies, the coil's parasitic capacitances that exist between turns of wire are so small to be completely negligible
- Check the Thiele-Small parameters for the drivers in the beginning of the chapter and find these values.

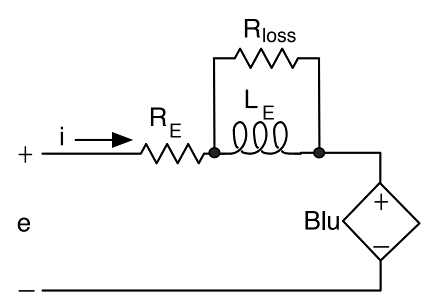


Figure 6.6: The Electrical Model of the Moving Coil Driver

Mechanical

- The cone or diaphragm has the most significant mass, $\mathbf{M_{MD}}$ (grams)

- The suspension system, made up of the surround and spider, provides a spring component called **C_{MS}** (meters/Newton)
- The suspension system also has a mechanical friction component **R_{MS}** (Newton seconds/meter) since they do not behave like ideal springs
- All but R_{MS} are given in the driver specifications at the beginning of the chapter

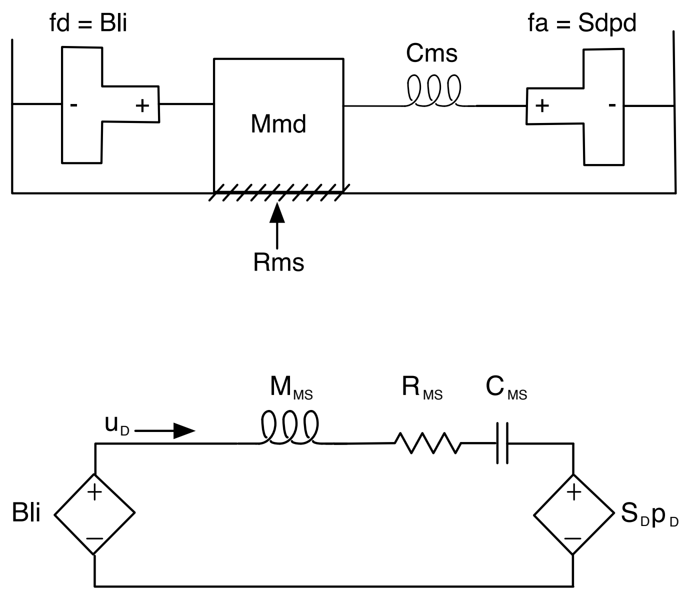


Figure 6.7: The Mechanical Models of the moving coil driver (top is mechanical model, bottom is circuit)

Acoustical

- The Acoustic Model ultimately depends on the enclosure the driver is mounted in since this will determine the acoustic load the front and back of the cone sees.
- We can generalize the circuit by calling the air loads Z_{AB} and Z_{AF} for the back and front sides respectively

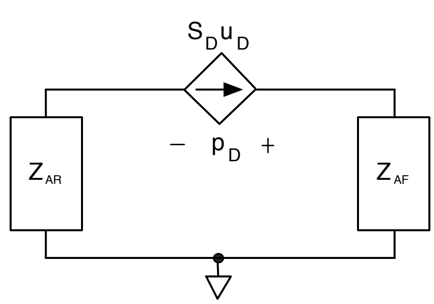


Figure 6.8: Generalized Acoustic Model of the Driver

6.6 The Infinite Baffle Enclosure

The Response Plots of the two driver data-sheets we looked at in the beginning of the chapter had some fine print below them. These responses were measured in an Infinite Baffle. An Infinite Baffle is a wall or other large surface that isolates the front acoustic load from the rear acoustic load. Additionally, the air masses on each side of the baffle are so large that their compliance is infinite. One example of an Infinite Baffle system consists of loudspeakers mounted in the ceiling with no box or other enclosure on the back side of the driver. There is no pneumatic air-spring on either side of the driver, just an air load. Another example is a speaker enclosure that is huge (think refrigerator sized) so that the compliance is so high that a driver mounted in it will feel no air-spring behind it.

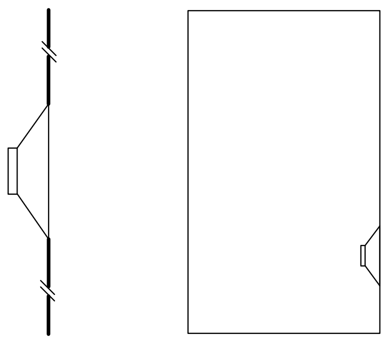


Figure 6.9: Two infinite baffles; on the left is a wall or ceiling mount and on the right is an enclosure that is so huge in comparison with the excursion of the driver that it presents no compliance load and appears to be the same load as on the front side of the driver.

In the case of the large enclosure, one question is "how do you know when the enclosure is so big we can consider it to be an infinite baffle?" The answer is found in another Thiele-Small parameter called $\mathbf{V_{AS}}$ or the **Volume Compliance** of the driver. This value is the volume of air (in liters or cubic feet) that has the same compliance as the driver's suspension system. The Compliance Ratio (α) is defined as the ratio of the Volume Compliance to the Box Volume (V_{AB}):

$$\alpha = \frac{V_{AS}}{V_{AB}}$$

When $\alpha < 1$(ie when the box volume is greater than the volume compliance) we have an infinite baffle enclosure.

We saw the circuit that models the acoustic air load for a driver in an Infinite Baffle in Chapter 2. The Acoustic Model of the driver in the enclosure would use that model for Z_{AB} and Z_{AF}. So, the complete model for a driver mounted in an infinite baffle would look like this:

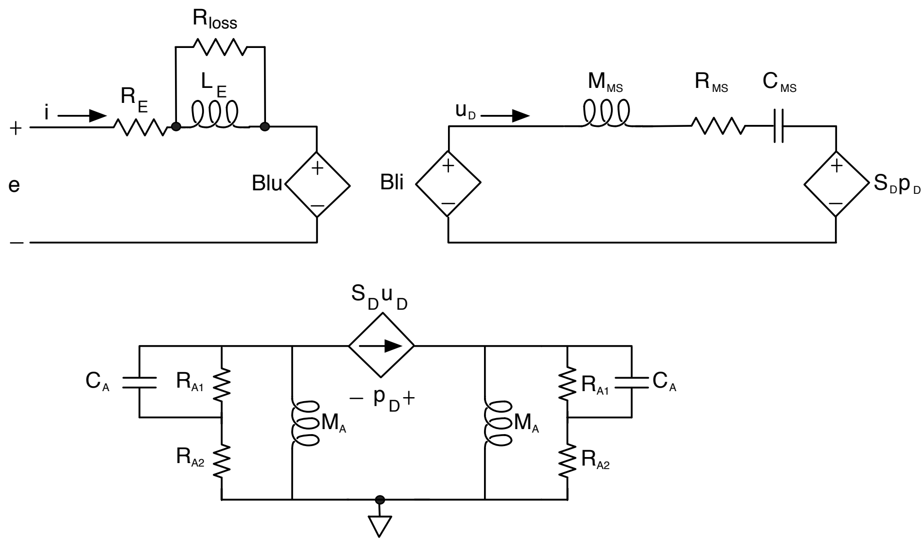


Figure 6.10: The complete electrical model of a driver in an Infinite Baffle Enclosure.

This trio of dependent circuits can be combined together into one analogous circuit using either Thevenin or Norton Equivalent circuits and combining dependencies. The Infinite Baffle Analogous Circuit looks like this:

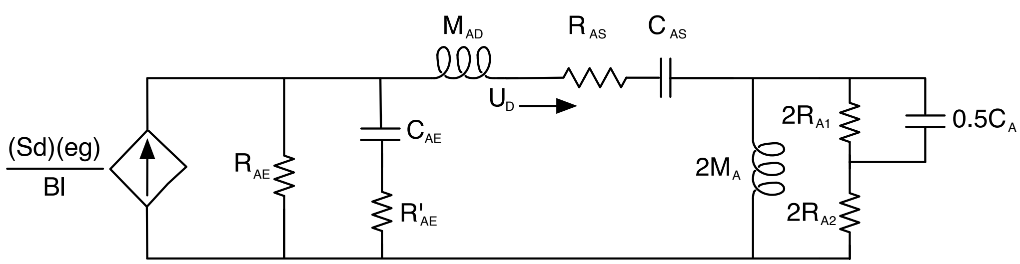


Figure 6.12: The Combination Analogous Circuit (Norton Form) contains three sections from left to right that lump and model the electrical (left), mechanical (center) and acoustic (right) combinations.
The circuit elements are:

$$R_{AE} = \frac{(Bl)^2}{S_D^2 R_E} \qquad C_{AE} = \frac{S_D^2 L_E}{(Bl)^2} \qquad R'_{AE} = \frac{(Bl)^2}{S_D^2 R_{LOSS}}$$

$$M_{AD} = \frac{M_{MD}}{S_D^2} \qquad R_{AS} = \frac{R_{MS}}{S_D^2} \qquad C_{AS} = S_D^2 C_{MS}$$

M_A, R_{A1}, R_{A2}, C_A see Chapter 2

6.7 Low Frequency Solution

At very low frequencies, the analogous circuit can be simplified and combined further into:

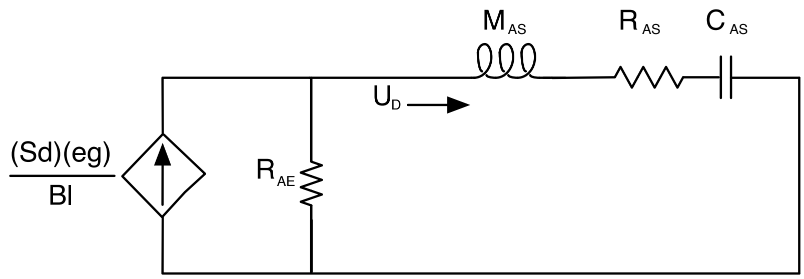


Figure 6.11: The Low Frequency Analogous Circuit for the driver in an Infinite Baffle

$$M_{AS} = M_{AD} + 2M_A = \frac{M_{MD}}{S_D^2} + 2\frac{8\rho_o}{3\pi^2 a}$$

After some math we can solve for the volume velocity U_D emitted by the system as:

$$U_D = \frac{S_D e_g}{Bl}\frac{R_{AE}}{R_{AT}}\frac{(1/Q_{TS})\left(s/\omega_s\right)}{\left(s/\omega_s\right)^2 + (1/Q_{TS})\left(s/\omega_s\right) + 1}$$

$$R_{AT} = R_{AE} + R_{AS} = \frac{(Bl)^2}{S_D^2 R_E} + \frac{R_{MS}}{S_D^2}$$

$$\omega_s = 2\pi f_s = \frac{1}{\sqrt{M_{MS}C_{MS}}} \qquad Q_{TS} = \frac{1}{R_{AT}}\sqrt{\frac{M_{AS}}{C_{AS}}}$$

The first thing to notice is that the Volume Velocity function is a Band Pass Filter transfer function. The peak Volume Velocity value will occur at the natural resonant frequency of the driver, f_s.

The last two terms, f_s and Q_{TS} are Thiele-Small Parameters that represent the Driver Resonant Frequency (f_s) and Total System Q (Q_{TS}). The Total System Q is also called the Resonance Magnification Factor. This factor is a combination of the two resonances in the electrical and mechanical circuits. Those resonance factors are named Q_{MS} (mechanical Q) and Q_{ES} (electrical Q) and the relationship is the algebraic mean of the two:

$$Q_{TS} = \frac{Q_{MS}Q_{ES}}{Q_{MS} + Q_{ES}} \qquad Q_{MS} = \frac{1}{R_{MS}}\sqrt{\frac{M_{MS}}{C_{MS}}} \qquad Q_{ES} = \frac{1}{R_{AE}}\sqrt{\frac{M_{AS}}{C_{AS}}}$$

So, even though the manufacturer did not specify R_{MS}, we can calculate it knowing Q_{MS}, C_{MS}, and M_{MS} which are all given! Finally, the Volume Compliance can be calculated as:

$$V_{AS} = \rho_o c^2 S_D^2 C_{MS}$$

6.8 High Frequency Solution

The upper limit of the frequency response is dominated by the electrical and mechanical portions of the overall circuit model. They produce a 1st Order Low Pass Filter with a transfer function of:

$$T_u(s) = \frac{1}{1 + {s}/{\omega_u}}$$

$$\omega_u = \frac{M_{MS} R_E}{M_{MD} L_E}$$

6.9 Combined Solution

Plotting both the LF and HF solutions is helpful in predicting the on-axis pressure transfer function because we know that volume velocity and pressure are directly proportional.

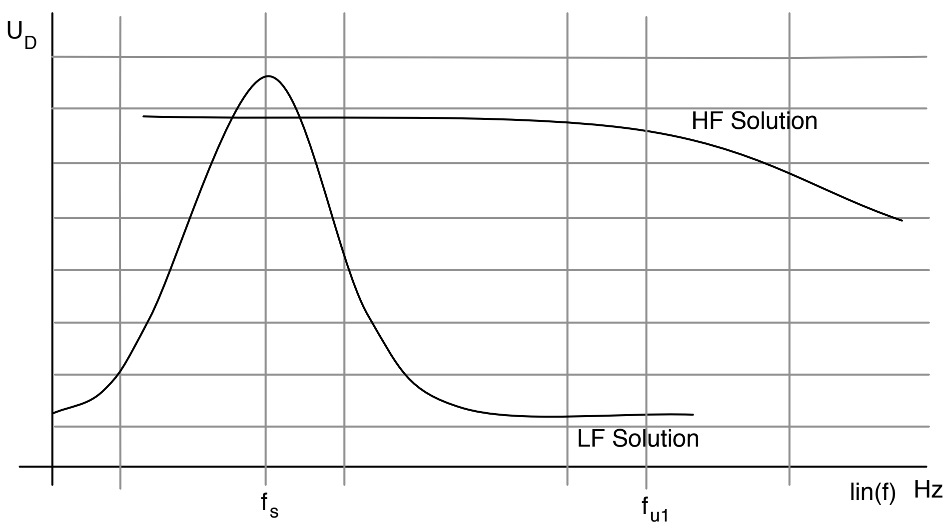


Figure 6.12: The two solutions plotted together foreshadow what we expect to see for our final on-axis pressure response in db_{SPL}.

6.9 On Axis Pressure Transfer Function

The on-axis pressure transfer function can be found by solving p = UZ in the combination analogous circuits, first for the LF solution and then for the HF. After a bunch of math, you get:

$$p = \frac{\rho_o}{2\pi}\frac{Ble_g}{S_D R_E M_{AS}}\frac{\left(s/\omega_s\right)^2}{\left(s/\omega_s\right)^2 + \left(1/Q_{TS}\right)\left(s/\omega_s\right) + 1}T_u(s)$$

$$G(s) = \frac{\left(s/\omega_s\right)^2}{\left(s/\omega_s\right)^2 + \left(1/Q_{TS}\right)\left(s/\omega_s\right) + 1}$$

then

$$p = \frac{\rho_o}{2\pi}\frac{Ble_g}{S_D R_E M_{AS}}G(s)T_u(s)$$

This equation reveals a 2nd Order High Pass Filter with a cutoff frequency of ω_s and a resonant quality factor of Q_{TS} for low frequencies and a first order high-pass filter for high frequencies. Their cutoffs are found with the above equations. This explains the two different frequency response plots of the 1028K and LAB12 drivers at the beginning of the chapter. Go back and look at the Thiele-Small parameter for ω_s (f_s) and Q_{TS}. Then look at the low frequency (HPF) edge of the responses. Compare with the predicted responses in Figure 6.13. Also, compare with the LF and HF solutions for the volume velocity.

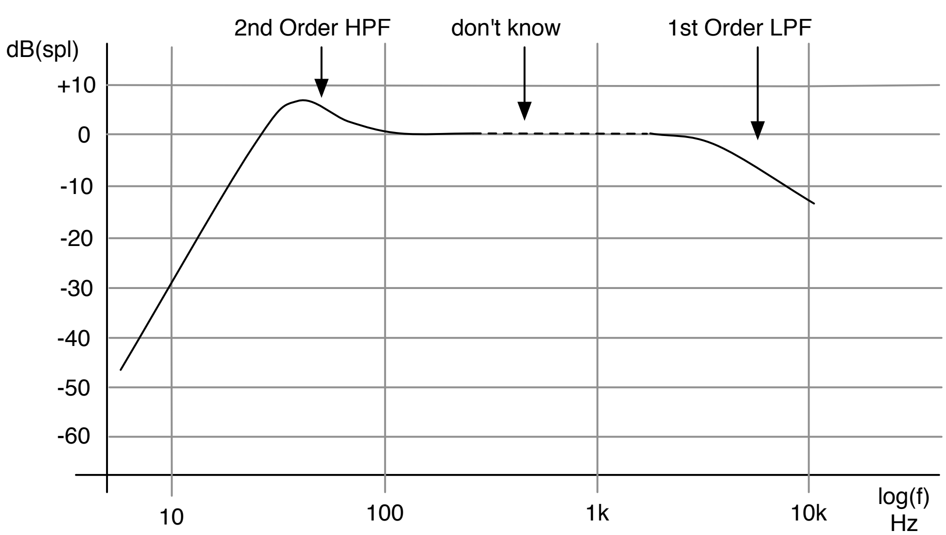


Figure 6.13: the predicted on-axis pressure response for a driver. The low frequency cutoff and Q are set by the driver's f_s and Q_{TS}. The high frequency edge is due to the electro-mechanical filter described above.

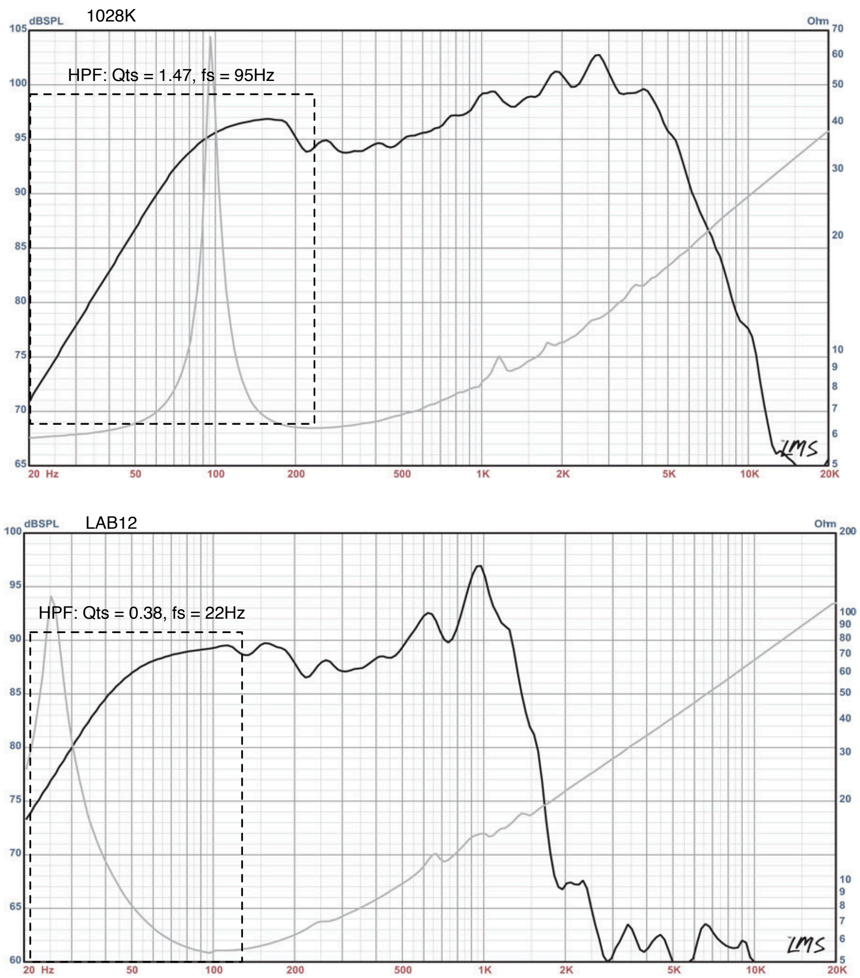


Figure 6.14: The Thiele-Small Parameters tell us immediately what the low-frequency cutoff and resonances are going to look like.

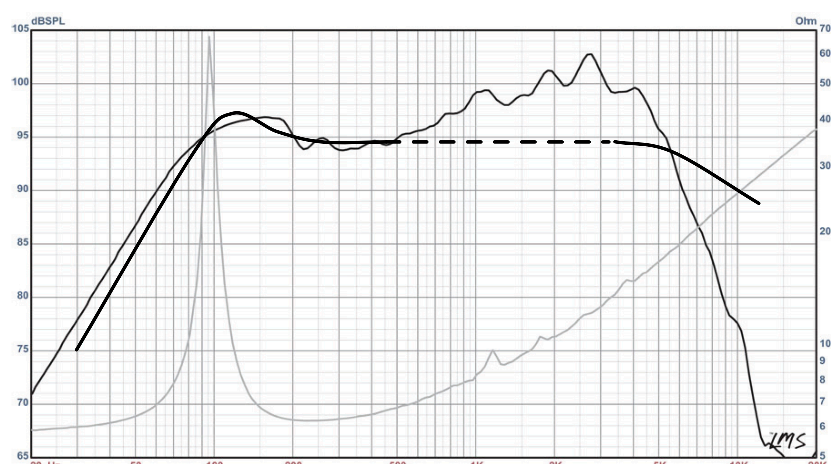


Figure 6.15: The predicted curve superimposed over the 1028K response; the low frequency approximation is more accurate. We also can't predict the magnitude of the resonances.

If $Q_{TS} > 0.707$ a resonant peak will occur. The peak frequency can be found as:

$$f_{peak} = f_s \frac{Q_{TS}}{\sqrt{Q_{TS}^2 - 0.5}}$$

When $Q_{TS} = 0.707$, the low cutoff frequency (the -3dB frequency) equals the driver resonant frequency, f_s. For other values, the low frequency (f_L) is found as:

$$f_L = f_s \left[\left(\frac{1}{2Q_{TS}^2} - 1 \right) + \sqrt{\left(\frac{1}{2Q_{TS}^2} - 1 \right)^2 + 1} \right]^{1/2}$$

This reveals that the low cutoff frequency is proportional to $1/Q_{TS}$ so as you increase the resonance (to get more bass) you sacrifice the low cutoff edge. This is the standard tradeoff in resonant filter design.

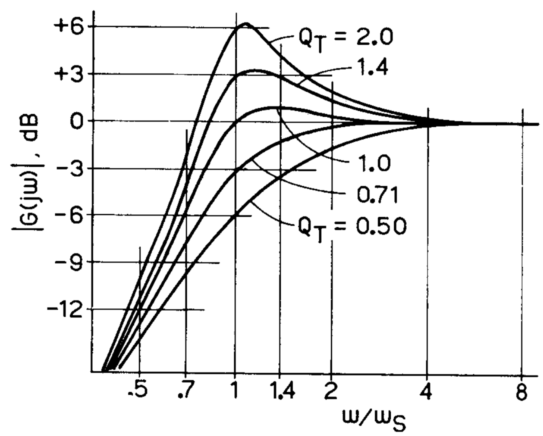


Figure 6.16: Normalized 2nd Order HPF Band Edge plots for various values of Q_{TS} (in this figure, Q_T is Q_{TS}.)

6.10 On Axis Displacement Transfer Function

The Displacement (x) of the driver can also be calculated using the relationship between Volume Velocity and Displacement from Chapter 1. It can be found as:

$$X(s) = \frac{1}{(C_{AS}M_{AS})s^2 + \sqrt{C_{AS}M_{AS}}\left(1/Q_{TS}\right)s + 1}$$

or

$$X(s) = e_g \left(\frac{V_{AS}}{\rho_o c^2 S_D^2 R_E \omega_s Q_{ES}} \right)^{1/2} \frac{1}{\left(s/\omega_s\right)^2 + \left(1/Q_{TS}\right)s/\omega_s + 1}$$

This equations reveal a 2nd Order Low Pass Filter response.

The frequency of the maximum excursion can be found as:

$$f_{x\max} = \frac{f_s}{Q_{TS}}\sqrt{Q_{TS}^2 - 0.5}$$

not defined for $Q_{TS} \le 0.707$

When plotted for various values of Q_{TS} for a given driver with a fixed f_s we see a resonant LPF system:

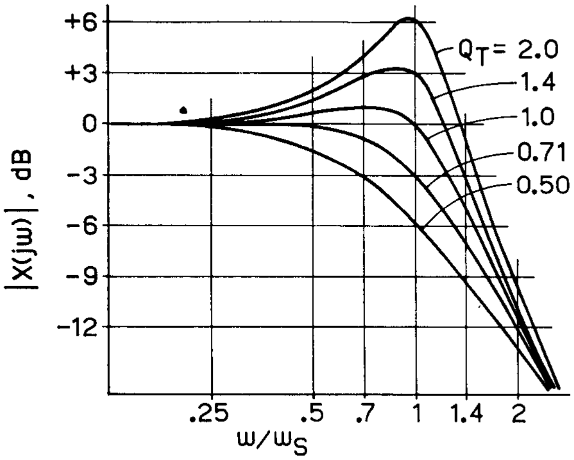


Figure 6.17: The normalized driver Displacement plot for various values of Q_{TS}; for values above 0.707 we observe a peak in the displacement curves. It makes sense that the excursion would follow this peak/curve.

6.11 On Axis Pressure Sensitivity

The Thiele-Small parameter called Sensitivity is the Pressure Sensitivity; it is the magnitude of the mid-band on-axis pressure measured at a distance of 1 meter and applying $1V_{RMS}$ across the voice-coil.

$$p_{sens}^{1V} = \frac{\sqrt{2\pi\rho_o}}{c} f_s^{3/2} \left(\frac{V_{AS}}{R_E Q_{ES}} \right)^{1/2}$$

$$p_{sens}^{1V}(dB) = 20\log\left(\frac{p_{sens}^{1V}}{2x10^{-5}\,Pa} \right)$$

The convention is to convert this pressure to dB_{SPL} and the two drivers at the beginning of the chapter have very different values (97.4 for the 1028K and 89.2 for the LAB12). The 1028K has a higher output for $1V_{RMS}$ across the voice coil, but this does not reflect the power efficiency because we don't know how much (or little) current it took to get that 1V drop. In order to figure out how efficient the driver is, we need to find the acoustic output power limit and the electrical input power limit. But, you sometimes see the sensitivity specified for 1W of input power to the driver. This can be found by multiplying the 1V pressure sensitivity by the square root of R_E.

$$p_{sens}^{1W}(dB) = 20\log\left(\frac{p_{sens}^{1V}\sqrt{R_E}}{2x10^{-5}\,Pa} \right)$$

6.12 Acoustic Output Power

The Acoustic Output Power can be calculated knowing the Volume Velocity and the Radiation Impedance. It is calculated as the power radiated into the front air-load only. The Acoustic Output Power is not constant across frequencies. Since we know that Acoustic Power and pressure are related proportionally, it would follow that the Acoustic Output Power response is similar to the pressure response. This is true up to a point; for the Acoustic Output Power we find a second break-point in the high frequency response, f_{u2}. NOTE: since this second HF breakpoint is dependent on the driver's physical size, it may wind up being above or below the first HF breakpoint, f_{u1}. Here, it is shown above the first HF breakpoint but that won't aways be the case.

$$f_{u2} = \frac{\sqrt{2}c/a}{2\pi}$$

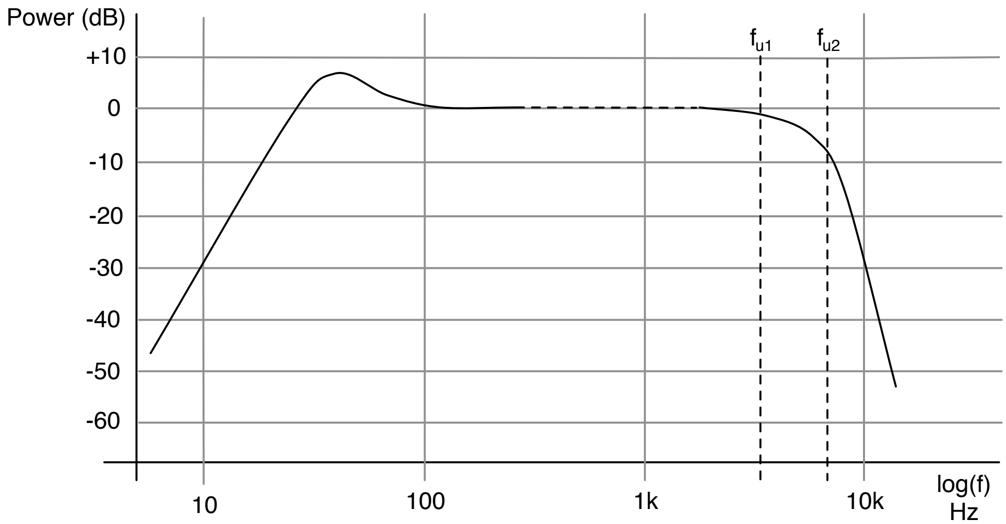


Figure 6.18: Predicted Acoustic Output Power Response shows the second HF breakpoint and steeper roll-off; this tends to match more closely the pressure responses we observed in the commercial drivers.

6.13 Reference Efficiency

The Reference Efficiency (η) is the ratio of the acoustic power output to the electrical power input. This value is typically around 2% or so, meaning that 98% of the input power was lost during the transduction. Can you think of where this would get lost?

$$\eta_o = \frac{P_{AR}}{P_E} = \frac{4\pi^2}{c^3}\frac{f_s^3 V_{AS}}{Q_{ES}}$$

6.14 Displacement Limited Electrical Input Power

The Electrical Power limit of the driver is dependent on the maximum excursion, x_{max}. The power limit is found as:

$$P_{E(MAX)} = \frac{1}{2}\frac{\rho_o c^2 \omega_s Q_{ES}}{V_{AS}} V_D^2 \left[\frac{Q_{TS}^2 - 0.25}{Q_{TS}^4}\right]$$

6.15 Displacement Limited Power Rating

Ultimately, the maximum excursion or X_{MAX} will limit the output power; this power limit can be found by combining the $P_{E(MAX)}$ and reference efficiency equations together. The result is valid for mid-band frequencies that are in between the LF and HF cutoff points.

$$P_{AR(MAX)} = \frac{1}{2} U_D^2 \,\mathrm{Re}\left[Z_{AF}(j\omega) \right]$$

$$= \frac{4\pi^3 \rho_o f_s^4}{c} V_D^2 \left[\frac{Q_{TS}^2 - 0.25}{Q_{TS}^4} \right]$$

$$V_D = S_D x_{\max}$$

NOTE: this is valid for $Q_{TS} >= 0{,}707$, if $Q_{TS} < 0.707$, use $Q_{TS} = 0.707$ instead

6.16 Voice Coil Impedance

The Voice Coil Impedance responses from the drivers in the beginning of the chapter certainly reveal that the impedance varies drastically over frequencies. This is one of the things that makes designing a high-power amplifier challenging; the load is complex and not a simple resistor. The voice coil impedance Z_{VC} is found by looking into the electrical terminals of the driver:

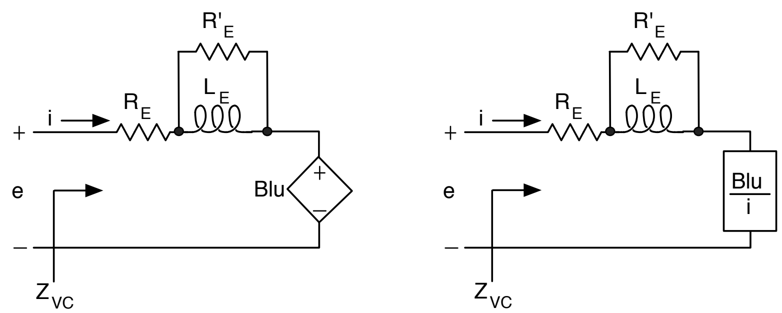


Figure 6.19: Z_{VC} is found by looking into the electrical terminals of the driver; note that the impedance of the dependent source can be replaced by its voltage over current (Blu/i).

Z_{VC} can be solved with some math. We can see by looking at the figure that it will be the sum of R_E and R'_E in parallel with L_E plus the impedance of the dependent source. That source is dependent on the coil velocity and is found in the mechanical circuit, so some of these parameters will show up in the final circuit.

$$Z_{VC} = R_E + j\omega L_E \,//\, R_E' + R_{ES} \frac{(1/Q_{MS})\left(s/\omega_s \right)}{\left(s/\omega_s \right)^2 + (1/Q_{MS})\left(s/\omega_s \right) + 1}$$

$$R_{ES} = R_E \frac{Q_{MS}}{Q_{ES}}$$

The equivalent circuit that can give this impedance is shown in Figure 6.20.

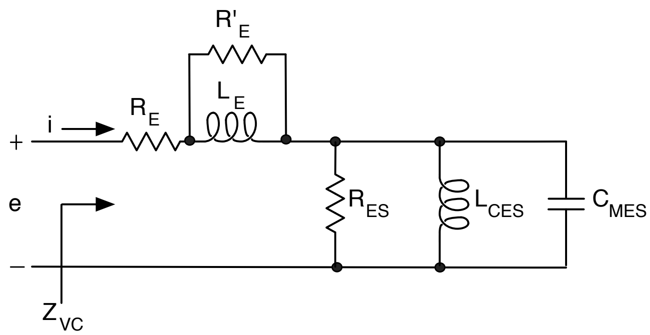


Figure 6.20: A voice-coil simulation circuit reveals a parallel RLC network; this implies a resonant frequency of some kind, formed when the impedances of the inductor and capacitor cancel out.

$$L_{CES} = (Bl)^2 C_{MS} = \frac{R_E}{2\pi f_s Q_{ES}} \qquad C_{MES} = \frac{M_{MS}}{(Bl)^2} \frac{Q_{ES}}{2\pi f_s R_E}$$

So, we can get all the component values we need from the Thiele-Small parameters. In order to understand the overall impedance plot, you have to create simplified equivalent circuits according to frequency range:

For VLF (Very Low Frequencies) the caps are open and the inductors are shorts. This leaves only R_E. Thus, R_E is the DC resistance alone. For 8-ohm speakers, this value is usually around 6-ohms instead. As the frequency rises, we get into the LF range. Here the parallel RLC circuit appears as the impedances of the reactive components change; the inductor's impedance is rising while the capacitor's is falling. This creates an increase in impedance.

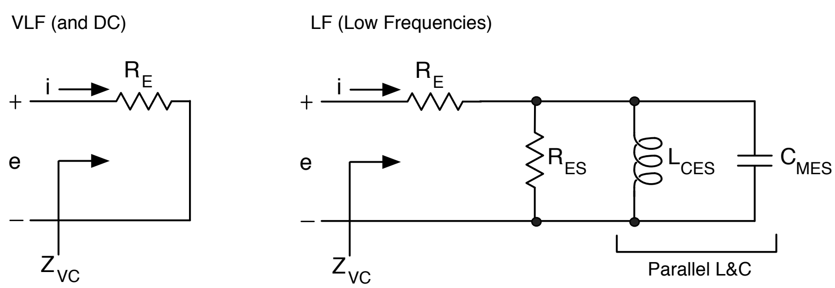


Figure 6.21: VLF and LF Circuits. For LF, the parallel L&C impedance is rising.

At some frequency, the parallel inductor and capacitor will cancel out:

$$R//L//C = Z_T$$

$$\frac{1}{Z_T} = \frac{1}{R} + \frac{1}{j\omega L} + j\omega C$$

$$1/j = -j$$

$$\frac{1}{Z_T} = \frac{1}{R} - j\frac{1}{\omega L} + j\omega C$$

The frequency at which these cancel is the resonant frequency of the voice-coil and the driver's f_s. When they do cancel out, the impedance reaches a maximum value of $R_E + R_{ES}$.

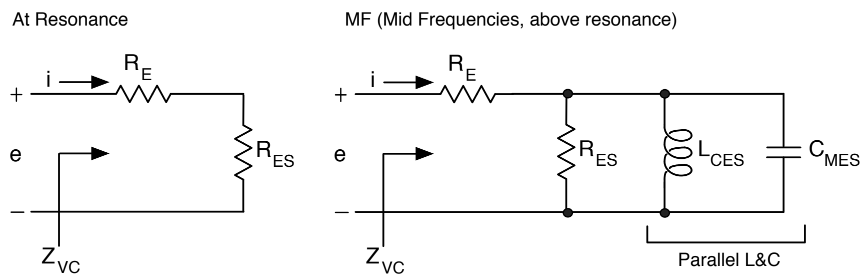


Figure 6.22: At resonance, the combined impedance is $R_E + R_{ES}$ while above that frequency, the parallel L&C show up again, this time their impedances moving in the opposite direction.

For high frequencies far above resonance, the coil impedance becomes the dominant component and we are left with the circuit in Figure 6.23:

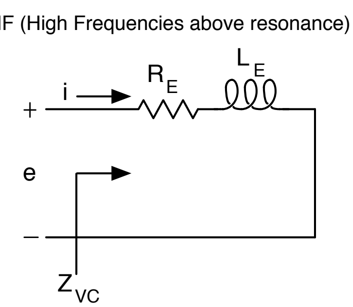


Figure 6.23: At high frequencies the coil impedance dominates. We expect to see a linear rise in impedance.

The impedance response shows the combination of these circuits across frequency:

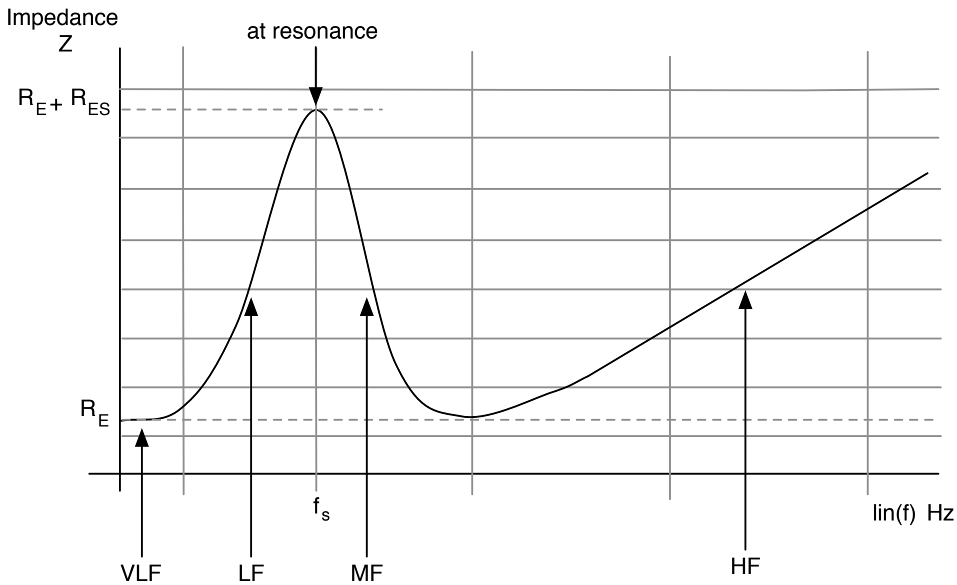


Figure 6.24: The combined responses of each circuit.The intersection point with 0Hz is the location of R_E

Figure 6.24 shows the final composite plot. We observe the proper values at DC (0Hz) and see the high frequency inductive rise (since impedance is linearly related to inductance). We also see the band-pass section around resonance formed by the parallel RLC circuit. But you might be thinking about the resonant frequency's impedance -- why is it a maximum rather than a minimum? Since the resonant frequency of a system is the frequency where it moves the easiest and with the greatest excursion, why would the impedance go up? The answer is Back EMF. Sure, the driver moves easier and has more excursion but this generates the maximum Faraday-induced Back EMF which shows up as the spike on our graph.

Go back and look at the Impedance plots from the 1028K and LAB12 drivers. You will see that they peak right at the driver resonant frequency, f_s. However, notice that these plots are doing with a log frequency axis; the left edge of the graph is not DC, but rather 20Hz. If you continued that log axis backwards by a decade, you would hit the 2Hz line and continuing back, the 0.2Hz and 0.02Hz lines; in fact you will never get back to 0Hz on a log frequency plot! So you are not seeing R_E on the plots, you are seeing the impedance at 20Hz.

Infinite Baffle Design Guide

The design has only one degree of freedom: choose a driver. After that, everything else can be calculated or predicted.

(1) Choose Driver. Remember that the driver specs will dictate everything, therefore if you want decent bass response, choose a driver with a relatively high Q_{TC} of 0.7 to 1.2 or so. Get the following Thiele-Small Parameters:

T-S Parameter		Value
f_s		Driver Resonant Frequency
Q_{TS} Q_{ES} Q_{MS}		Resonant Quality Factors (Total, Electrical and Mechanical)
V_{AS}		Volume Compliance
M_{MS}		Mechanical Mass Equivalent
R_E		DC Coil Resistance
L_E		Coil Inductance
S_D		Surface area of cone
x_{MAX}		Maximum peak displacement

(2) Predict Frequency Response:

$$M_{MD} = M_{MS} - 2S_D^2 M_A \qquad M_A = \frac{8\rho_o}{3\pi^2 a}$$

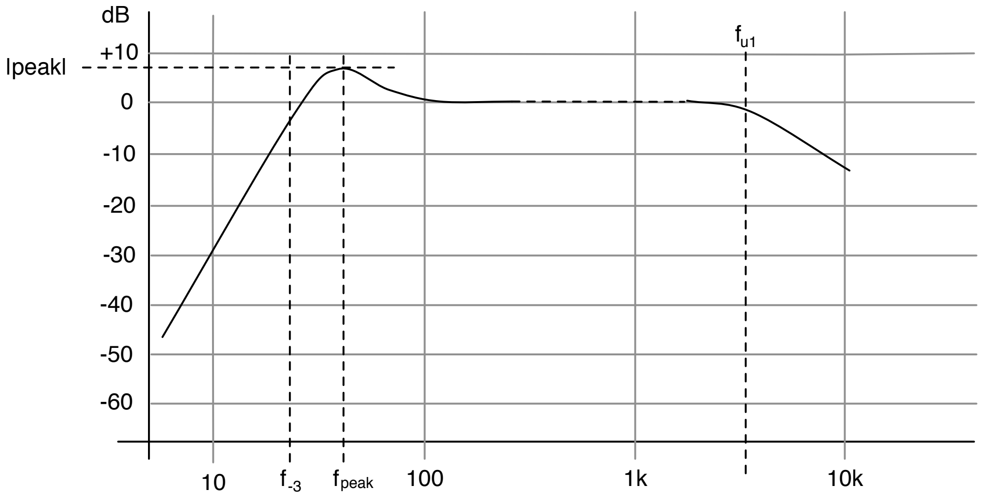
$$M_{MS} = \frac{1}{(2\pi f_s)^2 C_{MS}} \qquad C_{MS} = \frac{V_{AS}}{\rho_o c^2 S_D^2}$$

$$LF \qquad f_{-3} = \left[\beta + \sqrt{\beta^2 + 1}\right]^{1/2} f_s \qquad \beta = \frac{1}{2Q_{TS}^2} - 1$$

$$f_{peak} = \frac{Q_{TS} f_s}{\sqrt{Q_{TS}^2 - 0.5}} \qquad |peak| = \frac{Q_{TS}^2}{\sqrt{Q_{TS}^2 - 0.25}}$$

$$HF \quad f_{u1} = \frac{R_E M_{MS}}{2\pi L_E M_{MD}}$$

(3) Plot Predicted Response



Check Power Limits & Efficiency

$$P_{E(MAX)} = \frac{1}{2}\frac{\rho_o c^2 \omega_s Q_{ES}}{V_{AS}} V_D^2 \left[\frac{Q_{TS}^2 - 0.25}{Q_{TS}^4}\right]$$

$$P_{AR(MAX)} = \frac{4\pi^3 \rho_o f_s^4}{c} V_D^2 \left[\frac{Q_{TS}^2 - 0.25}{Q_{TS}^4}\right]$$

$$\eta_o = \frac{P_{AR}}{P_E} = \frac{4\pi^2}{c^3}\frac{f_s^3 V_{AS}}{Q_{ES}}$$

Check Sensitivity

$$p_{sens}^{1V} = \frac{\sqrt{2\pi\rho_o}}{c} f_s^{3/2}\left(\frac{V_{AS}}{R_E Q_{ES}}\right)^{1/2}$$

$$p_{sens}^{1V}(dB) = 20\log\left(\frac{p_{sens}^{1V}}{2x10^{-5}\,Pa}\right)$$

$$p_{sens}^{1W}(dB) = 20\log\left(\frac{p_{sens}^{1V}\sqrt{R_E}}{2x10^{-5}\,Pa}\right)$$

7 The Acoustic Suspension Enclosure

The next enclosure to examine is the Acoustic Suspension o Closed Box loudspeaker. To make this enclosure, you mount a driver in a sealed box so that no air can escape. Your concern in the design is to calculate the proper volume for the box; this is one more degree of freedom than we had in the Infinite Baffle case.

The front of the driver radiates into space while the back of the driver feels the impedance of the compliance formed by the closed box. The closed box is partially filled with a fibrous material. The filling has several consequences on the system design. First, the filling will help dampen acoustic standing waves that happen in the upper frequency ranges; we can't avoid these standing waves because we have an enclosure. At lower frequencies, the filling has the property of increasing the compliance of the air. This means that the box volume appears to be larger than it actually is. There is debate as to why this is observed. A good argument is a thermodynamic one: as the air in the enclosure compresses, its temperature rises. The filling absorbs the heat, cooling the air which causes the pressure to drop. Thus the enclosure has a larger apparent volume. This only works at low frequencies. The filling also adds mass-loading to the rear of the driver. This is not completely understood but could be due to the air particles trapped in the interstitial fiber cells; they act like millions of tiny tubes. Finally, the filling adds dampening to the low frequencies as well as a constant resistance factor. The typical filling amount is 20-50% of the enclosure volume. In theory, the apparent increase in box volume can not exceed 40% - a conservative number to use when designing is that you get about a 20-25%% increase in volume.

We define the compliance ratio (α) as the ratio of V_{AS} to V_{AB} which is:

$$\alpha = \frac{V_{AS}}{V_{AB}}$$

For an Infinite Baffle: $\alpha < 1$
For an Acoustic Suspension: $\alpha > 3$

Values of α between 1 and 3 are not useable as their models overlap too much. This means the volume of the enclosure must be at most 1/3 of the volume compliance of the driver.

7.1 Circuit Models for the Acoustic Suspension

To derive the circuit model for the Acoustic Suspension we only need to modify the acoustic circuit by adding the new components to the back side of the diaphragm:

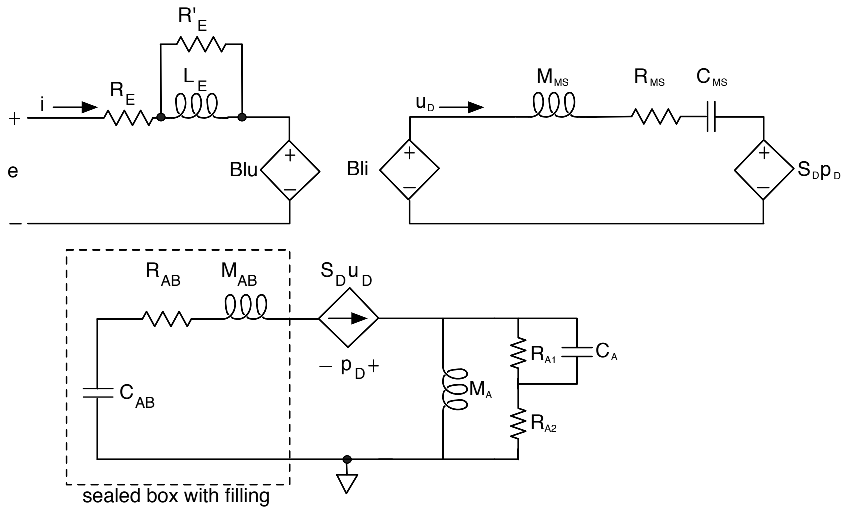


Figure 7.1: The model for the Acoustic Suspension Enclosure

7.2 Low Frequency Solution

This trio of dependent circuits can be combined together into one analogous circuit by using Thevenin or Norton equivalents and several pages of paper. For low-frequencies, this reduces to the circuit in Figure 7.2 (hint: compare this circuit with the analogous circuits for the Infinite Baffle). The component value equations are shown below. Examination of the circuit reveals a series RLC bandpass filter. The bandpass center frequency (maximum) occurs when the inductor and capacitor impedances cancel out. Compare with the Infinite Baffle case from the last chapter.

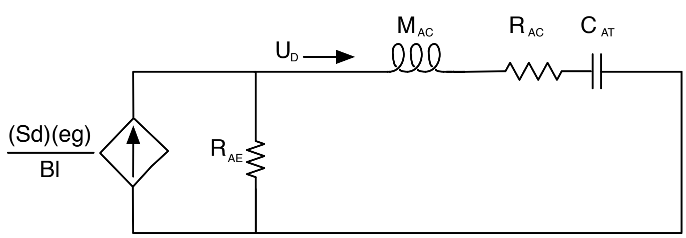


Figure 7.2: Low Frequency Analogous circuit

$$R_{AE} = \frac{(Bl)^2}{S_D^2 R_E} \qquad C_{AS} = S_D^2 C_{MS} \qquad M_{AD} = \frac{M_{MD}}{S_D^2}$$

$$R_{AS} = \frac{R_{MS}}{S_D^2} \qquad C_{AB} = \frac{V_{AB}}{\rho_O c^2} \qquad M_{AB} = \frac{B\rho}{\pi a}$$

$$R_{AC} = R_{AS} + R_{AB} \qquad C_{AT} = \frac{C_{AS} C_{AB}}{C_{AS} + C_{AB}} \qquad M_{AC} = M_{AD} + M_{AB} + M_A$$

$$M_A = \frac{8\rho_o}{3\pi^2 a}$$

$B =$ mass loading factor

$\rho =$ density of filling

Now the compliance ratio can be rewritten:

$$\alpha = \frac{V_{AS}}{V_{AB}} = \frac{C_{AS}}{C_{AB}}$$

$$C_{AT} = \frac{C_{AS}}{1+\alpha}$$

Next, we define the Total Volume Compliance V_{AT} as the volume of air having the same compliance as the system total compliance C_{AT}:

$$V_{AT} = \frac{V_{AS} V_{AB}}{V_{AS} + V_{AB}} = \frac{V_{AS}}{1+\alpha}$$

As before, we can solve for the volume velocity U_D using Ohm's Law. After some more math, we get:

$$U_D = \frac{S_D e_g}{Bl} \frac{R_{AE}}{R_{ATC}} \frac{(1/Q_{TC})\left(s/\omega_c \right)}{\left(s/\omega_c \right)^2 + (1/Q_{TC})\left(s/\omega_c \right) + 1}$$

$$R_{ATC} = R_{AE} + R_{AC}$$

$$\omega_c = 2\pi f_c = \frac{1}{\sqrt{M_{AC} C_{AT}}} \qquad Q_{TS} = \frac{1}{R_{ATC}} \sqrt{\frac{M_{AC}}{C_{AT}}}$$

One again the LF solution is a bandpass filter response. If you examine these equations, you will see they are nearly identical to the volume velocity equations of the last chapter except the subscripts have changed from S to C and TS to TC. “C” stands for “closed box.” To find the closed box resonant frequency and Q, you need to use the compliance ratio:

$$f_c \simeq \sqrt{1+\alpha} f_s$$
$$Q_{MC} \simeq \sqrt{1+\alpha} Q_{MS}$$
$$Q_{EC} \simeq \sqrt{1+\alpha} Q_{ES}$$
$$Q_{TC} = \frac{Q_{MC}Q_{EC}}{Q_{MC}+Q_{EC}}$$

$$5 \le Q_{MC} \le 10 \text{ for unfilled enclosures}$$
$$2 \le Q_{MC} \le 5 \text{ for filled enclosures}$$

It is common to let Q_{MC} = 7.5 for unfilled enclosures and Q_{MC} = 3.5 for the filled enclosures.

7.3 High Frequency Solution

The upper limit of the frequency response is dominated by the electrical and mechanical portions of the overall circuit model. It is identical to the solution for the Infinite Baffle. You will notice only the subsrips on the mass elements have changed to reflect the new analogous circuit. This produces a 1st Order Low Pass Filter with a transfer function of:

$$T_u(s) = \frac{1}{1+ s/\omega_u}$$
$$\omega_u = \frac{M_{AC}R_E}{M_{AD}L_E}$$

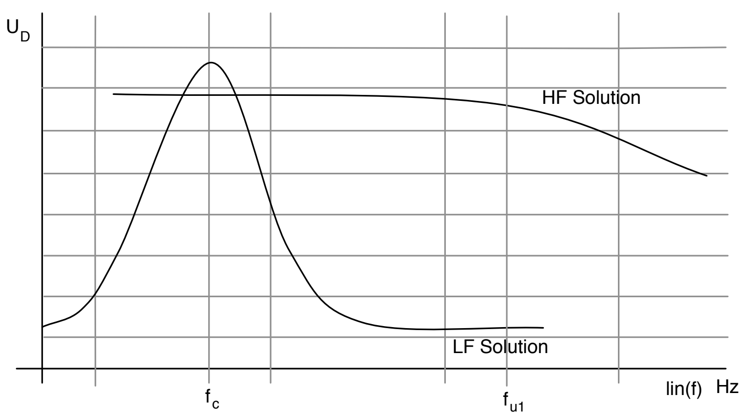


Figure 7.3: The two solutions can be plotted together again to foreshadow the predicted on-axis pressure response.

7.4 On-Axis Pressure Transfer Function

The on-axis pressure transfer function can be found by solving p = UZ in the combination analogous circuits, first for the LF solution and then for the HF. After a bunch of math, you get:

$$p = \frac{\rho_o}{2\pi} \frac{Bl e_g}{S_D R_E M_{AC}} \frac{\left(s/\omega_c \right)^2}{\left(s/\omega_c \right)^2 + \left(1/Q_{TC}\right)\left(s/\omega_c \right) + 1} T_u(s)$$

$$G(s) = \frac{\left(s/\omega_c \right)^2}{\left(s/\omega_c \right)^2 + \left(1/Q_{TC}\right)\left(s/\omega_c \right) + 1}$$

then

$$p = \frac{\rho_o}{2\pi} \frac{Bl e_g}{S_D R_E M_{AS}} G(s) T_u(s)$$

This reveals a 2nd Order HPF band edge on the low end. This is the same as the Infinite Baffle except that we have control over the Q_{TC} since we control the box volume. The relationship between Q_{TC} and Q_{TS} is:

$$Q_{TC} \simeq \sqrt{1+\alpha}\, Q_{TS} \simeq \frac{Q_{TS}}{f_s} f_c$$

Since we know that the compliance ratio must be greater than 1 then ***this means we can only control the Q_{TC} in the upwards direction, that is to say we can only increase the resonance of the system, not decrease it***. Remember also the equation for f_c:

$$f_c = \frac{1}{2\pi\sqrt{M_{AC} C_{AT}}}$$

We control (mainly) C_{AT} by controlling the box volume. Thus we can come up with the following overview:

Large Box Volume:

- C_{AT} increases
- f_c decreases
- Q_{TC} decreases

Small Box Volume:

- C_{AT} decreases
- f_c increases
- Q_{TC} increases

So, making the box smaller will increase the overall system Q - this can be used to make small loudspeaker enclosures sound like they are bass-y. The tradeoff is at the f_{-3} point; as we increase the Q, this lower frequency value also increases. This equation is the same as for the Infinite Baffle except for the new subscripts "c" for the closed box parameters.

$$f_{-3} = f_c\left[\left(\frac{1}{2Q_{TC}^2}-1\right)+\sqrt{\left(\frac{1}{2Q_{TC}^2}-1\right)^2+1}\right]^{1/2}$$

7.5 Impedance Response

The Acoustic Suspension Impedance Response looks essentially identical in shape to the Infinite Baffle except that the peak impedance value occurs at the resonant frequency of the closed box system, f_c. The equivalent circuit is the same except the component values depend on system Q's and resonant frequencies.

$$Z_{VC} = R_E + j\omega L_E\,//R_E^{'} + R_{ES}\frac{(1/Q_{MC})\left(s/\omega_c\right)}{\left(s/\omega_c\right)^2+(1/Q_{MC})\left(s/\omega_c\right)+1}$$

$$R_{ES} = R_E\frac{Q_{MC}}{Q_{EC}}$$

The equivalent circuit that can give this impedance is shown in Figure 6.19.

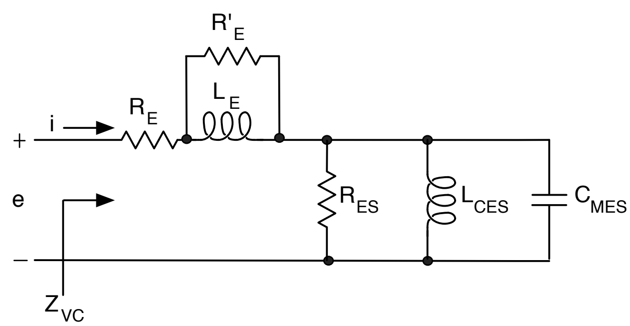


Figure 7.4: A voice-coil simulation circuit reveals a parallel RLC network; this implies a resonant frequency of some kind, formed when the impedances of the inductor and capacitor cancel out

The Impedance Response is important for designing closed box systems; it can be tested electronically and requires no microphone or other sophisticated equipment. You can test and verify that your design is correct by measuring the impedance and finding the maximum value. Then, adjust the box volume until the Impedance Plot is correct. When it looks correct, the frequency response plot will also look correct.

$$L_{CES} = (Bl)^2 C_{MS} = \frac{R_E}{2\pi f_c Q_{EC}} \qquad C_{MES} = \frac{M_{MS}}{(Bl)^2}\frac{Q_{EC}}{2\pi f_c R_E}$$

The same VLF, LF, resonance, MF and HF circuits hold true for the analysis of the Acoustic Suspension Impedance Response with the adjusted component values.

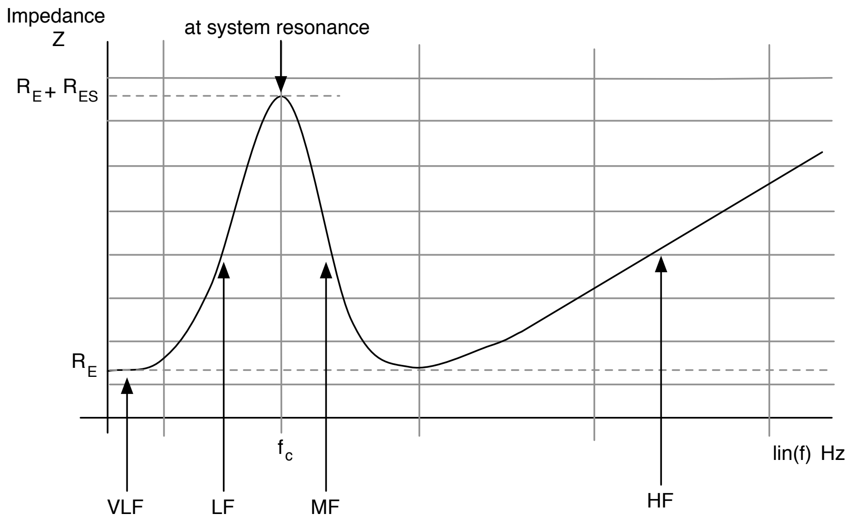


Figure 7.5: The Impedance Plot is the same as Infinite Baffle except the maximum value is now at the system resonant frequency, f_c

7.5 Alignments and Transient Response

We ultimately have control over the final Q_{TS} so we can choose what we want the LF curve to look like. Our choice is called an ***Alignment***. You can *almost* think of this as our manipulation of the alignment of the speaker's resonant frequency and Q to the box's resonant frequency and Q.

Q_{TC}	Name	Abbv.	f_{-3}/f_c	Notes
0.2 - 0.5	over-damped	OD2	varies	best transient response, poorest bass
0.500	critically-damped	CD2	1.554	good transient response, poorer bass
0.577	Bessel	BL2	1.272	maximally flat delay response, poor bass
0.707	Butterworth	B2	1.0	lowest f_{-3}, maximally flat response
> 0.707	Chebychev	C2	varies >1.0	resonant peaking, transient response rings Q_{TC} = 1.0 is a favorite C2 alignment

When examining the resonance (Q) vs. cutoff (f_{-3}) we see that as we increase the resonance, we decrease the lower cutoff - this is a common tradeoff in filter design. The tradeoffs are obvious for B2 and C2 alignments where we relate the cutoff frequency to the resonant peaking.

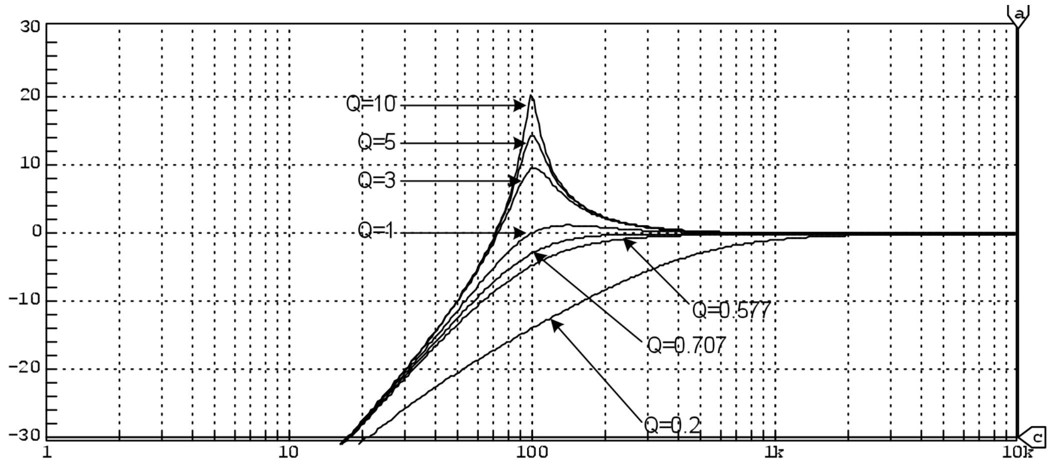


Figure 7.6: comparison of values for Q_{TC} from overdamped ($Q_{TC} = 0.2$) to extremely resonant ($Q_{TC} = 10$)

You can also see the drop in the f_{-3} point as Q_{TC} increases in the original paper's solution plot:

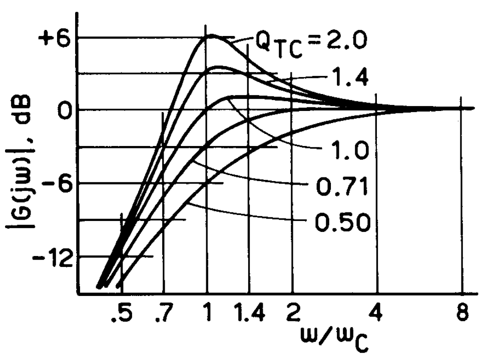


Figure 7.7: the normalized responses from Richard Small's Closed Box Design paper, zoomed in with a max $Q_{TC} = 2$

Transient Response

For over-damped systems, we will get a better transient response. As the resonance increases, the transient response overshoots and rings:

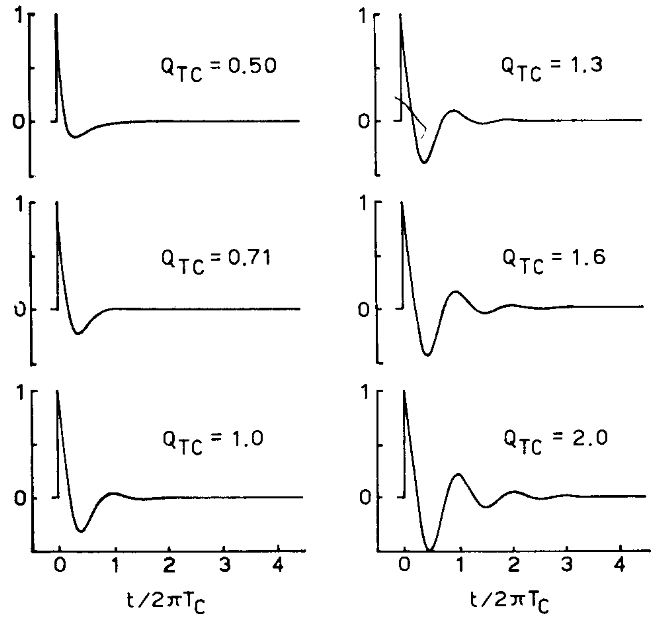


Figure 7.8: Step Responses for various values of Q_{TC}: we observe ringing when the system Q exceeds 0.707

7.6 On Axis Pressure Sensitivity

The Thiele-Small parameter called Sensitivity is the Pressure Sensitivity; it is the magnitude of the mid-band on-axis pressure measured at a distance of 1 meter and applying $1V_{RMS}$ across the voice-coil. These equations are the same as for the Infinite Baffle except the subscripts have been changed to reflect the new system.

$$p_{sens}^{1V} = \frac{\sqrt{2\pi\rho_o}}{c} f_c^{3/2} \left(\frac{V_{AT}}{R_E Q_{EC}} \right)^{1/2}$$

$$p_{sens}^{1V}(dB) = 20\log\left(\frac{p_{sens}^{1V}}{2x10^{-5}\,Pa} \right)$$

$$p_{sens}^{1W}(dB) = 20\log\left(\frac{p_{sens}^{1V}\sqrt{R_E}}{2x10^{-5}\,Pa} \right)$$

7.7 Reference Efficiency

The Reference Efficiency (η) is the ratio of the acoustic power output to the electrical power input. These equations are the same as for the Infinite Baffle except the subscripts have been changed to reflect the new system.

$$\eta_o = \frac{P_{AR}}{P_E} = \frac{4\pi^2}{c^3}\frac{f_c^3 V_{AT}}{Q_{EC}}$$

$$V_{AT} = \frac{V_{AS}V_{AB}}{V_{AS}+V_{AB}} = \frac{V_{AS}}{1+\alpha}$$

The Reference Efficiency can also be written as follows:

$$\eta_o = k_\eta f_{-3}^3 V_{AB}$$

$$k_\eta = \text{ an efficiency constant}$$

$$k_\eta = k_{\eta(Q)}k_{\eta(C)}k_{\eta(G)}$$

$$k_{\eta(Q)} = \text{ losses due to system Qs (box seal)}$$

$$k_{\eta(C)} = \text{ losses due to system compliances}$$

$$k_{\eta(Q)} = \text{ losses due to system frequency response}$$

The individual equations for the loss factors are:

$$k_{\eta(Q)} = \frac{Q_{TC}}{Q_{EC}}$$

$$k_{\eta(C)} = \frac{V_{AT}}{V_{AB}}$$

$$k_{\eta(G)} = \frac{4\pi^2}{c^3}\frac{1}{\left(f_{-3}/f_c\right)^3 Q_{TC}}$$

Examining the middle term for compliance loss we observe something interesting; if we design an enclosure with no fill, we will get a certain efficiency. Adding fill to the box will increase its apparent box volume V_{AB} which will ***decrease*** the loss constant k. This is an argument for the idea that adding fill to the enclosure ultimately makes it more efficient. Do you agree with that argument?

7.8 Displacement Limited Electrical Input Power

The Electrical Power limit of the driver is dependent on the maximum excursion, x_{max}. These equations are the same as for the Infinite Baffle except the subscripts have been changed to reflect the new system.
The power limit will be the amount of power the coil must dissipate at maximum excursion and is found as:

$$P_{E(MAX)} = \frac{1}{2}\frac{\rho_o c^2 \omega_c Q_{EC}}{V_{AT}} V_D^2 \left[\frac{Q_{TC}^2 - 0.25}{Q_{TC}^4 C}\right]$$

or

$$P_{E(MAX)} = \frac{P_{AR(MAX)}}{\eta_o}$$

7.9 Displacement Limited Power Rating

Ultimately, the maximum excursion or X_{MAX} will limit the output power; this power limit can be found by combining the $P_{E(MAX)}$ and reference efficiency equations together. The result is valid for mid-band frequencies that are in between the LF and HF cutoff points. These equations are the same as for the Infinite Baffle except the subscripts have been changed to reflect the new system.

$$P_{AR(MAX)} = \frac{4\pi^3 \rho_o f_c^4}{c} V_D^2 \left[\frac{Q_{TC}^2 - 0.25}{Q_{TC}^4}\right]$$

$$V_D = S_D x_{\max}$$

NOTE: this is valid for Q_{TC} > = 0,707, if Q_{TC} < 0.707, use Q_{TC} = 0.707 instead

> NOTE: Enclosure designs are for bass response only and therefore only apply to woofers. Tweeters and midrange drivers have such tiny excursions that practically any enclosure for a woofer will appear as an Infinite Baffle for these components, so we do not bother designing enclosures for them. That said, some of these components will come with a sealed enclosure already designed and connected. The enclosure may only be there to protect the back of the cones from vibrations from the woofer.

Acoustic Suspension Design Guide

The end result of our design with be a box volume; that volume ultimately controls the overall Q_{TC} of the system. To design the enclosure there are two basic methods; one of them is less accurate because it does not take into account the filling in the enclosure. Enclosure design is an iterative process. You can either start with a given driver and then choose either Q_{TC} or your desired f_c or f_{-3} and work back to the Q_{TC} or you can try to specify the final system responses, Q_{TC} and f_{-3} and find a driver that will meet these specifications. Don't forget that the driver must actually fit in the enclosure!
Remember that we can only increase the resonance of our box system so you need to choose a driver with a resonance lower than the desired. You can use the alignment table for selecting your target Q_{TC} and calculating your final f_{-3} point.

(1) Choose Driver. Get the following Thiele-Small Parameters:

T-S Parameter		Value
f_S		Driver Resonant Frequency
Q_{TS} Q_{ES} Q_{MS}		Resonant Quality Factors (Total, Electrical and Mechanical)
V_{AS}		Volume Compliance
x_{MAX}		Maximum peak displacement
S_D		Surface area of cone
R_E		DC Resistance of the coil
L_E		Inductance of the coil
M_{MS}		Mechanical Mass of the Suspension

(2) Predict Frequency Response:

Vance Dickason, Simplified (from *The Loudspeaker Design Cookbook*)

1) decide on the final Q_{TC} of the system and select a driver that has a Q_{TS} that is ***lower*** than the desired response remember we can only increase the system Q
2) Calculate the compliance ratio (α) and then the box volume and the predicted resonant frequency; this design guesstimates the increase to the box volume as a 20% gain

$$\alpha = \left(\frac{Q_{TC}}{Q_{TS}}\right)^2 - 1$$

$$V_{AB} = \frac{V_{AS}}{\alpha} \text{ unfilled}$$

$$V_{AB} = \frac{V_{AS}}{1.25\alpha} \text{ filled 50\%}$$

$$f_c \simeq \sqrt{1+\alpha}\, f_s$$

Richard Small, Complete (from AES Paper)

1) decide on the final Q_{TC} of the system and select a driver that has a Q_{TS} that is ***lower*** than the desired response remember we can only increase the system Q
2) Guesstimate the mechanical quality factor Q_{MS} using the rules of thumb:

$$5 \le Q_{MC} \le 10 \text{ for unfilled enclosures}$$
$$2 \le Q_{MC} \le 5 \text{ for filled enclosures}$$

3) Calculate QEC, the compliance ratio (α) and then the box volume and the predicted resonant frequency

$$Q_{EC} = \frac{Q_{MC}Q_{TC}}{Q_{MC} + Q_{TC}}$$
$$\alpha = \left(\frac{Q_{EC}}{Q_{ES}}\right)^2 - 1$$
$$V_{AB} = \frac{V_{AS}}{\alpha} \text{ unfilled}$$
$$V_{AB} = \frac{V_{AS}}{1.25\alpha} \text{ filled 50\%}$$
$$f_c \simeq \sqrt{1+\alpha}\, f_s$$

In both designs, the f_{-3} is found with:

$$f_{-3} = f_c\left[\left(\frac{1}{2Q_{TC}^2} - 1\right) + \sqrt{\left(\frac{1}{2Q_{TC}^2} - 1\right)^2 + 1}\right]^{1/2}$$

Both designs produce a final Box Volume. To design the box, we first have to estimate the volume the driver itself takes up:

$$V_{driver}(ft) \approx 6x10^{-6} d^4 ft^3$$
$$V_{driver}(L) \approx 170x10^{-6} d^4 L$$
$$d = \text{ advertised diameter of the driver in inches}$$

Enclosure Dimensions

Once that is done, you need to decide on the dimensions of the enclosure. Two popular rules-of-thumb exist for the relationship between length, width and height:

Golden Ratio: 0.6 x 1.0 x 1.6

Squarish: 0.8 x 1.0 x 1.25

Remember that standing waves will be set up in the enclosure so the dimensions will alter those modes. Perfectly cube and perfectly spherical enclosures are typically (but not always) avoided.

Build the enclosure and measure the input impedance of the speaker. This is done with an AC input source (oscillator), low power amplifier, and a power resistor that you must choose; its value should be in the range of the impedances you expect; 32 ohms is a good place to start. To find the impedance, apply the oscillator to the circuit below and measure the AC input V_{IN} as well as V_x for various frequencies. The impedance at a given frequency is found by back-solving the resistor divider equation:

$$Z_B = \frac{R_X}{V_{IN}/V_X - 1}$$

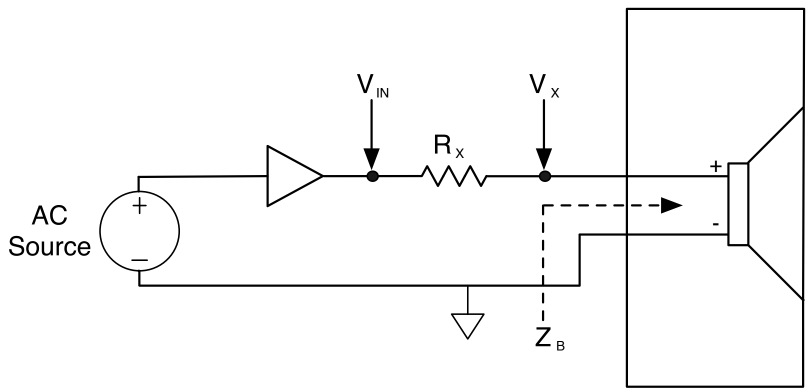


Figure 7.9: Test setup for measuring the input impedance a loudspeaker

Check your f_c by finding the impedance maximum. Adjust the volume of the box until your f_c is tuned properly.

Plot the responses:

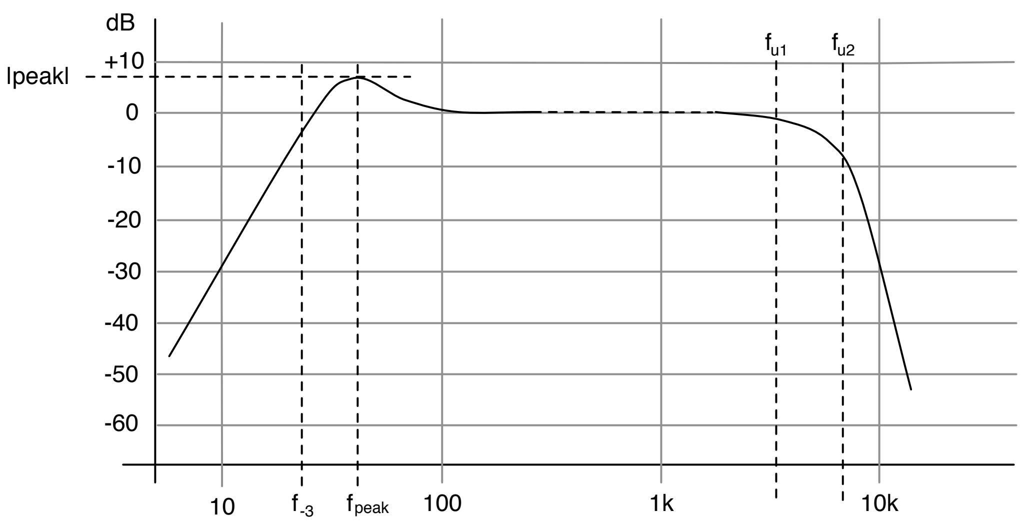


Figure 7.10: Plot the predicted Frequency Response

$$LF \qquad f_{-3} = \left[\beta + \sqrt{\beta^2 + 1} \right]^{1/2} f_s \qquad \qquad \beta = \frac{1}{2Q_{TC}^2} - 1$$

$$f_{peak} = \frac{Q_{TC} f_c}{\sqrt{Q_{TC}^2 - 0.5}} \qquad \qquad |peak| = \frac{Q_{TC}^2}{\sqrt{Q_{TC}^2 - 0.25}}$$

NOTE: Using simpler Infinite Baffle equations here since M_{AB} is very difficult to predict, especially with fill material.

$$HF \qquad f_{u1} = \frac{R_E M_{MS}}{2\pi L_E M_{MD}} \qquad \qquad f_{u2} = \frac{c\sqrt{2}}{2\pi a}$$

$$M_{MD} = M_{MS} - 2S_D^2 M_A \qquad \qquad M_A = \frac{8\rho_o}{3\pi^2 a}$$

(4) Check Power Limits & Efficiency

$$P_{AR(MAX)} = \frac{4\pi^3 \rho_o f_c^4}{c} V_D^2 \left[\frac{Q_{TC}^2 - 0.25}{Q_{TC}^4} \right]$$

$$V_{AT} = \frac{V_{AS} V_{AB}}{V_{AS} + V_{AB}} = \frac{V_{AS}}{1 + \alpha}$$

$$\eta_o = \frac{P_{AR}}{P_E} = \frac{4\pi^2}{c^3} \frac{f_c^3 V_{AT}}{Q_{EC}}$$

$$P_{E(MAX)} = \frac{P_{AR(MAX)}}{\eta_o}$$

(5) Check Sensitivity

$$p_{sens}^{1V} = \frac{\sqrt{2\pi\rho_o}}{c} f_c^{3/2} \left(\frac{V_{AT}}{R_E Q_{EC}} \right)^{1/2}$$

$$p_{sens}^{1V}(dB) = 20\log\left(\frac{p_{sens}^{1V}}{2x10^{-5}\,Pa} \right)$$

$$p_{sens}^{1W}(dB) = 20\log\left(\frac{p_{sens}^{1V}\sqrt{R_E}}{2x10^{-5}\,Pa} \right)$$

Design Example:

Design an Acoustic Suspension enclosure for the Eminence LAB12 Driver from Chapter 6. The alignment is B2. Use a 50% Acousta-Stuf Fill.

From the data-sheet:

$$
\begin{aligned}
f_s &= 22Hz \\
Q_{TS} &= 0.38 \\
Q_{MS} &= 13.32 \\
Q_{ES} &= 0.39 \\
V_{AS} &= 125.2L / 4.4 ft^3 \\
x_{MAX} &= 13.00mm \\
S_D &= 506.7cm^2 \\
R_E &= 8 ohms \\
L_E &= 1.48mH \\
M_{MS} &= 146gm
\end{aligned}
$$

- For B2 Alignment: $Q_{TC} = 0.707$
- Let $Q_{MC} = 3.5$ = a guess for filled enclosures
- Calculate QEC, the compliance ratio (α) and then the box volume and the predicted resonant frequency

$$Q_{EC} = \frac{Q_{MC}Q_{TC}}{Q_{MC} + Q_{TC}} = 0.588$$

$$\alpha = \left(\frac{Q_{EC}}{Q_{ES}}\right)^2 - 1 = 1.274$$

$$V_{AB} = \frac{V_{AS}}{1.25\alpha} = 2.76 ft^3$$

$$f_c \simeq \sqrt{1+\alpha} f_s = 33.2Hz$$

- Find f_{-3} (NOTE: we can use the alignment table $f_{-3}/f_c = 1.0$ for B2 alignments as well as the equation)

$$f_{-3} = f_c \left[\left(\frac{1}{2Q_{TC}^2} - 1 \right) + \sqrt{\left(\frac{1}{2Q_{TC}^2} - 1 \right)^2 + 1} \right]^{1/2} = 33.2Hz$$

- Final box volume:

$$V_{driver}(ft) \approx 6x10^{-6} d^4 ft^3 = 0.124 ft^3$$
$$V_B = V_{AB} + V_{driver} = 2.884 ft^3$$

- Dimensions: use the Golden Ratio and let height be largest length followed by width and depth:

Golden Ratio: 0.6 x 1.0 x 1.6

We can find the 1.0 distance by multiplying the ratios together

$$(0.6x)(x)(1.6x) = 0.96x^3$$
$$x = \sqrt[3]{\frac{2.884}{0.96}} = 1.44 ft = 17.2"$$

We know the diameter of the driver is 12" so the box has to be at least this wide; adding an extra inch on each side gives w = 14" = 1.17ft and since the width is the second largest, it's scaling is 1.0 so we get:

l = (1.6)(1.44) = 2.30ft = 27.6"

w = 1.44ft = 17.2"

d = (0.6)(1.44) = 0.864ft = 10.36"

Is this satisfactory? We need to check the dimensions of the driver to make sure it will fit: the depth is 6.44 inches so we are good to go.

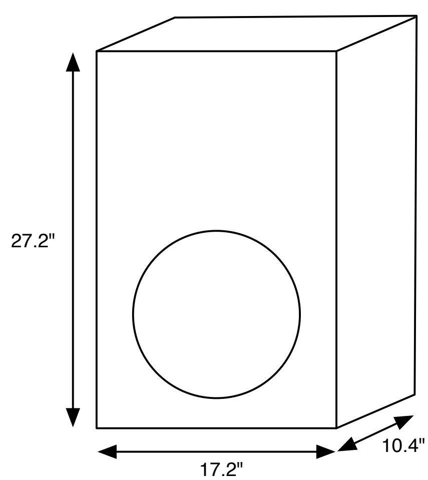


Figure 7.11: Scale drawing of the enclosure (so far)

Since we have a B2 alignment we know there is no peak frequency or magnitude. Our predicted Frequency Response is completed with the upper cutoff:

$$HF \quad f_{u1} = \frac{R_E M_{MS}}{2\pi L_E M_{MD}}$$

$$a = 12cm = 0.12m$$

$$M_A = \frac{8\rho_o}{3\pi^2 a} = 2.65kg$$

$$M_{MD} = M_{MS} - 2S_D^2 M_A = 0.146kg - 2(0.05067)^2(2.65) = 132gm$$

$$f_{u1} = \frac{R_E M_{MS}}{2\pi L_E M_{MD}} = \frac{8(0.146)}{2\pi(0.00148)(0.132)} = 948Hz$$

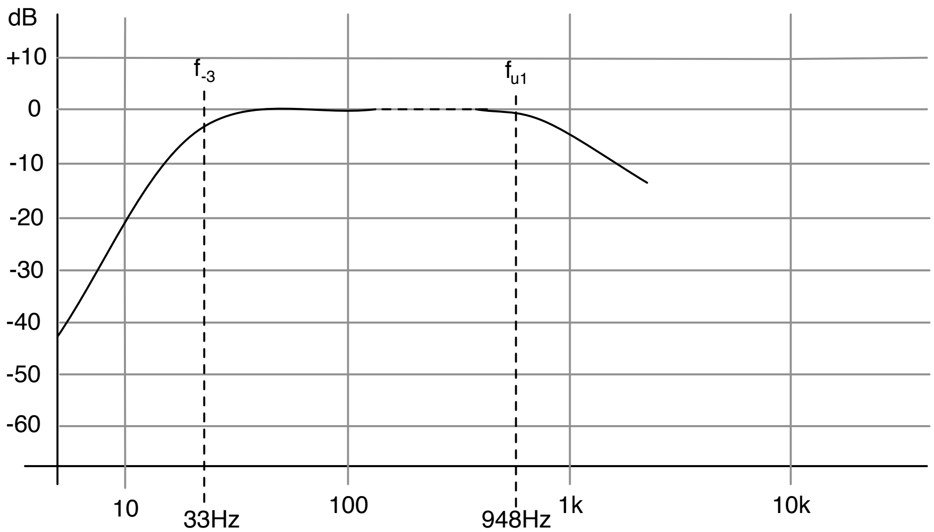


Figure 7.12: The predicted LF response for our Acoustic Suspension system

- Spec Power Limits

$$V_D^2 = \left(x_{\max}\pi a^2\right)^2 = \left(588x10^{-6}\right)^2 = 345.9x10^{-9}m^4$$

$$P_{AR(MAX)} = \frac{4\pi^3\rho_o f_c^4}{c} V_D^2 \left[\frac{Q_{TC}^2 - 0.25}{Q_{TC}^4}\right] = \frac{4\pi^3(1.18)33^4}{345} 345.9x10^{-9}\left[\frac{(0.707)^2 - 0.25}{(0.707)^4}\right] = 0.174W_{Acoustic}$$

$$V_{AT} = \frac{V_{AS}V_{AB}}{V_{AS} + V_{AB}} = \frac{V_{AS}}{1+\alpha} = \frac{125.4}{1+1.274} = 55.1L$$

$$\eta_o = \frac{P_{AR}}{P_E} = \frac{4\pi^2}{345^3}\frac{33^3 0.0551}{0.588} = 0.00323 = 0.3\% \quad \text{note: converted L to m}^3$$

$$P_{E(MAX)} = \frac{0.174}{0.00323} = 54.38W$$

- Spec Sensitivity:

$$p_{sens}^{1V} = \frac{\sqrt{2\pi\rho_o}}{c} f_c^{3/2} \left(\frac{V_{AT}}{R_E Q_{EC}} \right)^{1/2} = \frac{\sqrt{2\pi(1.18)}}{345} 33^{3/2} \left(\frac{0.0551}{8(0.588)} \right)^{1/2} = 0.162$$

$$p_{sens}^{1V}(dB) = 20\log\left(\frac{p_{sens}^{1V}}{2x10^{-5} Pa} \right) = 78.1dB$$

8 The Bass-Reflex Enclosure

The Bass-Reflex Enclosure augments the Closed-Box with a tube, port or vent. This enclosure goes by the following names:

- Bass Reflex
- Ported
- Vented
- Thiele-Small

This clearly un-seals the enclosure. Vented boxes typically only have a thin layer of acoustic filling (1-2”) glued to the inner walls to prevent standing waves. But too much filling can interfere with the operation of the tube. The fundamental principle of operation of this enclosure is to use the tube/enclosure as a Helmholts Resonator at very low frequencies to extend the low frequency response of a speaker system. This means we are adding a resonator to our existing enclosure design. This adds two another reactive component (the mass in the tube) and another degree of freedom in design. Great care must be taken in the design and fabrication of these enclosures - not for beginners. The iterative process may go on for quite some time before the final version is tuned and fully tested as ready for use.

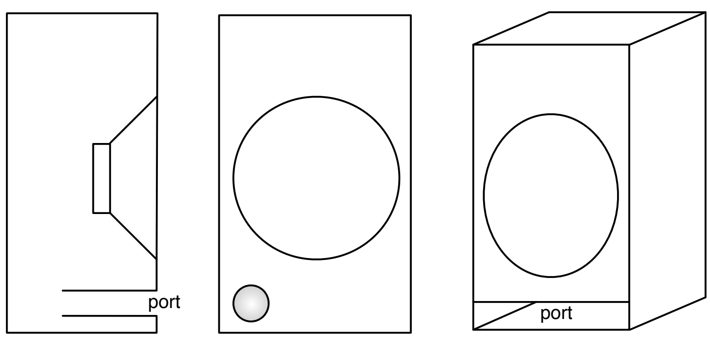


Figure 8.1: The Bass-Reflex Enclosure features a port or vent; the port does not have to be circular, though it usually is for non-PA type speakers

The advantages and disadvantages over a Closed-Box System are in table 8.1

Pros		Cons
can have a lower f_{-3} than a closed-box with the same efficiency η		but, the enclosure will be larger than the closed-box
can have a better efficiency η than a closed-box with the same volume		but the f_{-3} will usually be the same or higher than the close-box
extends response with Helmholtz Resonator		the resonator can cause unwanted audible effects

Table 8.1: some pros and cons of the Bass Reflex Enclosure

8.1 Modeling The Bass-Reflex Enclosure

The model for a Bass-Reflex Enclosure consists of three components: the driver, the port, the chamber (compliance of enclosure) and the losses due to leaks in the enclosure (seal) and losses due to absorption. This is the only enclosure where we try to take these losses into account; the construction is more difficult than a closed-box design. Richard Small starts with a basic assumption that each of the components has an associated volume velocity, U:

- The Driver Volume Velocity: U_D
- The Port Volume Velocity: U_P
- The Box Volume Velocity: U_B
- The Lossy-Leak Volume Velocity: U_L
- The Lossy-Absorption Volume Velocity: U_A

The value of U_L is determined by the quality of the carpentry/construction of the enclosure. Each of these Volume Velocities has an associated Q for the resonance of the Volume Velocity:

- The Driver Q: Q_D
- The Port Q: Q_P
- The Box Q: Q_B
- The Lossy-Leak Q: Q_L
- The Lossy-Absorption Q: Q_A

The value of Q_L is determined by the quality of the carpentry/construction of the enclosure. The complete circuit is shown in Figure 8.2.

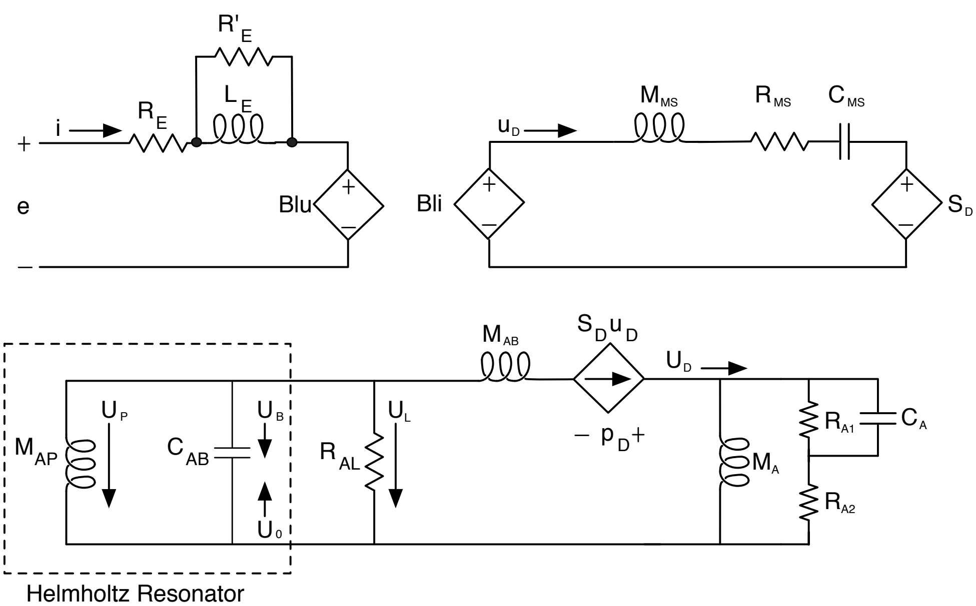


Figure 8.2: The complete electro-mechanico-acoustical model circuit for the Bass Reflex Enclosure; a Helmholtz Resonator (HH) is connected to the back side of the driver.

$$M_{AP} = \frac{\rho_o}{S_P}\left(L_P + 1.462\sqrt{\frac{S_P}{\pi}} \right)$$

$S_P =$ cross sectional area of port

$L_P =$ total length of port including flanged and unflanged corrections

$$C_{AB} = \frac{V_{AB}}{\rho_o c^2}$$

The resistive air loss R_{AL} has to be guesstimated. M_{AB} is the mass of air trapped in the filling in the box and must also be estimated. You will notice that all three volume velocity components M_{AP}, C_{AB}, and R_{AL} connect to the other side of the air load because the port opens to the outside connecting the air load on the front to the air load on the back of the cone.

8.2 The LF Combination Analogous Circuit

Applying the Norton Equivalent Circuit modeling technique with some math crunches the three circuits into one analogous circuit for low frequencies. The M_{AC} component models the combined masses on the diaphragm.

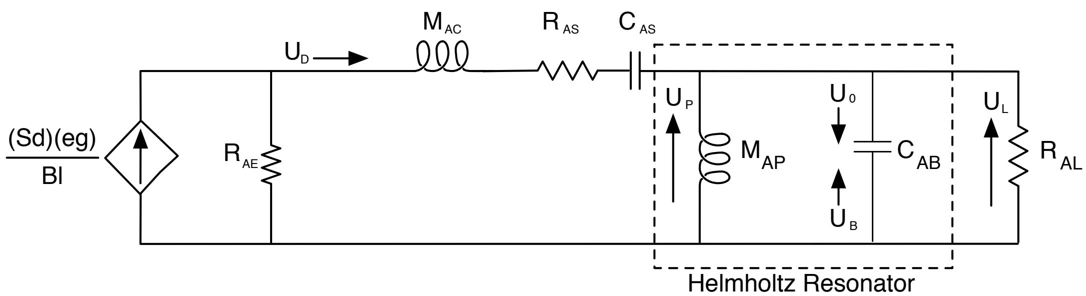


Figure 8.3: The LF Combination Analogous Circuit for the Bass-Reflex Enclosure; notice there are 4 reactive components

$$M_{AC} = M_{AD} + M_{AB} + M_A$$

8.3 Bass-Reflex Operation

The Bass-Reflex Enclosure is interesting because it combines a Helmholtz Resonator (the tube + enclosure chamber) to augment the bass response of the driver. The Helmholtz Resonator has its own resonant frequency f_B and quality factor Q_B. To understand the behavior, we break the problem down into three parts - below f_B at f_B and above f_B.

Below f_B

Below the HH resonant frequency, the driver experiences an effect called ***unloading***. The driver moves back and air blows out through the port. The driver moves out and air is sucked into the port. Therefore, the driver feels no real acoustic load from the enclosure. This can be a problem in Bass-Reflex designs; in some cases a HPF is used to prevent very low frequencies from getting to the driver possibly causing damage from exceeding x_{MAX}. The Resonator is not producing any output.

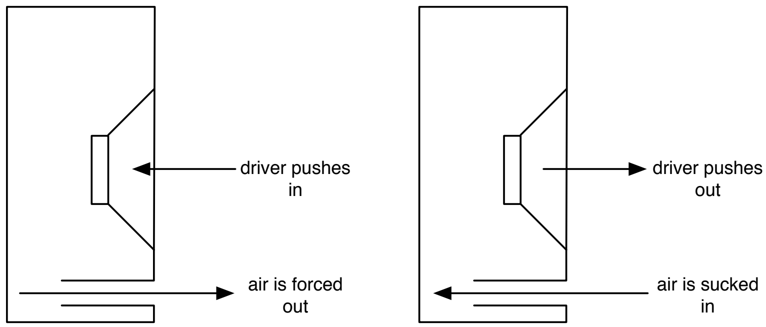


Figure 8.4: Below the HH frequency, unloading occurs as air moves freely from outside to inside the enclosure and back. All of the output of the system comes from the driver as the HH resonator is effectively "off."

Above f_B

Above the HH resonant frequency, the driver's excursion is small and the box behaves as an infinite baffle. The Resonator is not producing any output.

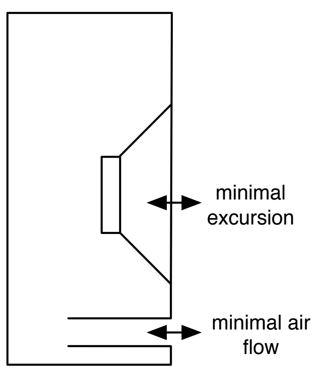


Figure 8.5: Above the HH frequency, excursion is limited, All of the output of the system comes from the driver as the HH resonator is effectively "off"

At f_B

At the HH resonant frequency is where the magic happens. Something very interesting occurs. We are energizing the HH resonator from the back side. The amount of delay time it takes for the acoustic wave to reach the opening of the port on the front is enough to exactly invert its phase 360 degrees, placing the output of the HH resonator in-phase with the front side of the cone. The HH resonator is in phase with the driver!

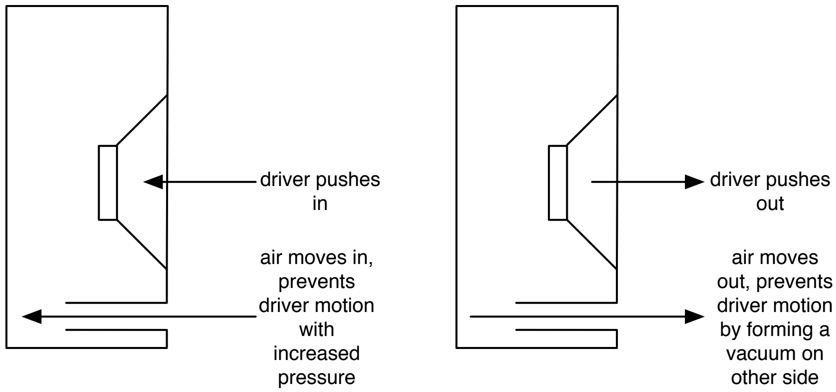


Figure 8.6: At the HH frequency, the driver does not move at all in theory; 100% of the audio output is from the port.

So what follows is quite amazing: when the driver tries to push outward, air is flowing out of the port at the same time preventing it from moving out and when the driver tries to push inward, air is flowing into the port, stopping the driver from moving in. This means that at the HH resonant frequency, the driver is not moving at all and 100% of the system output is coming from the port! However, the driver is really trying hard to move. This causes excess mechanical stress on the driver suspension at the HH frequency. This presents another problem with these designs; the drivers undergo a large amount of mechanical stress at the HH frequency and this can cause mechanical failure.

8.4 Driver Excursion & Resonator Output

We can plot the driver excursion and resonator output together on the same graph. You see in Figure 8.7 that the Driver Excursion reaches a theoretical minimum at the HH frequency.

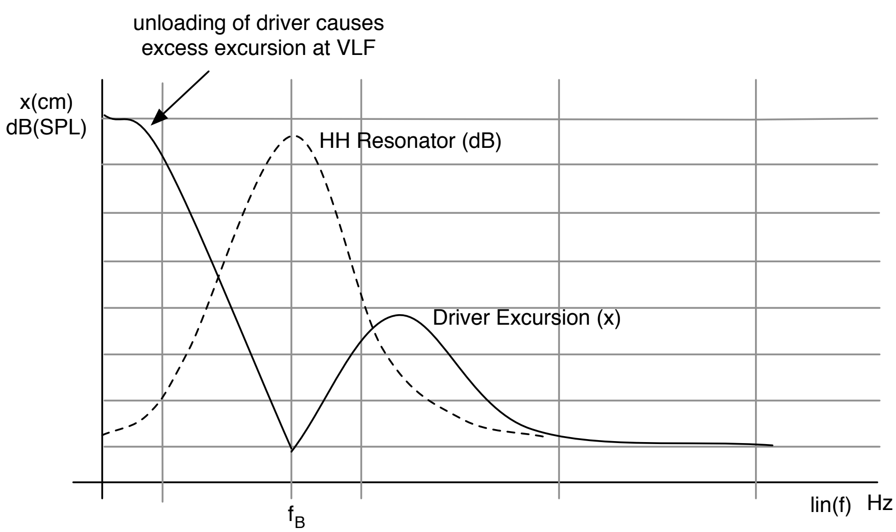


Figure 8,7: Plotting the driver excursion (x) and the HH output (dB) reveals how the Bass-Reflex Enclosure operates.

The null in the excursion is at the HH frequency; below that the excursion is excessive due to unloading and above it, the excursion is naturally very small. Figure 8.8 shows some different combinations of aligning the box resonant frequency with the driver response. Changing the alignment can create over, under, or just damped curves. You can see that minor errors in the alignment of the resonances could have disastrous results in the frequency response as unwanted dips or peaks outside the driver's resonant range.

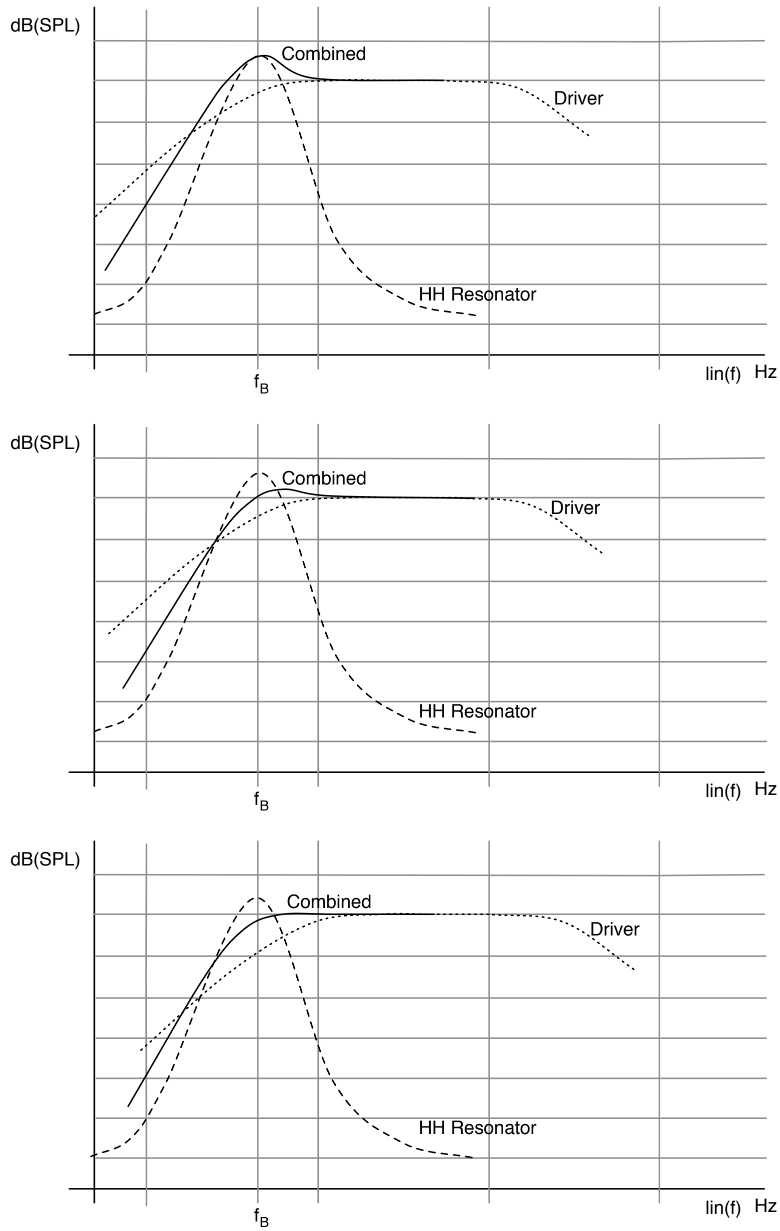


Figure 8.8: Three possible alignments between the driver and the HH resonator. The upper 2 graphs exhibit peaking in the response so we might label them Chebychev while the lower one appears to be maximally flat (Butterworth).

8.5 Problems with the Vent

We discussed that the vent will cause unloading and therefore mechanical stresses on the driver. In addition, at the HH frequency when the driver's excursion is 0, there is no back EMF being generated so the input impedance is R_E alone; this creates excess current flowing at the resonant frequency. Therefore at the HH frequency, the driver must dissipate much more power as heat than at the system resonant frequency of an acoustic suspension enclosure.

Chuffing/Windage

Figure 8.9 shows a condition that must be avoided; the back of the vent must be at least 3” from a wall of the enclosure. When this occurs, audible noise is created due to the flow of air around the vent opening. This is sometimes called “*chuffing*” or “*windage*.” Figure 8.9 also shows several solutions. You can extend the port outside the enclosure (in fact, there’s nothing illegal about the whole length of the port being outside the enclosure other than aesthetics). You can also curve the vent to provide the same port length but with an easier geometry. Alternatively multiple ports can be used.

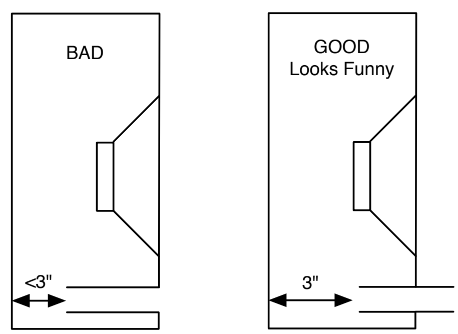


Figure 8.9: Improper location of the back of the vent near enclosure surfaces can result in audible noise; two solutions are shown.

Organ Pipe Resonances

If the tube length turns out to be very long, standing waves can be set up in the tube at harmonics of the tube length. These can cause peaks or notches in the response. The location of the vent relative to the driver is also a factor because of the mutual coupling that will occur between the two. The resonances are very unpredictable and given a complete frequency response plot, might be difficult to separate from mechanical resonances and standing waves on the surface of the cone.

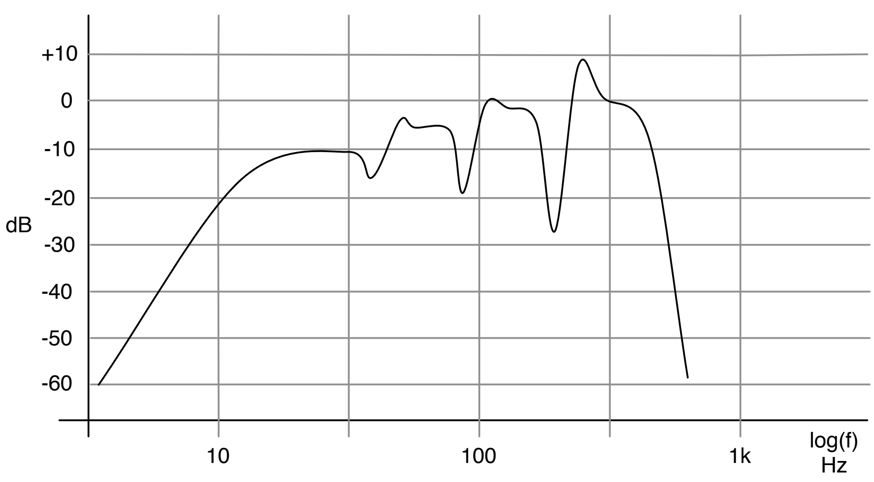


Figure 8.10: This greatly exaggerated plot shows what pipe-resonances alone wold look like; in this case they get worse as the frequency gets higher.

8.6 On-Axis Pressure Transfer Function

The total Volume Velocity U_0 is the sum of the three Volume Velocities we discussed in the beginning of the chapter.

$$U_0 = U_D + U_L + U_P = -U_B$$

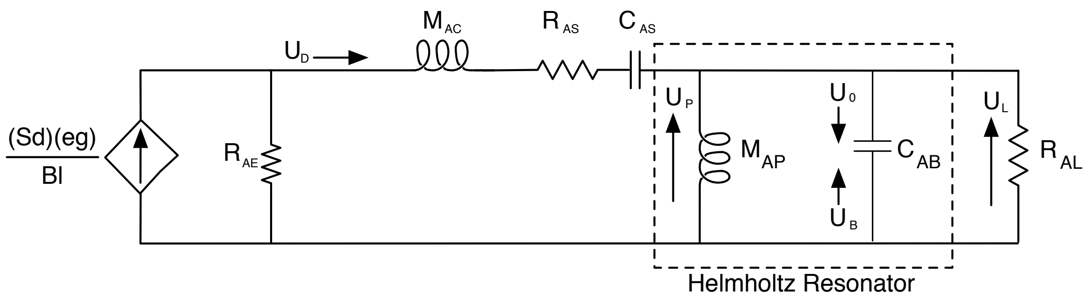


Figure 8.3 (again): The combination analogous circuit

Deriving the total Volume Velocity requires a large amount of algebra. We wind up with two resonances, f_B the resonant frequency of the box/port combination and f_s the resonant frequency of the driver. The final equation is:

$$U_0(s) = \frac{Ble_g}{S_D R_E M_{AS}} \frac{s^3/\omega_0^4}{\left(s/\omega_0\right)^4 + a_3\left(s/\omega_0\right)^3 + a_2\left(s/\omega_0\right)^2 + a_1\left(s/\omega_0\right) + 1}$$

$$\omega_0 = \sqrt{\omega_S \omega_B}$$

$$a_1 = \frac{1}{Q_L\sqrt{h}} + \frac{\sqrt{h}}{Q_{TS}} \qquad a_2 = \frac{\alpha+1}{h} + h + \frac{1}{Q_L Q_{TS}} \qquad a_3 = \frac{1}{Q_{TS}\sqrt{h}} + \frac{\sqrt{h}}{Q_L}$$

$$Q_L = R_{AL}\sqrt{\frac{C_{AB}}{M_{AP}}}$$

Take a look at the Q_L term here - normally for RLC resonant circuits the Q is defined like this:

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

which is the opposite of the Q_L here. For this design, the Q_L is inverted which means that higher values of Q_L represent less-lossy enclosures.

Notice the Volume Velocity is a function of s (j ω). We define the following terms to be used in our designs:

Tuning Ratio $h = \frac{\omega_B}{\omega_S} = \frac{f_B}{f_S}$

Compliance Ratio $\alpha = \frac{V_{AS}}{V_{AB}} = \frac{C_{AS}}{C_{AB}}$

Inverse Compliance Ratio $S = \frac{1}{\alpha} = \frac{V_{AB}}{V_{AS}}$

The On-Axis Pressure Response is:

$$p = \frac{\rho_o}{2\pi} j\omega U_0$$

After some more algebra we can represent the solution as:

$$p = \frac{\rho_o}{2\pi} \frac{Ble_g}{S_D R_E M_{AS}} \frac{(s/\omega_0)^4}{(s/\omega_0)^4 + a_3(s/\omega_0)^3 + a_2(s/\omega_0)^2 + a_1(s/\omega_0) + 1}$$

$$= \frac{\rho_o}{2\pi} \frac{Ble_g}{S_D R_E M_{AS}} G(s)$$

$$G(s) = \frac{(s/\omega_0)^4}{(s/\omega_0)^4 + a_3(s/\omega_0)^3 + a_2(s/\omega_0)^2 + a_1(s/\omega_0) + 1}$$

The pressure is also a function of s and the filter response is a 4th order HPF. The coefficients a_1, a_2, and a_3 determine the shape of the HPF varying from over-damped to resonant (Chebychev). When we design an enclosure we are really finding the dimensions of the box and tube which create the coefficients we need for a given filter response. This all depends on the driver we select and our carpentry skills: notice we only design with the Q_L term.

8.7 Alignments

NOTE: Enclosure designs are for bass response only and therefore only apply to woofers. Tweeters and midrange drivers have such tiny excursions that practically any enclosure for a woofer will appear as an Infinite Baffle for these components, so we do not bother designing enclosures for them. That said, some of these components will come with a sealed enclosure already designed and connected. The enclosure may only be there to protect the back of the cones from vibrations from the woofer.

The Alignments are based on the magnitude squared version of the pressure transfer function's frequency dependent term, G(s). See ***Vented-Box Loudspeaker Systems Part IV: Appendices*** by Richard Small for the derivations of the alignments that follow. We start with the generalized magnitude squared transfer function:

$$\left|G_V(s)\right|^2 = \left|G_V(j2\pi f)\right|^2 = \frac{\left(f/f_0\right)^8}{\left(f/f_0\right)^8 + A_3\left(f/f_0\right)^6 + A_2\left(f/f_0\right)^4 + A_1\left(f/f_0\right)^2 + 1}$$

$$A_1 = a_1^2 - 2a_2$$
$$A_2 = a_2^2 + 2 - 2a_2a_3$$
$$A_3 = a_3^2 - 2a_2$$

For any given Alignment the solution is found by setting the magnitude squared transfer function to 0.5 (half power or -3dB in magnitude) and solving for the roots. This is done with the intermediate variable d:

$$d = \left(f_{-3}/f_0\right)^2$$
and
$$\left|G_V(j2\pi f)\right|^2 = \frac{1}{2}$$

solve for the roots of the resulting equations

$$d^4 - A_3d^3 - A_2d^2 - A_1d - 1 = 0$$
and
$$r^4 - (a_3Q_L)r^3 + (a_1Q_L)r - 1 = 0$$

The solutions are related to our design parameters as follows (these are the answers!)

$$h = \frac{f_B}{f_S} = r^2 \qquad q = \frac{f_{-3}}{f_S} = r\sqrt{d} \qquad Q_{TS} = \frac{r^2Q_L}{a_1rQ_L - 1}$$

$$\alpha = \frac{V_{AS}}{V_B} = r^2\left(a_2 - \frac{1}{Q_LQ_{TS}} - r^2\right) - 1$$

There are three common alignments however the Chebychev alignments contain a family of responses which generate various amounts of ripple in passband and a resonant hump at the edge of the stop-band. The alignments for Bass-Reflex are Q_{TS} dependent as with the Acoustic Suspension. However, since we are also including the Helmholtz Resonator in the design, our target Q_{TS} value changes; instead of hinging on the 0.707 value, it now revolves around a range of values depending on the loss value, Q_L.

Q_L ranges in value from 3 to infinity; these are empirical measurements made on actual cabinets and drivers. The higher the number the less-lossy the enclosure. A Q_L of infinity would represent a perfect, lossless enclosure. Small notes "the enclosures tested were well built and appeared to be quite leak-free. In fact, some of the more serious leaks were traced to the drivers. These leaks were caused by imperfect gasket seals and/or by leakage of air through a porous dust cap." We typically use Q_L = 7 as a starting point then tune the final system to this value. **For Q_L = 7, a Q_{TS} of 0.405 will give the 4th Order Butterworth response**; the others are based on it's Q_{TS} value.

This table is for $Q_L = 7$:

Q_{TS}	Name	Abbv.	q: f_{-3} vs. f_s	h: f_B vs. f_s
< 0.405	Quasi-Butterworth	QB3	$q > 1$: $f_{-3} > f_S$	$h > 1$: $f_B > f_S$
0.405	Butterworth	B4	$q = 1$: $f_{-3} = f_S$	$h = 1$: $f_B = f_S$
> 0.405	Chebychev	C4	$q < 1$: $f_{-3} < f_S$	$h < 1$: $f_B < f_S$

For the Butterworth Design here is the Q_L and Q_{TS} relationship (remember, the higher the Q_L the leakier the enclosure). The other designs are based on it.

Q_L	Q_{TS}
infinity	0.383
20	0.39
10	0.398
7	0.405
5	0.414
3	0.439

8.8 The B4 Alignment

This seems to be the most sought after alignment because it has the best tradeoff of good transient response and low-end response. There is no resonant peaking. The 4th Order Butterworth Response is:

$$\left|G_V(j2\pi f)\right|^2 = \frac{\left(f/f_0\right)^8}{\left(f/f_0\right)^8 + 1}$$

Thus $A_3 = A_2 = A_1 = 0$ and:

$$a_2 = 2 + \sqrt{2}$$
$$a_3 = a_1 = \sqrt{2a_2}$$

NOTES:

- The B4 Alignment is the only one that has $q = 1$ for all values of Q_L so its f_{-3} will always equal the driver resonant frequency f_S
- The B4 Alignment has $h = 1$ for all values of Q_L so the box resonance f_B is exactly aligned with the driver resonance f_S
- The Butterworth Response is maximally flat through out the passband and has no peaking; it is 4th order so it rolls off at 24dB/octave
- It is a good tradeoff between transient response and low end
- For a given Q_L there is only one value of Q_{TS} that can produce a B4 alignment; this is not true for the Acoustic Suspension where any Q_{TS} could be used as long as it was below the target Q_{TC}

8.9 The QB3 Alignment

The Quasi-3rd Order Butterworth alignment is still technically a 4th order system, but its slope rolls off less steeply at frequencies far from the cutoff; the name Quasi-Butterworth came from Thiele.

$$\left|G_V(j2\pi f)\right|^2 = \frac{\left(f/f_0\right)^8}{\left(f/f_0\right)^8 + B^2\left(f/f_0\right)^2 + 1}$$

The value of B determines the response; when B = 0 we get the B4 Alignment. For the QB3 Alignment you specify B > 0 and solve for the roots. Comparing the equation to our generalized magnitude squared function reveals that:

$A_2 = A_3 = 0$

$A_1 = B^2$

therefore:

$$B^2 = a_1 - 2a_2 2$$

$$a_2 > 2 + \sqrt{2}$$

$$a_3 = \sqrt{2}a_2$$

$$a_1 = \frac{a_2^2 + 2}{2a_3}$$

NOTES:

- The QB3 Alignment will require a Q_{TS} that is ***lower*** than the B4 Alignment
- The QB3 Alignment will have a higher f_{-3} than the B4
- The Butterworth Response is maximally flat through out the passband and has no peaking; it is Quasi-3th order so it rolls off at 18dB/octave far from the cutoff
- Its transient response is about the same as the B4; perhaps only slightly better

The two Butterworth Alignments are plotted here by holding the f_{-3} constant. This shows a tradeoff between bass just below the cutoff.

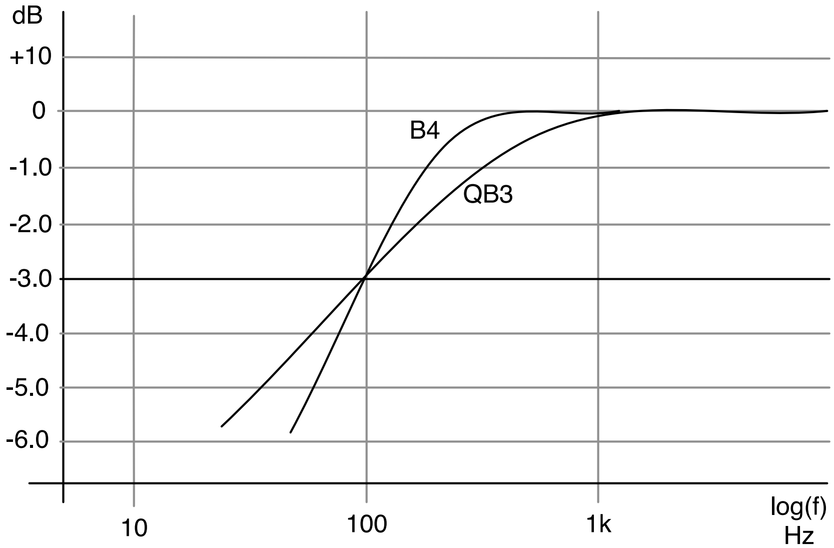


Figure 8.10: The B4 Alignment has more bass before cutoff but the QB3 Alignment has more bass after cutoff.

8.10 The C4 Alignment Family

Chebychev filters are characterized by their ripple in the passband. The ripple varies between 1 and 1+ ε^2 as shown in Figure 8.6 below:

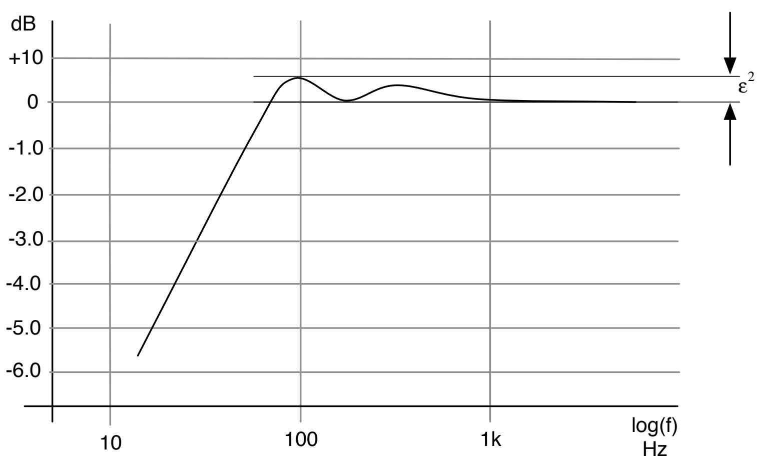


Figure 8.11: Chebychev responses are based on the amount of ripple in the passband.

Therefore, there are a family or Chebychev responses given by the magnitude squared function:

$$|G_V(j2\pi f)|^2 = \frac{1+\varepsilon^2}{\varepsilon^2\left(8\left(f_n/f\right)^4 - 8\left(f_n/f\right)^2+1\right)^2+1}$$

$$f_n = \frac{f_{-3}}{\sqrt{2}}\left[1+4\sqrt{2+\frac{1}{\varepsilon^2}}\right]^{1/2}$$

$$\varepsilon = \sqrt{10^{dB(ripple)/10}-1}$$

We define a factor k that is related to the ripple amount by:

$$k = \tanh\left[\frac{1}{4}\sinh^{-1}\left(\frac{1}{\varepsilon}\right)\right]$$

The family of Chebychev responses can now be related to the value of k for our designs. Figure 8.7 shows several of these Chebychev curves displaced so you can see the different ripple amount. As k decreases, the ripple amount increases:

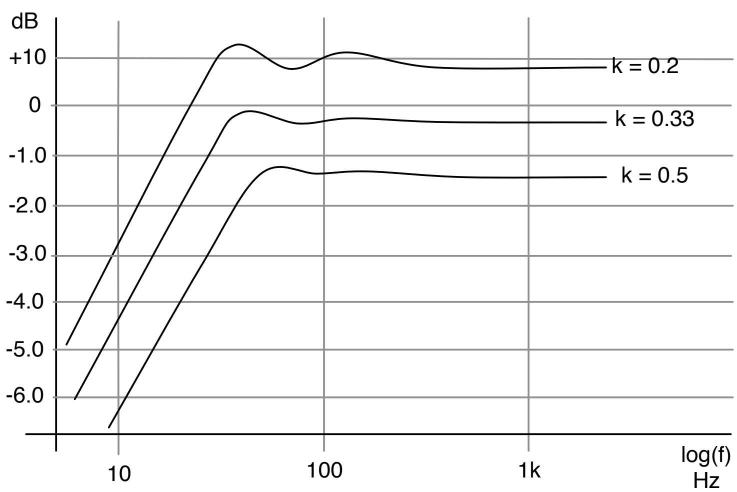


Figure 8.12: Several different responses for a few values of k.

To calculate the Chebychev Alignment data, you first specify the ripple in dB and calculate ε and k. Then:

$$D = \frac{k^4+6k^2+1}{8}$$

$$a_1 = \frac{k\sqrt{4+2\sqrt{2}}}{D^{1/4}} \qquad a_2 = \frac{1+k^2(1+\sqrt{2})}{D^{1/2}} \qquad a_3 = \frac{a_1}{D^{1/2}}\left(1-\frac{1-k^2}{2\sqrt{2}}\right)$$

NOTES:
- The C4 Alignment will require a Q_{TS} that is ***higher*** than the B4 Alignment
- The C4 Alignment will have a lower f_{-3} than the B4
- The Chebychev responses will reveal peaking and rippling in the passband
- Its transient response is not as good as the B4 Alignment

8.11 Alignment Comparisons

When plotted together you can see the differences in the alignments; the two curves at the right are both QB3 with various different values for the B factor:

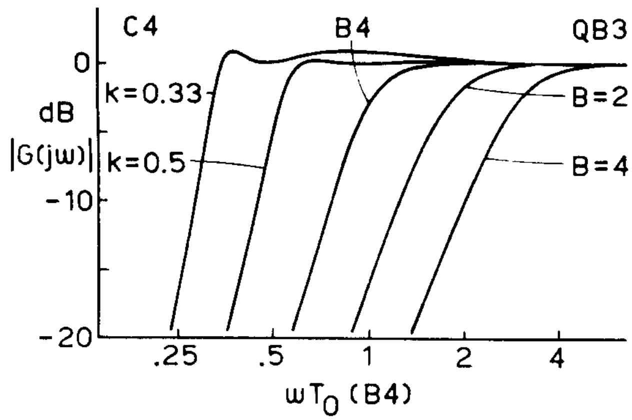


Figure 8.13: All the alignments plotted together and offset to see the differences in peaking and rolloff slope.

The Transient (Step) responses are shown below. These reveal ringing for all but the SC4 response. The KIEBS and SC4 (Sub-Chebychev 4th Order) are considered experimental.

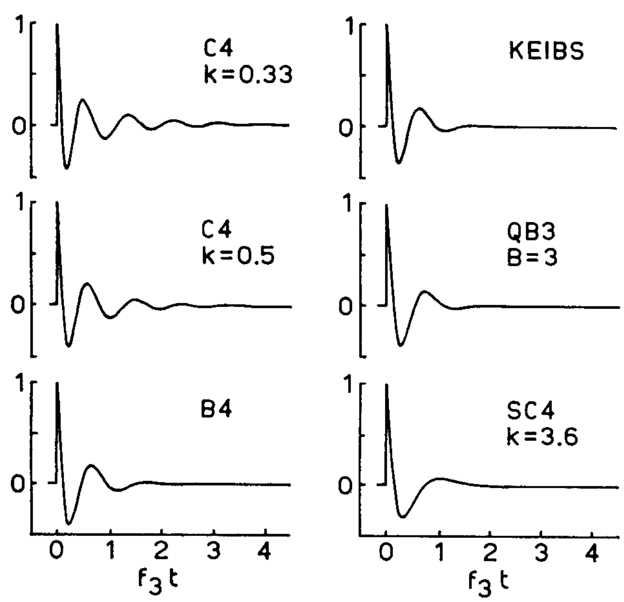


Figure 8.14: The normalized step responses for the various alignments.

8.12 Voice Coil Impedance

When looking into the voice coil terminals we see the effects of the unloading at low frequencies, the null motion of the driver at the HH resonant frequency, and the effect of the vent together. If we assume the air load on the diaphragm is the same as for an infinite baffle the voice coil impedance function is

$$Z_{VC}(s) = R_E + R_{ES}\frac{(1/Q_{MS})(s/\omega_S)\left[(s/\omega_B)^2 + (1/Q_L)(s/\omega_B)\right]}{(s/\omega_0)^4 + b_3(s/\omega_0)^3 + b_2(s/\omega_0)^2 + b_1(s/\omega_0) + 1}$$

$$R_{ES} = R_E\frac{Q_{MS}}{Q_{ES}}$$

$$b_1 = \frac{1}{Q_L\sqrt{h}} + \frac{\sqrt{h}}{Q_{MS}} \qquad b_2 = \frac{\alpha+1}{h} + h + \frac{1}{Q_L Q_{MS}} \qquad b_3 = \frac{1}{Q_{MS}\sqrt{h}} + \frac{\sqrt{h}}{Q_L}$$

The circuit that produces this impedance is doubly-resonant and is shown in Figure 8.12. It consists of both Series RLC and Parallel RLC networks, each of which produces a bandpass response. (Why?)

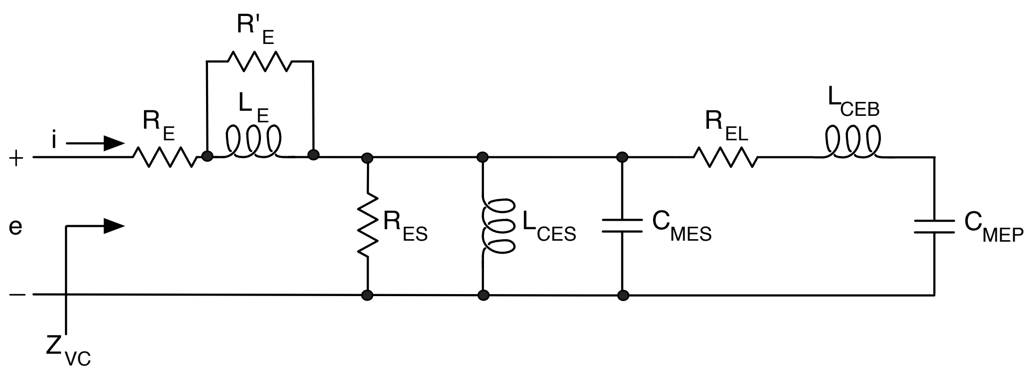


Figure 8.15: The voice-coil equivalent circuit.

$$R_{ES} = R_E\frac{Q_{MS}}{Q_{ES}} \qquad L_{CES} = (Bl)^2 C_{MS} \qquad C_{MES} = \frac{M_{MS}}{(Bl)^2}$$

$$R_{EL} = \frac{(Bl)^2}{S_D^2 R_{AL}} \qquad L_{CEB} = \frac{(Bl)^2 C_{AB}}{S_D^2} \qquad C_{MEP} = \frac{S_D^2 M_{AP}}{(Bl)^2}$$

The voice coil plot reveals two impedance maxima. The null between the two peaks is where the driver is not moving and there is no back EMF: this is at the Helmholtz Frequency. The twin peaks are caused by the two resonant sub-circuits. The null point between them occurs at the Helmholtz Frequency. The impedance measurement is one way to check the port tuning. In the case of Q_{TS} = 0.4 (approx) the two peaks will have the same height; this will usually correspond to a B4 alignment.

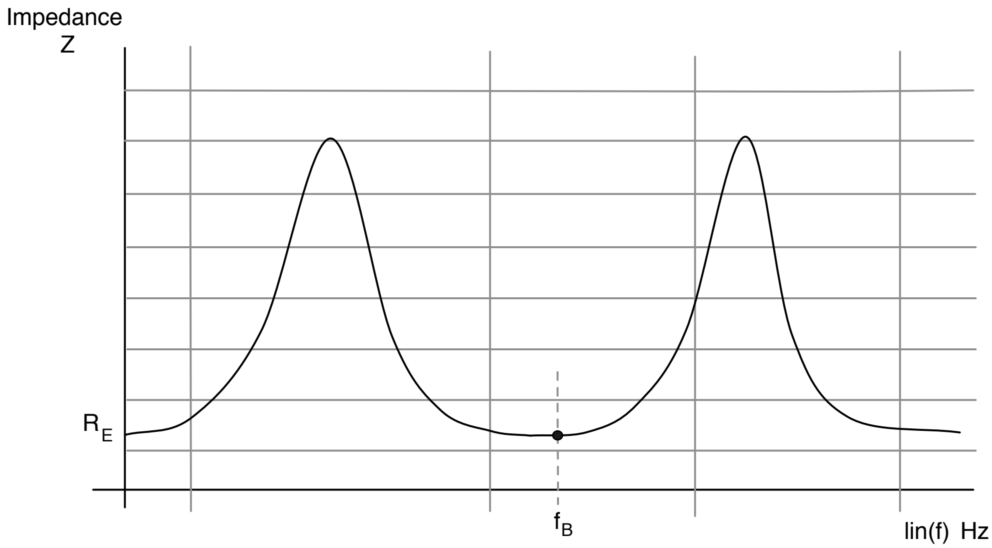


Figure 8.16: The predicted impedance plot for the voice coil equivalent circuit. The HH Frequency is the null between the peaks.

8.13 Power, Efficiency and Excursion

The excursion limited output power $P_{AR(\max)}$ is:

$$V_{D(\max)} = x_{\max}\pi a^2$$

$$P_{AR(\max)} = 3\left[V_{D(\max)} f_{-3}^2\right]^2$$

The reference efficiency and maximum electrical input power are:

$$\eta_0 = \frac{4\pi^2}{c^3}\frac{f_S^3 V_{AS}}{Q_{ES}}$$

$$P_{E(\max)} = \frac{P_{AR(\max)}}{\eta_0}$$

8.14 Enclosure Design with Nomographs

Bass Reflex Enclosure design is much more involved than the simple closed box case. It is also more limited. For the Acoustic Suspension, we are free to design any alignment we want as long as the final Q_{TC} is higher than the driver's Q_{TS} (ie we can only increase resonance). For the Bass Reflex design, the driver's Q_{TS} dictates the alignment instead. If you go back and think about all the equations/variables required for the three alignments you will see that you need:

For all three

- Q_{TS}
- Q_L
- α

- q
- h

For QB3
- B

For C4
- k

The resulting design solution are a box volume V_{AB} and a box frequency f_B from which the port dimensions are found. But there are far more variables and equations than these two solutions. One way to deal with this is to write a program for making all the calculations - you can do that with all the equations in this Chapter but you might have some problems with the analog filter coefficients if you do not have a background in analog filter design. An older, and more interesting solution is called a nomograph. It consists of solutions to all the equations in the form of axes and curves. You use a ruler to draw lines that cross the curves and axes and then read the values off of the graph. These have been used since the 1950's for solving engineering problems without using a calculator or computer. The equations only need to be solved once and then graphed. The method is easy to understand. The first thing to note is that we will need a separate nomograph for each Q_L we can specify. Since this is an estimate right from the beginning, we don't need a whole lot of graphs. Richard Small specified nomographs for the following values: Q_L = 3, 5, 7, 10, 20 and infinity (lossless).

Let's examine one of these nomographs first, the one we will usually start with Q_L = 7.

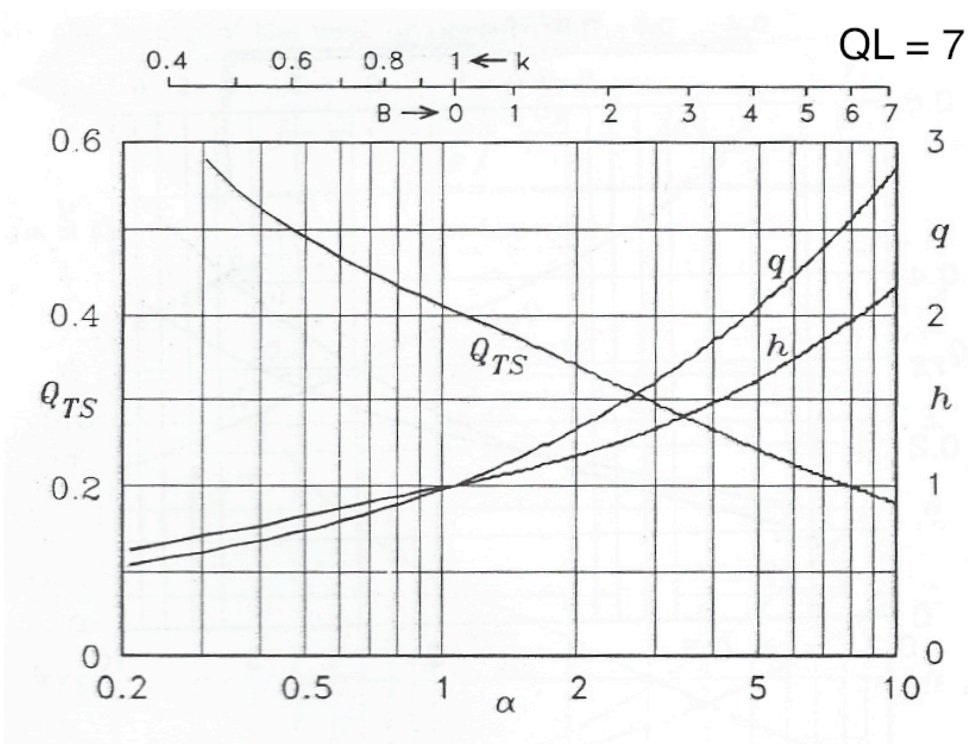


Figure 8.17: Nomograph for Q_L = 7

You can see that all the variables are represented; the right axis has values for both q and h while the top axis has both B (QB3) and k (C4) variables. You can also see that the range of values for Q_{TS} that will give acceptable designs is about 1.5 to 0.58 (the left axis) and the range of compliance ratios (α) is about 0.25 to 10 which is a fairly large range (this value sets the final box volume). You can design the enclosure in less than a minute by simply drawing two perpendicular lines on this graph.

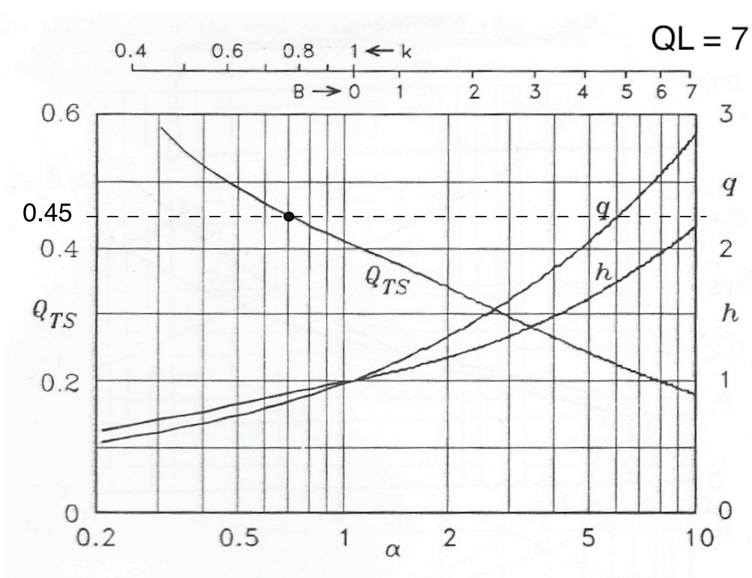
Bass Reflex Design Guide

(1) Choose Driver. Get the following Thiele-Small Parameters:

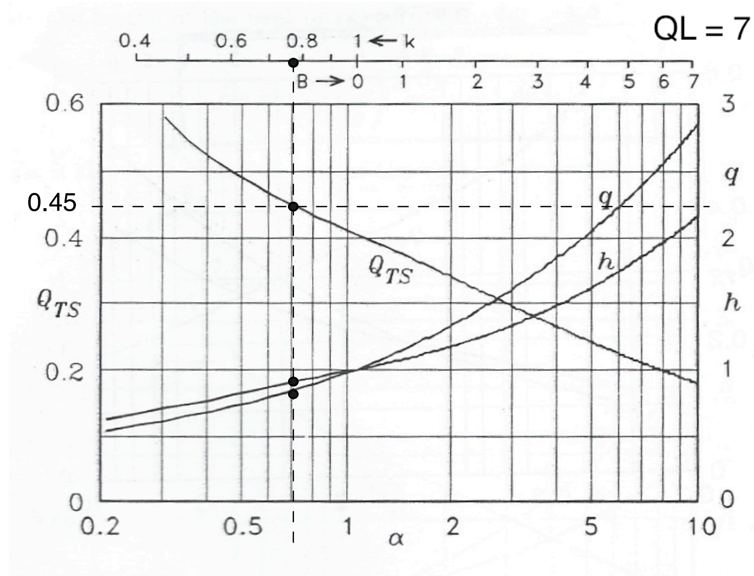
T-S Parameter		Value
f_S		Driver Resonant Frequency
Q_{TS} Q_{ES} Q_{MS}		Resonant Quality Factors (Total, Electrical and Mechanical)
V_{AS}		Volume Compliance
x_{MAX}		Maximum peak displacement
S_D		Surface area of cone
R_E		DC Resistance of the coil
L_E		Inductance of the coil
M_{MS}		Mechanical Mass of the Suspension

(2) Decide on your carpentry skills - choose a Q_L nomograph accordingly. Commonly we start with $Q_L = 7$ but you can also use other graphs then fine tune the design to fit. Once you have the graph, the steps are:

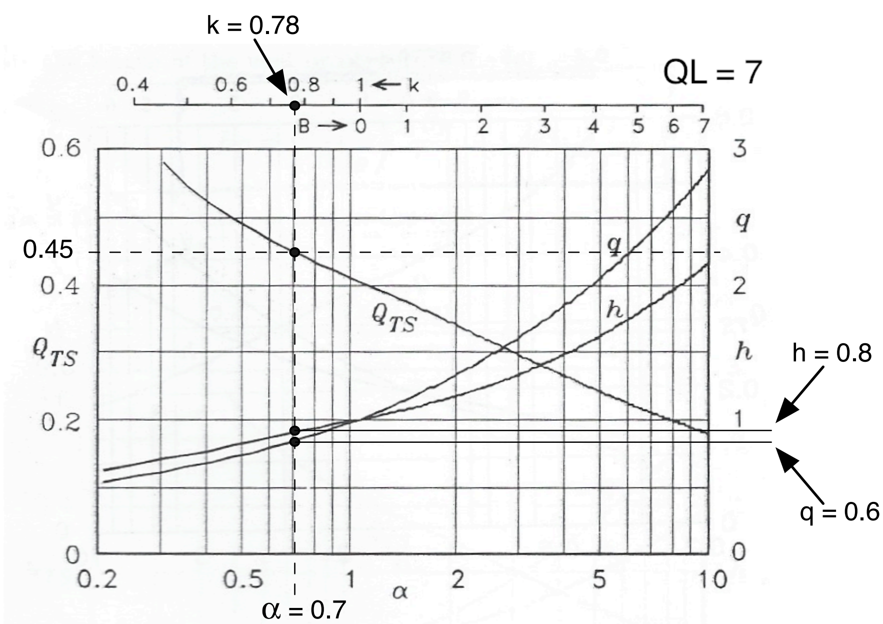
- find the Q_{TS} of your driver on the left axis. Draw a line from that point across the graph in the x-dimension. In this example, we'll use a Q_{TS} of 0.45; find the intersection point with the Q_{TS} curve.



- Draw a perpendicular line from the Q_{TS} intersection point through the y-dimension and note the intersection points with the q and h curves as well as the intersection of the two x-axes (top and bottom)



- Read all the design values directly off the intersection points/axes. Note the B-k and α axes are logarithmic so you need to be careful when estimating these points



- Check the intersection with the top axis: if B = 0 the alignment is B4; if B > 0 it is QB3 and if you intersect to the left of B = 0 where k increases, it is C4

For our design we get:

- k = 0.78 -- C4 Alignment, k is fairly large so ripple will be small
- q = 0.6
- h = 0.8
- α = 0.7

Now, use the equations to find the box volume and box tuning frequency:

$$V_{AB} = \frac{V_{AS}}{\alpha} \qquad f_{-3} = qf_S \qquad f_B = hf_S$$

- Find the box dimensions using either the Golden Ratio or squarish design (or your own):

Two popular rules-of-thumb exist for the relationship between length, width and height:

Golden Ratio: 0.6 x 1.0 x 1.6

Squarish: 0.8 x 1.0 x 1.25

- Use the tuning frequency to find the port length; the port diameter needs to be large for larger drivers.
- Find the minimum port diameter to prevent chuffing and wind noises

To calculate the minimum diameter of the port required to prevent port noises, you will also need to know the following:

Xmax = maximum linear displacement (mm)
Dia = Effective diameter of driver (cm)
Np = number of ports

Calculate the minimum port diameter from the following equations:

$$S_D = \frac{\pi\left(\frac{Dia}{100}\right)^2}{4}$$

$$V_D = \frac{S_D x_{\max}}{1000}$$

$$d_{\min}(cm) = \frac{100\left(20.3\frac{V_D^2}{f_B}\right)^{0.25}}{\sqrt{N_P}}$$

$$d_{\min}(in) = d_{\min}(cm)/2.54$$

M. Leach Method

- first choose a port radius, a:
- for multiple ports, combine the port cross sectional areas

$$a = \text{ port radius}$$

$$S_P = \pi a^2$$

$$L_P = \left(\frac{c}{2\pi f_B}\right)^2 \left(\frac{S_P}{V_{AB}}\right) - 1.463\sqrt{\frac{S_P}{\pi}}$$

T. Gravesen Method (metric)

- first choose a port diameter, d (cm):

$$L_P = \frac{N_P 23562.5 d^2}{f_B^2 V_{AB}} - kd$$

$$d = \text{ port diameter (cm)}$$

$$N_P = \text{ number of ports}$$

$$\mathrm{V}_{AB} = \text{ box volume (L)}$$

$$k = \text{ correction factor, } 0.732 \text{ for normal vent}$$

Rectangular Vent:

To use a rectangular (slot) vent, find the diameter of a tube that has the same cross sectional area as the vent and use it in the equations. The slot correction factor will be different from the tube, but that can be tuned later.

$$d_{equiv} = 2\sqrt{(w)(h)}$$

$$w = \text{ width of slot}$$

$$h = \text{height of slot}$$

- Check to see if this port length is do-able in the design (enclosure); if not choose a new radius and re-design the port.

- Build the enclosure and measure the input impedance of the speaker. This is done with an AC input source (oscillator), low power amplifier, and a power resistor that you must choose; its value should be in the range of the impedances you expect; 32 ohms is a good place to start. To find the impedance, apply the oscillator to the circuit below and measure the AC input V_{IN} as well as V_x for various frequencies. The impedance at a given frequency is found by back-solving the resistor divider equation:

$$Z_B = \frac{R_X}{V_{IN}/V_X - 1}$$

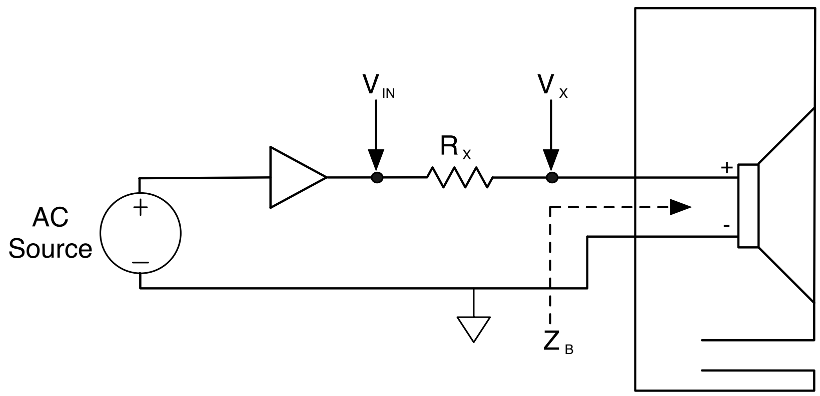
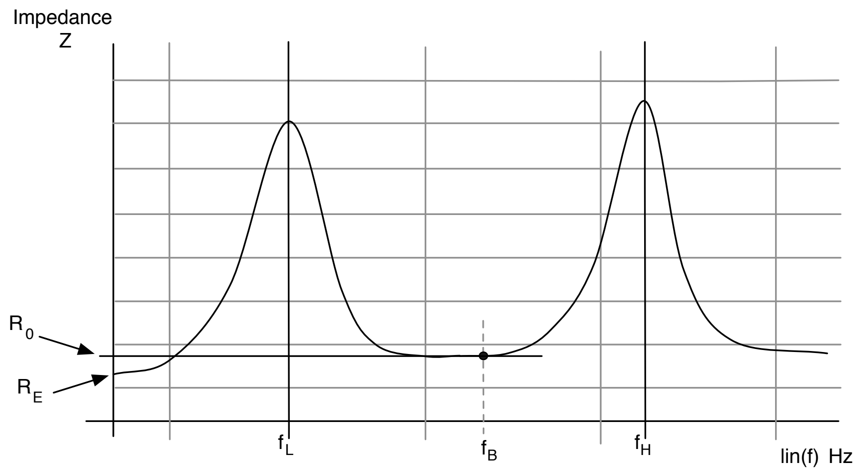


Figure 8.18: Test setup for measuring the input impedance a loudspeaker (does not have to be bass reflex)

- Tuning: On the impedance plot, find the following



- R_0
- R_E
- f_L
- f_H

Calculate the value for Q_L and see how it compares with the Q_L you designed with:

$$f_{SB} = \frac{f_L f_H}{f_B} \qquad r_m = \frac{R_0}{R_E}$$

$$h_a = \frac{f_B}{f_{SB}} \qquad \alpha' = \frac{\left(f_H^2 - f_B^2\right)\left(f_B^2 - f_L^2\right)}{f_H^2 f_L^2}$$

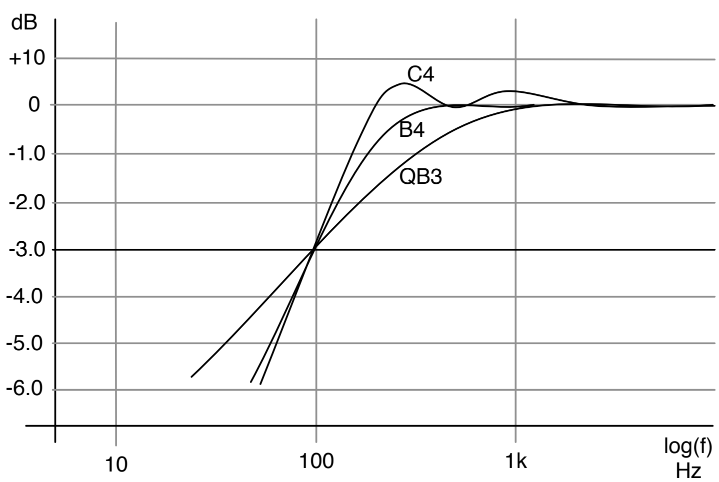
$$Q_{ESB} = \frac{f_S}{f_{SB}} Q_{ES} \qquad Q_{MSB} = \frac{f_S}{f_{SB}} Q_{MS}$$

$$Q_{LD} = \frac{h_a}{\alpha'}\left[\frac{1}{Q_{ESB}\left(r_m - 1\right)} - \frac{1}{Q_{MSB}}\right]$$

Now check:

- $Q_{LD} = Q_L$? done!
- $Q_{LD} < Q_L$? ***increase*** V_{AB} and test again
- $Q_{LD} > Q_L$? ***decrease*** V_{AB} and test again

- Plot your expected Frequency Response



- Check excursion, power and efficiency

$$V_{D(\max)}(m^2) = x_{\max} \pi a^2$$

$$P_{AR(\max)} = 3\left[V_{D(\max)} f_{-3}^2\right]^2$$

$$\eta_0 = \frac{4\pi^2}{c^3} \frac{f_S^3 V_{AS}}{Q_{ES}}$$

$$P_{E(\max)} = \frac{P_{AR(\max)}}{\eta_0}$$

Example:

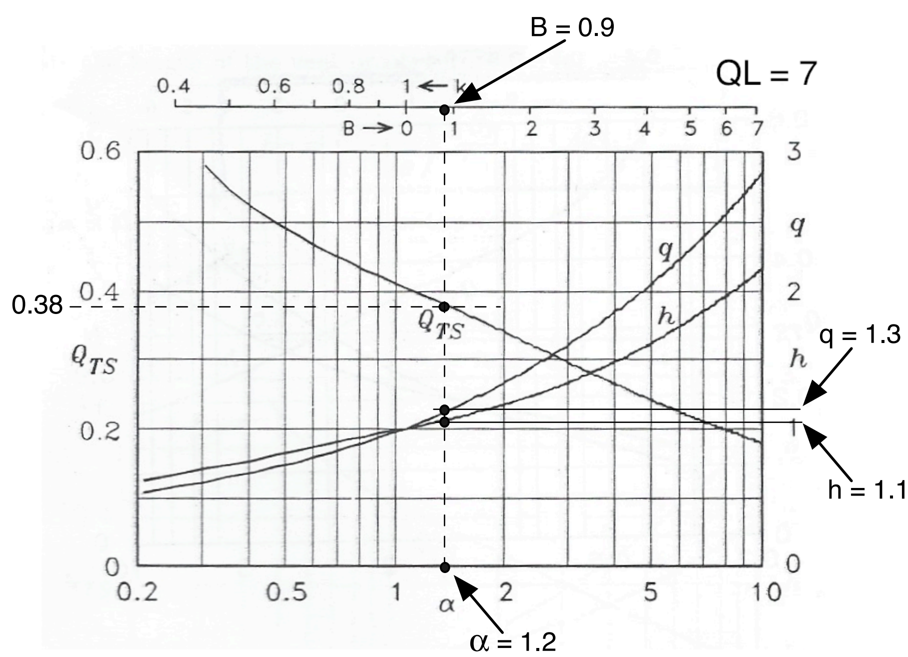
Design a Bass Reflex Enclosure for the Eminence LAB12 Driver from Chapter 6.

From the data-sheet:

$$f_s = 22Hz$$
$$Q_{TS} = 0.38$$
$$Q_{MS} = 13.32$$
$$Q_{ES} = 0.39$$
$$V_{AS} = 125.2L / 4.4ft^3$$
$$x_{MAX} = 13.00mm$$
$$S_D = 506.7cm^2$$
$$R_E = 8ohms$$
$$L_E = 1.48mH$$
$$M_{MS} = 146gm$$

Choose Q_L = 7; from the Alignment Table, we see that for a B4 alignment, Q_{TS} needs to be 0.41 and we are slightly below this so we are going to get a QB3 design (though it will be very close to a B4).

Plot Q_{TS} = 0.38 on the nomograph and find the intersection points.



The nomograph confirms this will be QB3 with a B > 0 value. The other values are read off the graph and applied to the equations:

$$V_{AB} = \frac{V_{AS}}{\alpha} = \frac{4.4\,ft^3}{1.2} = 3.7\,ft^3 \qquad f_{-3} = qf_S = 1.3(22) = 28.6\,Hz \qquad f_B = hf_S = 1.1(22) = 24.2\,Hz$$

$$V_{driver}(ft) \approx 6x10^{-6} d^4 ft^3 = 0.124\,ft^3$$
$$V_B = V_{AB} + V_{driver} = 3.82\,ft^3$$

Using the golden ratio and a width of 14" = 1.167ft (to accommodate the 12" diameter) we get the dimensions:

$$(0.6x)(x)(1.6x) = 0.96x^3$$
$$x = \sqrt[3]{\frac{3.82}{0.96}} = 1.58\,ft = 18.9"$$
$$\text{l} = (1.6)(1.58) = 2.53\text{ft} = 30.33"$$
$$\text{w} = 1.58\text{ft} = 18.9"$$
$$\text{d} = (0.6)(1.58) = 0.948\text{ft} = 11.4"$$

Calculate Port Length:

Calculate the minimum port diameter:

$$Dia = \text{port diameter (cm)}$$
$$S_D = \pi\left(\frac{Dia}{100}\right)^{1/2}$$
$$V_D = \frac{S_D x_{\max}}{1000}$$
$$d_{\min}(cm) = \frac{100\left(20.3\frac{V_D^2}{f_B}\right)^{0.25}}{\sqrt{N_P}}$$
$$d_{\min}(in) = d_{\min}(cm)/2.54$$
$$= 1.68"$$

So, a port with a diameter of 2" would yield a radius a = 1" = 0.0833ft but I'll choose a 3" diameter (a = 1.5" = 0.125ft) port to lessen the windage.

$$S_P = \pi a^2 = \pi(0.125)^2 = 0.049\,ft^2$$

$$L_P = \left(\frac{c}{2\pi f_B}\right)^2 \left(\frac{S_P}{V_{AB}}\right) - 1.463\sqrt{\frac{S_P}{\pi}}$$

$$= \left(\frac{1131}{2\pi(24.2)}\right)^2 \left(\frac{0.049}{3.82}\right) - 1.463\sqrt{\frac{0.049}{\pi}}$$

$$= 0.526\,ft = 6.32"$$

This port will be 6.32" long which works with my enclosure depth. NOTE: it is common to get hung up here with ports that are too long.

- Check excursion, power and efficiency

$$V_{D(\max)}(m^2) = x_{\max}\pi a^2 = 5.88x10^{-6}m^2$$

$$P_{AR(\max)} = 3\left[V_{D(\max)}f_{-3}^2\right]^2 = 0.242W$$

$$\eta_0 = \frac{4\pi^2}{c^3}\frac{f_S^3 V_{AS}}{Q_{ES}} = \frac{4\pi^2}{345^3}\frac{22^3 0.125}{0.39} = 0.00328 = 0.328\%$$

$$P_{E(\max)} = \frac{P_{AR(\max)}}{\eta_0} = \frac{0.242}{0.00328} = 73.75W$$

Ordinarily, you would now fabricate and build. Comparing this enclosure with the one we designed for the Acoustic Suspension, we get the following plots and mechanical drawings. Notice the vented enclosure is slightly larger than the acoustic suspension. They both have a Butterworth response (no peaking) but the vented enclosure is 4th order.

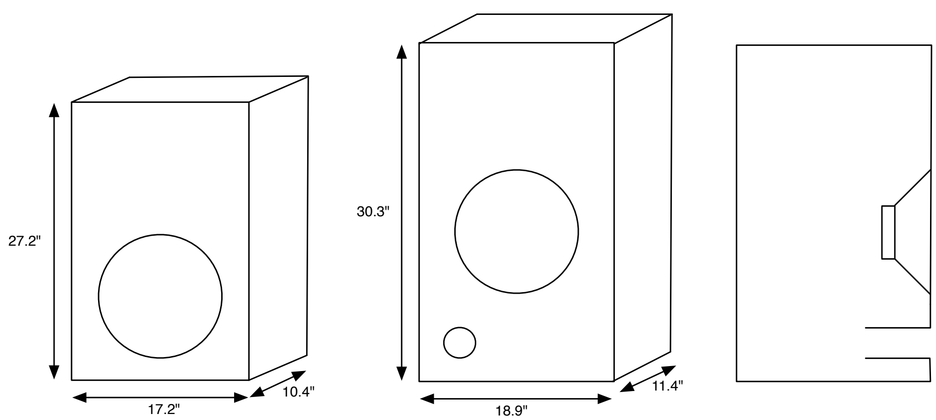


Figure 8.20: Comparison of the Acoustic Suspension system from the last chapter and this Bass Reflex Enclosure; the diagrams are to scale.

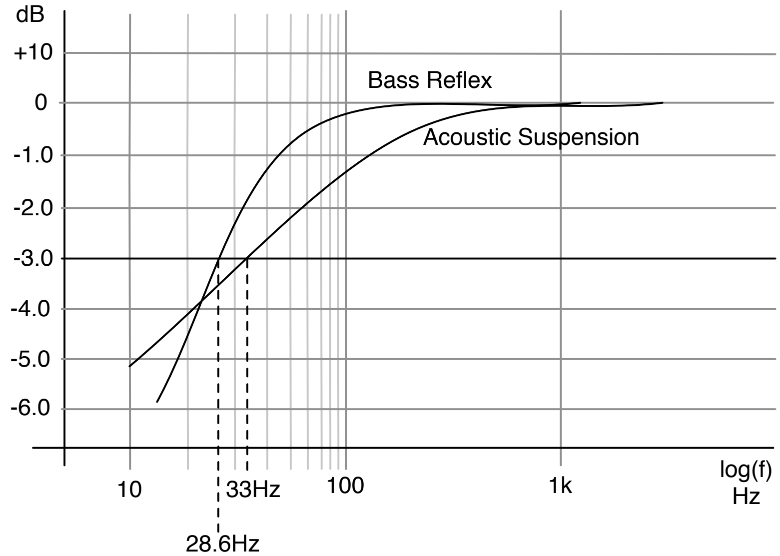
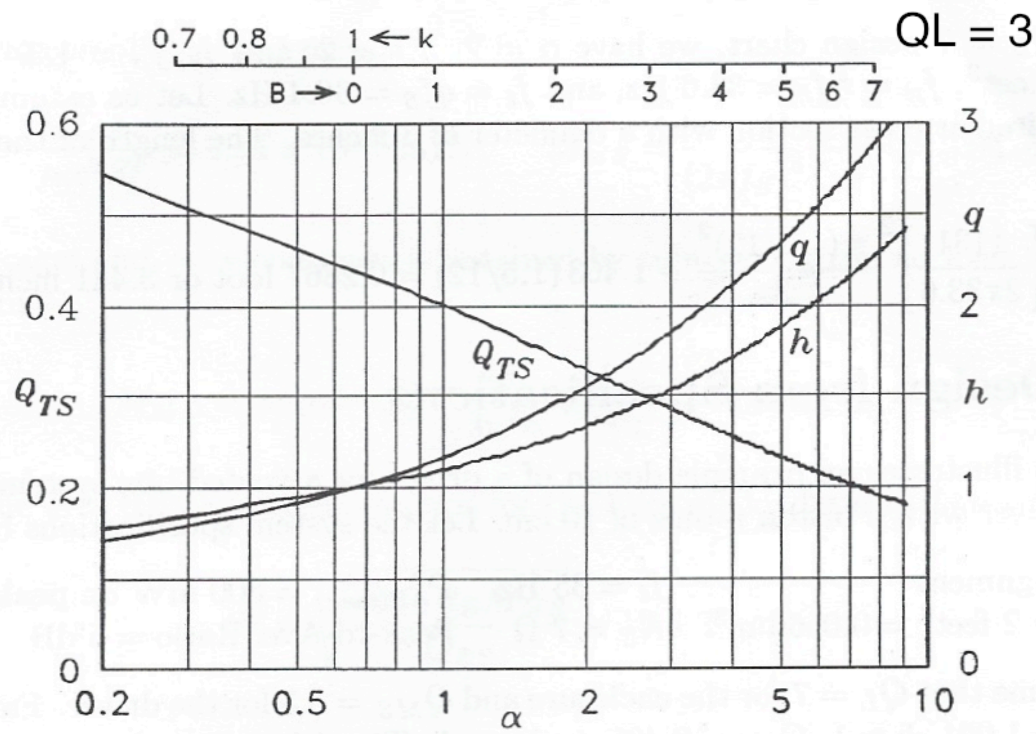
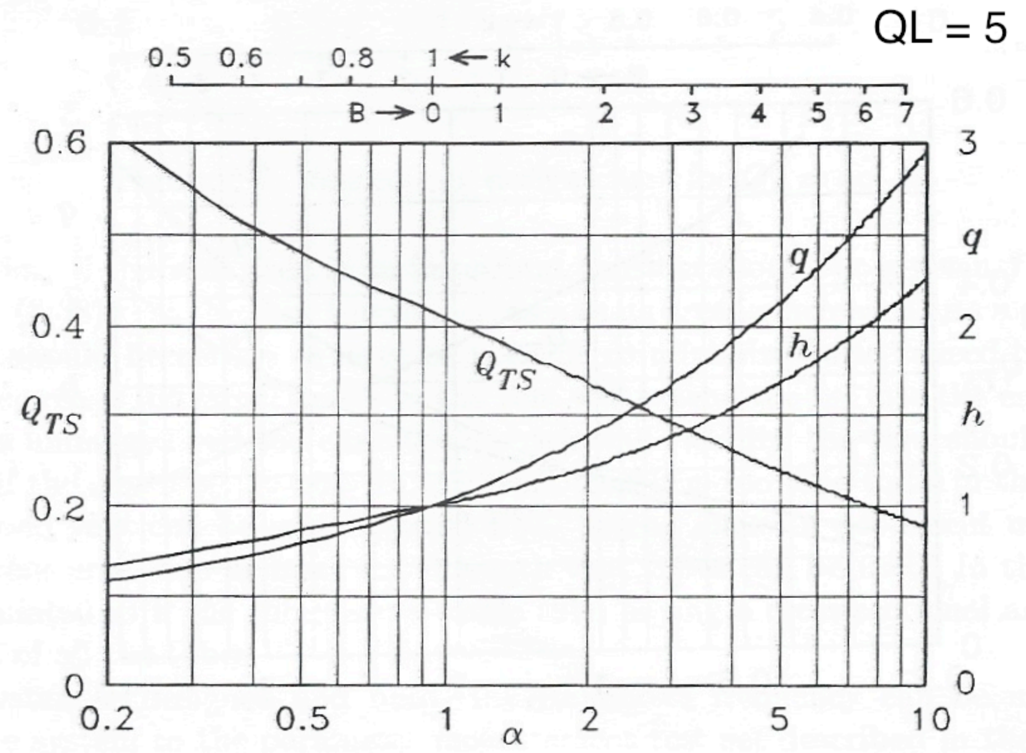
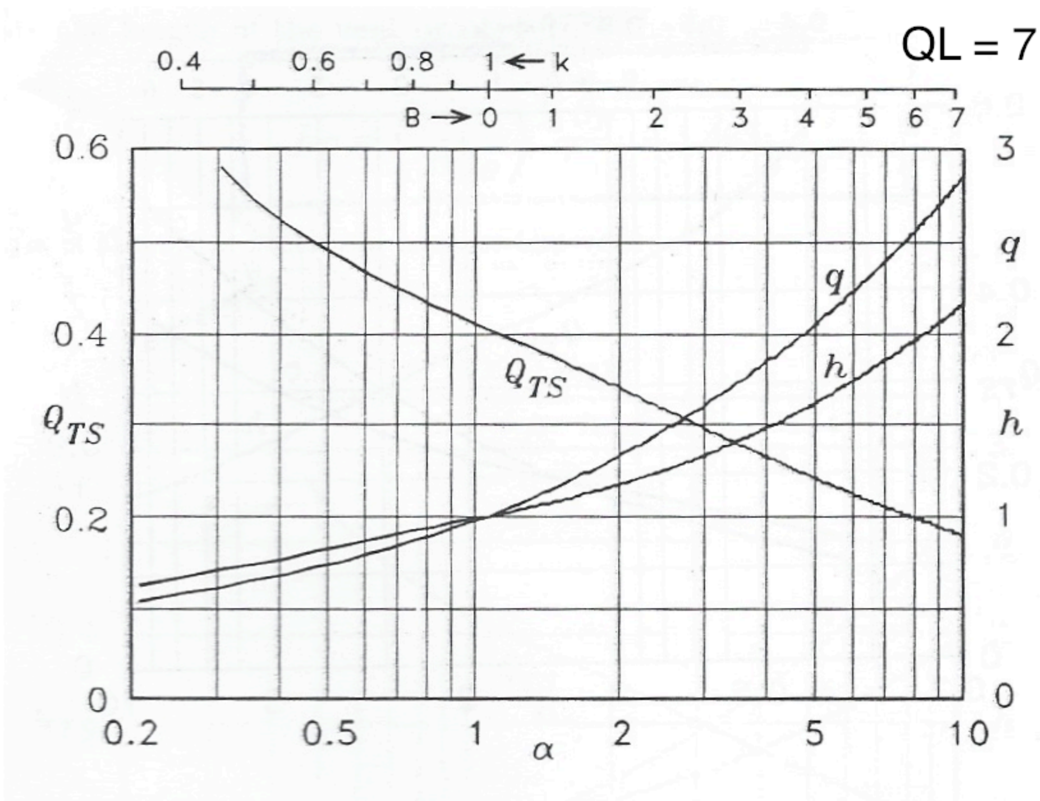


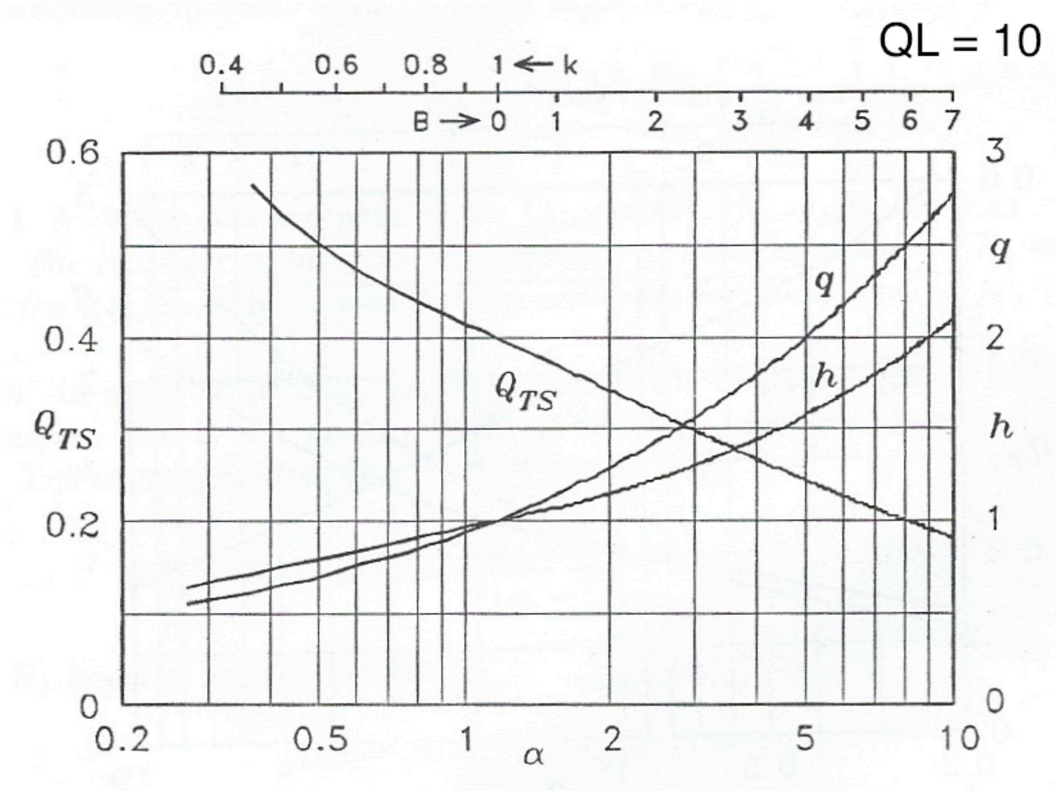
Figure 8.21: Comparing the low frequency responses of our two enclosure designs.

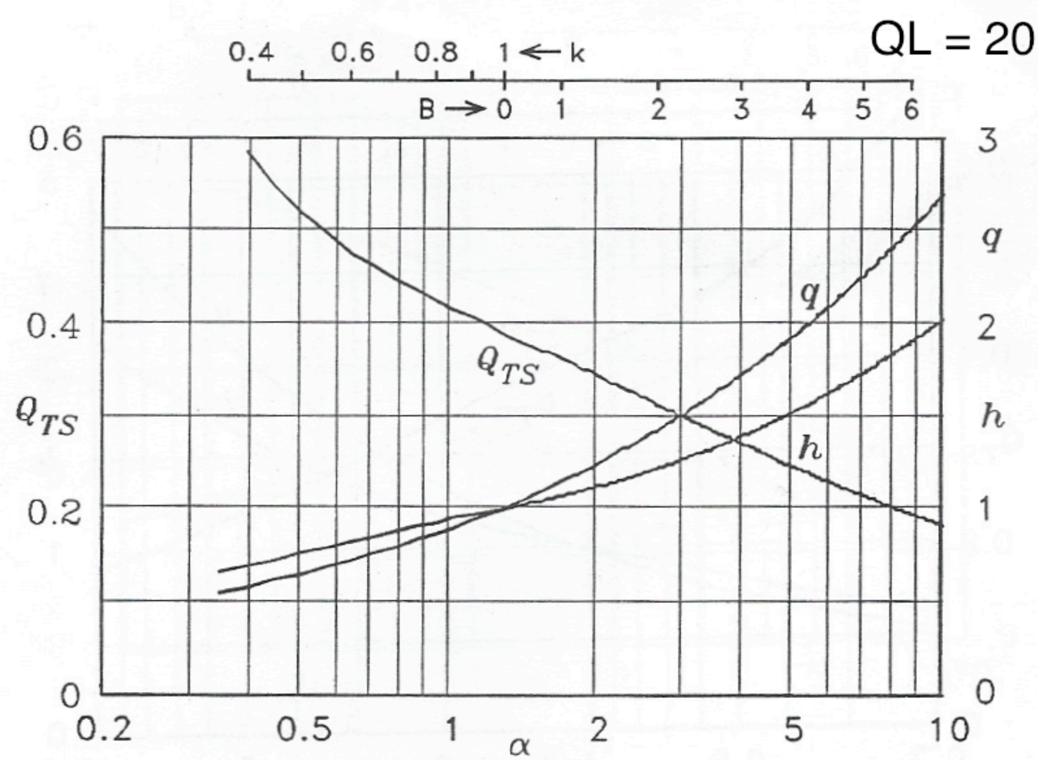
Full Sized Nomographs



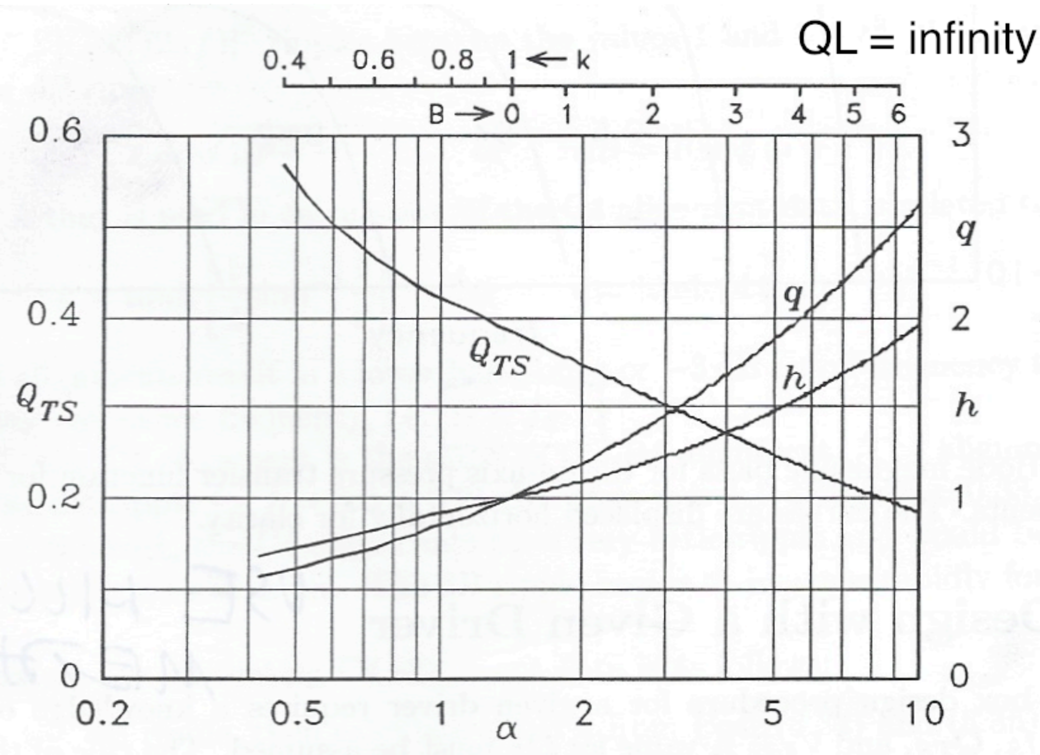



QL = 7
0.4
0.6
0.8
1 ← k
B → 0
1
2
3
4
5
6
7
0.6
0.4
0.2
0
Q_{TS}
3
q
2
h
1
0
q
h
Q_{TS}
0.2
0.5
1
2
5
10
α


QL = 10
0.4
0.6
0.8
1 ← k
B → 0
1
2
3
4
5
6
7
0.6
0.4
0.2
0
Q_{TS}
3
q
2
h
1
0
q
h
Q_{TS}
0.2
0.5
1
2
5
10
α



QL = 20
0.4
0.6
0.8
1 ← k
B → 0
1
2
3
4
5
6
0.6
0.4
0.2
0
Q_{TS}
3
q
2
h
1
0
q
Q_{TS}
h
0.2
0.5
1
2
5
10
α



QL = infinity
0.4
0.6
0.8
1 ← k
B → 0
1
2
3
4
5
6
0.6
0.4
0.2
0
Q_{TS}
3
q
2
h
1
0
q
Q_{TS}
h
0.2
0.5
1
2
5
10
α

9 Passive Radiator, Bandpass, Transmission Line Enclosures and Pure LC Crossovers

This chapter includes a collection of enclosure designs whose derivations are difficult. These include:

- Passive Radiator (PR) Enclosures
- Transmission Line (TL) Enclosures
- Bandpass (Subwoofer) Enclosures

9.1 Passive Radiator Enclosures

The idea behind a Passive Radiator Enclosure is to take a vented enclosure and replace the air mass in the port with a mechanical mass of a passive radiator; the passive radiator resembles a driver without the magnet motor assembly. An inexpensive way to make one involves cutting off the magnet assembly from the frame of a conventional driver. However, commercial passive radiators are available with interesting geometries including purely flat piston-like cones. Passive Radiators are specified the same as normal drivers except they have no Bl product, power ratings or efficiency calculations.

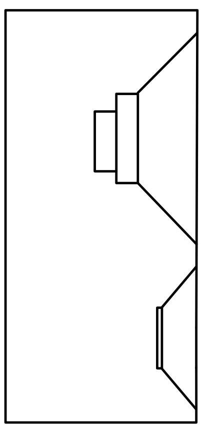


Figure 9.1: A PR Enclosure features a motor-less resonator system

By replacing the port/mass system with a mechanical mass the unwanted effects of the port (windage, chuffing, organ pipe resonances, impossible port length-designs) are eliminated. However, the main difference is that the passive radiator has a suspension and therefore a compliance and a resistance (damping) component. The electrical model is below. You can see that it is identical to the Bass Reflex model except the series addition of the compliance and resistance of the PR.

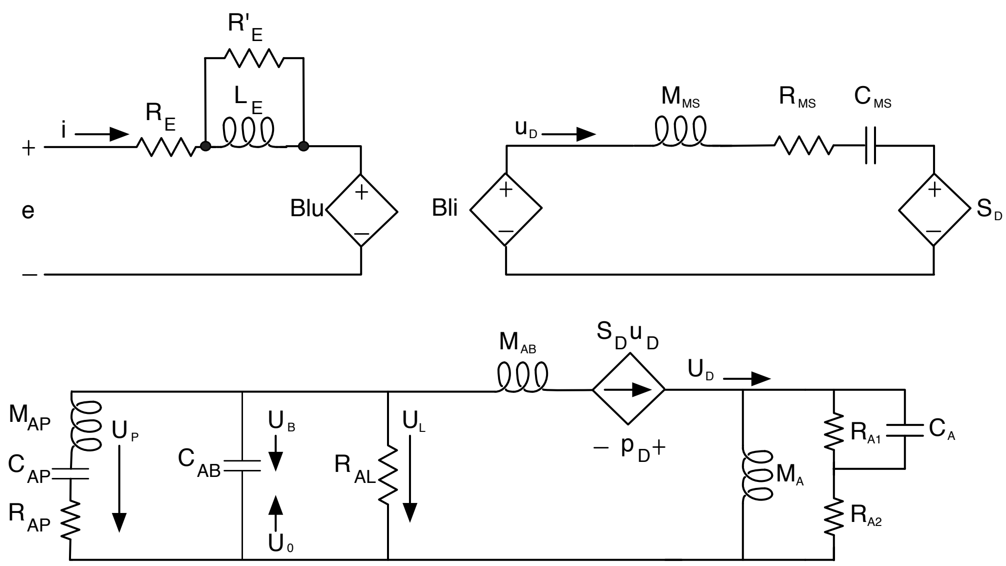


Figure 9.2: The Passive Radiator equivalent circuit includes the compliance (C_{AP}) and resistance (R_{AP}) of the passive radiator

The PR Enclosure requires a second compliance ratio, δ which is similar to the α ratio of the last two chapters:

$$\alpha = \frac{C_{AS}}{C_{AB}} = \frac{V_{AS}}{V_{AB}} \qquad \delta = \frac{C_{AP}}{C_{AB}}$$

It is common to simplify the design by forcing the two ratios to be equal. This can be done easily by using the same driver (with magnet removed) as the Passive Radiator. Alternatively, you can select a PR with the same compliance as the final box.

Likewise, another tuning ratio (y) is specified that relates the PR resonant frequency to the driver resonant frequency:

$$\omega_P = \frac{1}{\sqrt{M_{AP} C_{AP}}}$$

$$y = \frac{\omega_P}{\omega_S}$$

The Norton form combination equivalent circuit is:

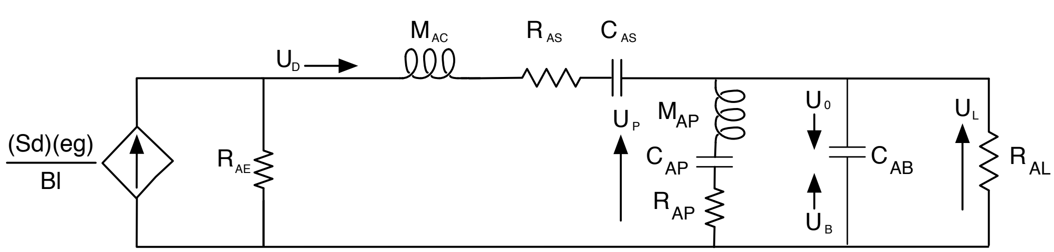


Figure 9.3: The Combination Analogous Circuit for the PR Enclosure

The total Volume Velocity U_0 that flows into the box is:

$$U_0 = \frac{S_D e_g}{Bl} \frac{R_{AE} j\omega C_{AB}}{1 + Z_{A1} Y_{A2}}$$

$$Z_{A1} = j\omega M_{AC} + R_{AE} + R_{AS} + \frac{1}{j\omega C_{AS}}$$

$$Y_{A2} = j\omega C_{AB} + \frac{1}{R_{AL}} + \frac{1}{j\omega M_{AP} + R_{AP} + 1/j\omega C_{AP}}$$

The On-Axis Pressure Function is:

$$p = \frac{\rho_o}{2\pi} \frac{Ble_g}{S_D R_E M_{AS}} G_P(s)$$

$$G_P(s) = \frac{(s/\omega_0)^2 \left[(s/\omega_0)^2 + (\omega_P/\omega_0)^2 \right]}{(s/\omega_0)^4 + b_3 (s/\omega_0)^3 + b_2 (s/\omega_0)^2 + b_1 (s/\omega_0) + 1}$$

$$\omega_0 = \omega_S \sqrt{y} (1 + \alpha + \delta)^{1/4}$$

$$b_3 = \frac{1}{(1+\alpha+\delta)^{1/4} \sqrt{y} Q_{TS}} \qquad b_2 = \frac{1}{(1+\alpha+\delta)^{1/2}} \left[(1+\delta) y + (1+\alpha)/y \right] \qquad b_1 = \frac{(1+\delta)\sqrt{y}}{(1+\alpha+\delta)^{3/4} Q_{TS}}$$

When the compliance of the passive radiator is infinite, it becomes the same as the vented box function. The filtering term is another 4th order analog filtering system and the coefficients b_1 - b_3 determine the shape of the response. The numerator's two zeros are not located at s = 0 as in the vented enclosure which produces a change in the frequency response. The zeros form a notch in the response. The goal is to get this notch as far away from the corner frequency as possible. Specifying alignments is tedious and difficult because this filtering term can not be put directly into the filtering forms we are used to. The closest filter type is called the Elliptic Filter which has zeros that are not at s = 0 however the elliptic version has two extra zeros. This is a quasi-elliptic transfer function.

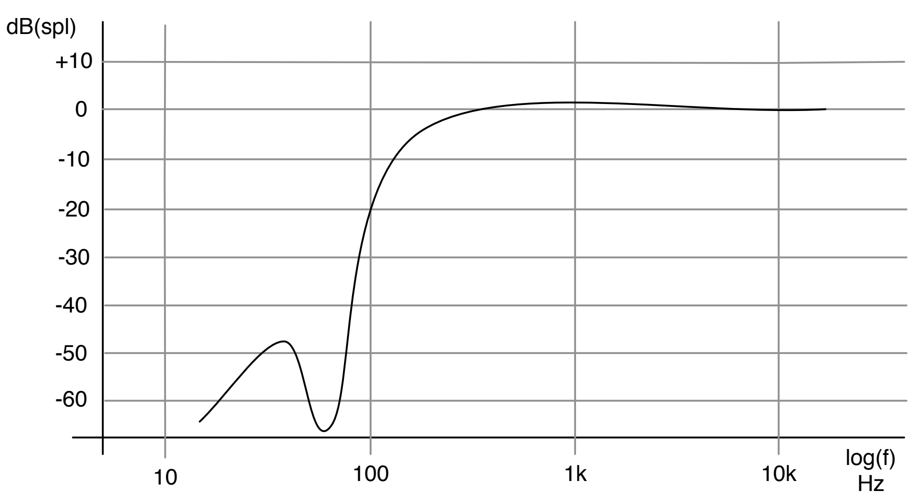


Figure 9.4: Response of the quasi-elliptical PR enclosure; note that like the Chebychev function there may be rippling in the passband. We observe the notch in the stop-band.The null occurs at the resonant frequency of the passive radiator, f_P.

The rippling/notch creates a slightly steeper cutoff than the Chebychev 4th order alignments from the last chapter. The overall amount of rippling in the passband as well as the steepness of the rolloff is determined by the PR tuning ratio, δ which is usually set to be equal to the normal tuning ratio, α.

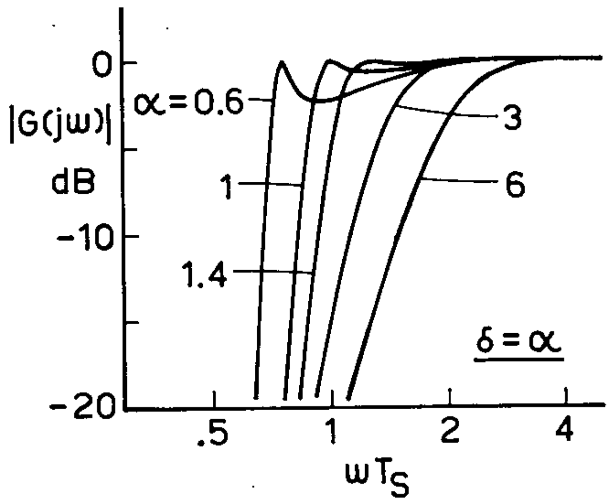
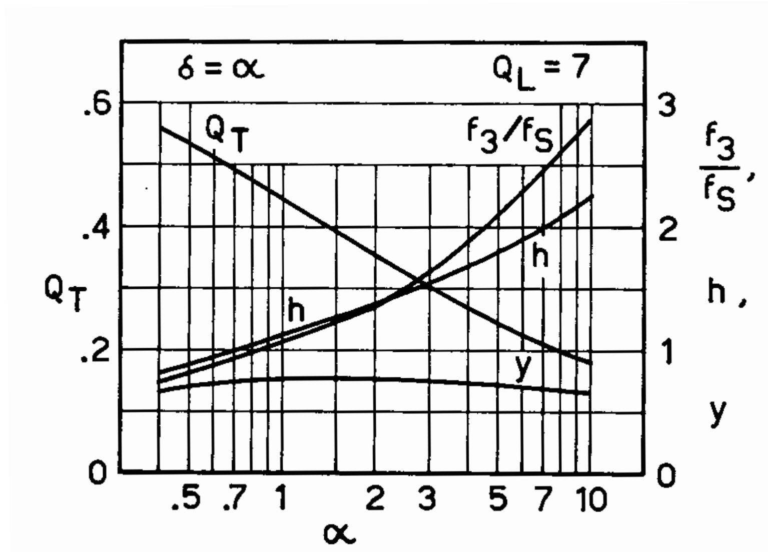
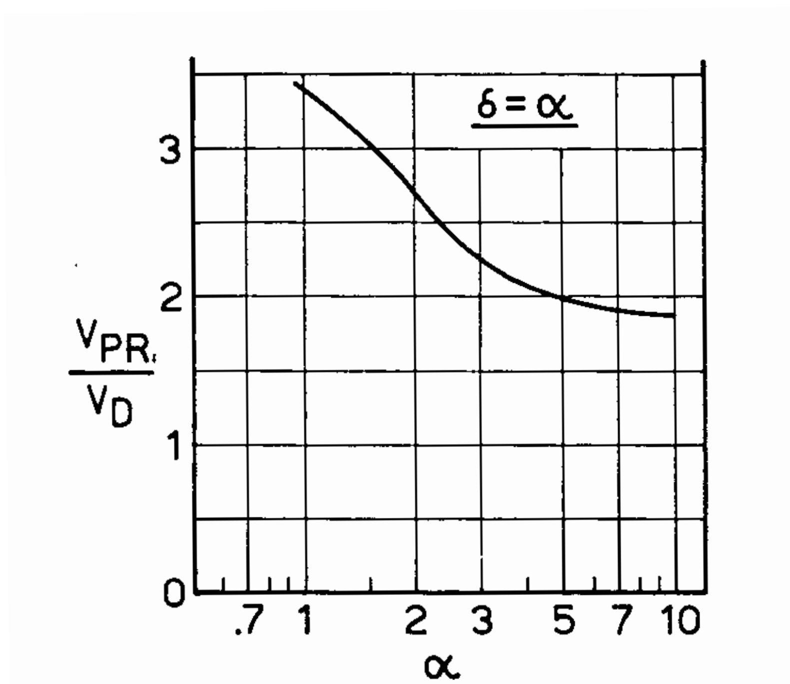


Figure 9.5: the rippling in the passband depends on the value of the tuning ratio δ = α here. As the ratio gets smaller the rippling increases as well as the steepness of the rolloff.

9.2 Passive Radiator Design with Nomographs

Because of the unpleasant algebra required and the fact that there are six variables to solve for, the nomograph method from the last chapter is often used. Another alternative is a tabulated version of the same graphs as found in Dickason[1]. In addition to calculating the box dimensions and tuning frequencies, you must also calculate the volume displacement of the passive radiator; this will then specify the surface area and x_{max} of the passive radiator. There are two nomographs; the second is the normally used graph, for $Q_L = 7$ and the second is for the volume displacement of the passive radiator (V_{PR}) versus the volume displacement of the driver V_D.





9.3 Other Passive Radiator Specifications

The other enclosure specifications can be assumed to be nearly identical to the Bass-Reflex Enclosure. these include:

- Impedance Plot
- P_{AR}(max)
- P_E(max)
- η_0

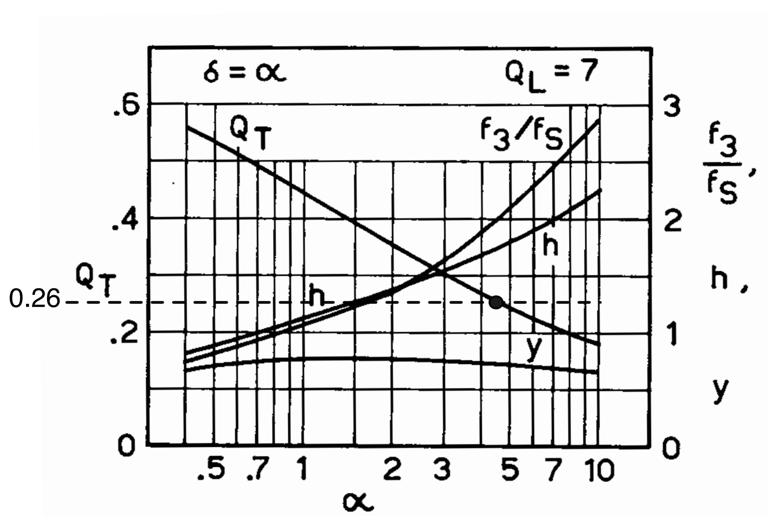
Passive Radiator Design Guide

(1) Choose Driver. Get the following Thiele-Small Parameters:

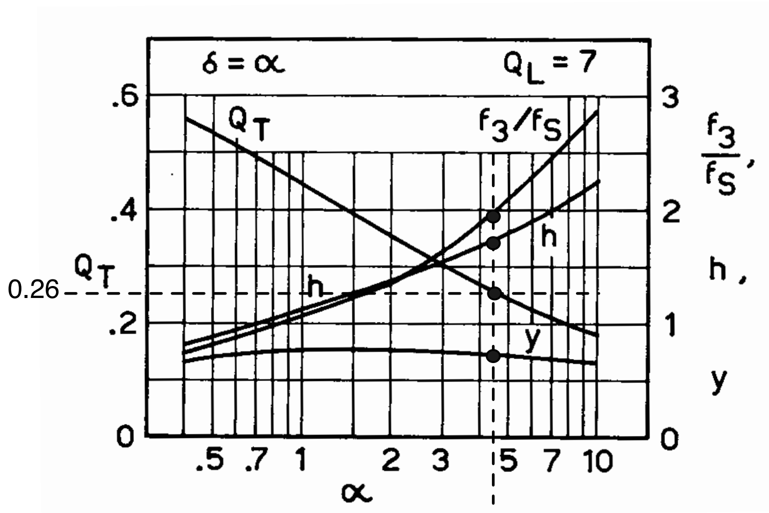
T-S Parameter		Value
f_s		Driver Resonant Frequency
Q_{TS} Q_{ES} Q_{MS}		Resonant Quality Factors (Total, Electrical and Mechanical)
V_{AS}		Volume Compliance
x_{MAX}		Maximum peak displacement
S_D		Surface area of cone
R_E		DC Resistance of the coil
L_E		Inductance of the coil
M_{MS}		Mechanical Mass of the Suspension

(2) Decide on your carpentry skills - choose a Q_L nomograph accordingly. Commonly we start with $Q_L = 7$ but you can also use other graphs then fine tune the design to fit. Once you have the graph, the steps are:

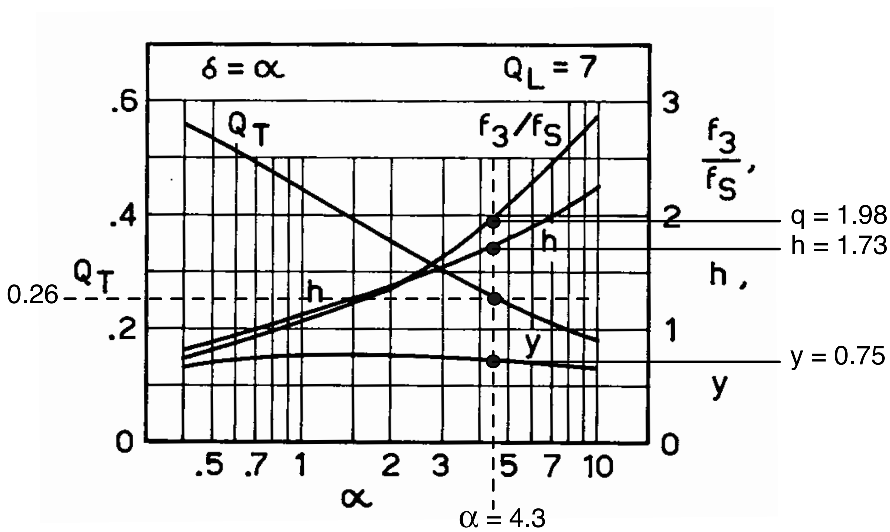
- find the Q_{TS} of your driver on the left axis. Draw a line from that point across the graph in the x-dimension. In this example, we'll use a Q_{TS} of 0.26; find the intersection point with the Q_{TS} curve.



- Draw a perpendicular line from the Q_{TS} intersection point through the y-dimension and note the intersection points with the q, h, and y curves as well as the intersection of the two x-axes (top and bottom)



- Read all the design values directly off the intersection points/axes.



For this design we get:

- y = 0.75
- q = 1.98
- h = 1.73
- α = 4.3

Now, use the equations to find the box volume and box tuning frequencies:

$$V_{AB} = \frac{V_{AS}}{\alpha} \qquad f_{-3} = qf_S \qquad f_B = hf_S \qquad f_P = yf_S$$

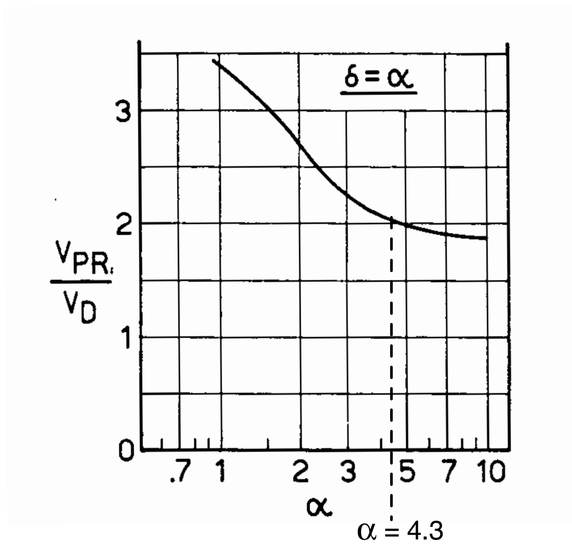
- Find the box dimensions using either the Golden Ratio or squarish design (or your own):

Two popular rules-of-thumb exist for the relationship between length, width and height:

Golden Ratio: 0.6 x 1.0 x 1.6

Squarish: 0.8 x 1.0 x 1.25

- Use the second nomograph to find the volume displacement of the passive radiator:

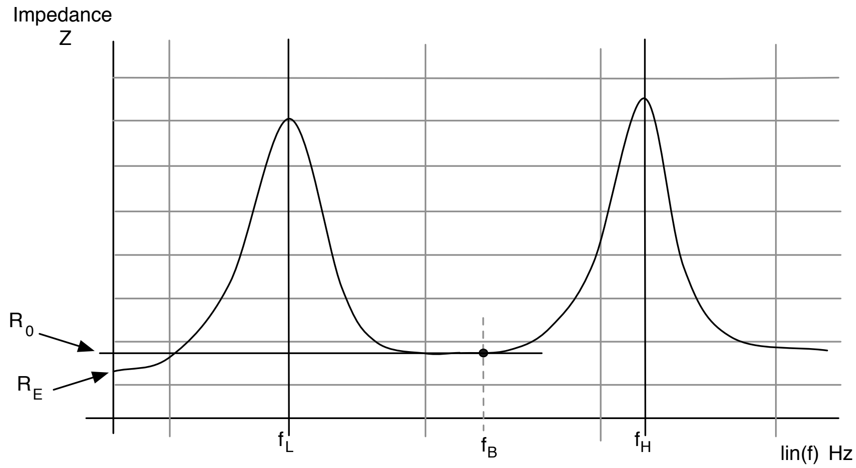


- Specify the Passive Radiator's Volume Displacement related to the volume displacement required. Start with the radiator's piston radius a and use the ratio from above to calculate.

$$V_{D(\max)}(m^2) = x_{\max} \pi a^2$$

$$V_{PR(\max)}(m^2) = x_{PR\max} \pi a_{PR}^2$$

Find the required Passive Radiator. Tuning the enclosure is done the same way as for the Vented Enclosure. The mass of the PR may be modified (spray paint, glue, molding clay) for tuning purposes.
Tuning: On the impedance plot, find the following



- R_0
- R_E
- f_L
- f_H

Calculate the value for Q_L and see how it compares with the Q_L you designed with:

$$f_{SB} = \frac{f_L f_H}{f_B} \qquad r_m = \frac{R_0}{R_E}$$

$$h_a = \frac{f_B}{f_{SB}} \qquad \alpha' = \frac{\left(f_H^2 - f_B^2\right)\left(f_B^2 - f_L^2\right)}{f_H^2 f_L^2}$$

$$Q_{ESB} = \frac{f_S}{f_{SB}} Q_{ES} \qquad Q_{MSB} = \frac{f_S}{f_{SB}} Q_{MS}$$

$$Q_{LD} = \frac{h_a}{\alpha'}\left[\frac{1}{Q_{ESB}\left(r_m - 1\right)} - \frac{1}{Q_{MSB}}\right]$$

Now check:

- $Q_{LD} = Q_L$? done!
- $Q_{LD} < Q_L$? ***increase*** V_{AB} and test again
- $Q_{LD} > Q_L$? ***decrease*** V_{AB} and test again

- Check excursion, power and efficiency

$$V_{D(\max)}(m^2) = x_{\max} \pi a^2$$

$$P_{AR(\max)} = 3\left[V_{D(\max)} f_{-3}^2\right]^2$$

$$\eta_0 = \frac{4\pi^2}{c^3}\frac{f_S^3 V_{AS}}{Q_{ES}}$$

$$P_{E(\max)} = \frac{P_{AR(\max)}}{\eta_0}$$

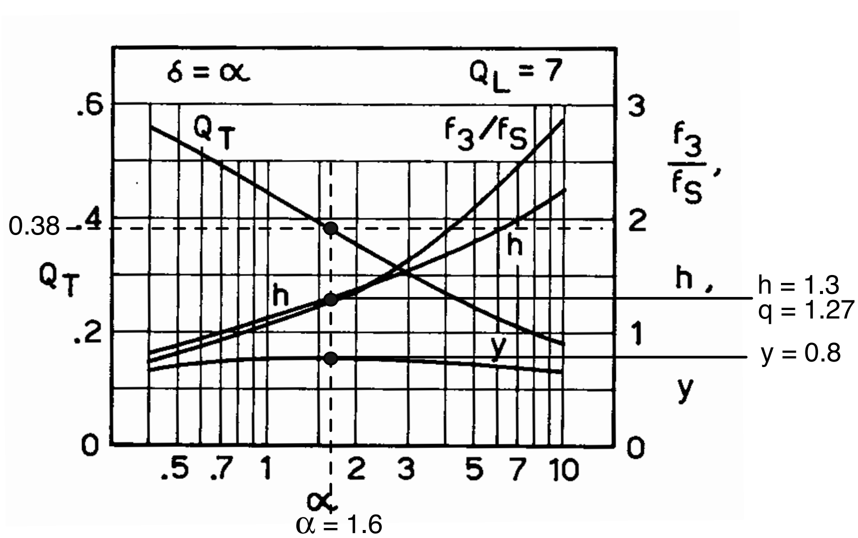
Example:

Design a Passive Radiator Enclosure for the Eminence LAB12 Driver from Chapter 6.

From the data-sheet:

$$f_s = 22Hz$$
$$Q_{TS} = 0.38$$
$$Q_{MS} = 13.32$$
$$Q_{ES} = 0.39$$
$$V_{AS} = 125.2L / 4.4 ft^3$$
$$x_{MAX} = 13.00mm$$
$$S_D = 506.7cm^2$$
$$R_E = 8ohms$$
$$L_E = 1.48mH$$
$$M_{MS} = 146gm$$

- Use the nomograph to get the parameters:



- Use the equations to calculate the parameters:

$$V_{AB} = \frac{V_{AS}}{\alpha} = \frac{4.4\,ft^3}{1.6} = 2.75\,ft^3$$
$$f_{-3} = qf_S = 1.27(22) = 27.9Hz$$
$$f_B = hf_S = 1.3(22) = 28.6Hz$$
$$f_P = yf_S = 0.8(22) = 17.6Hz$$

- Design the enclosure

$$V_{driver}(ft) \approx 6x10^{-6} d^4 ft^3 = 0.124\,ft^3$$
$$V_B = V_{AB} + V_{driver} = 2.87\,ft^3$$

Using the golden ratio and a width of 14" = 1.167ft (to accommodate the 12" diameter) we get the dimensions:

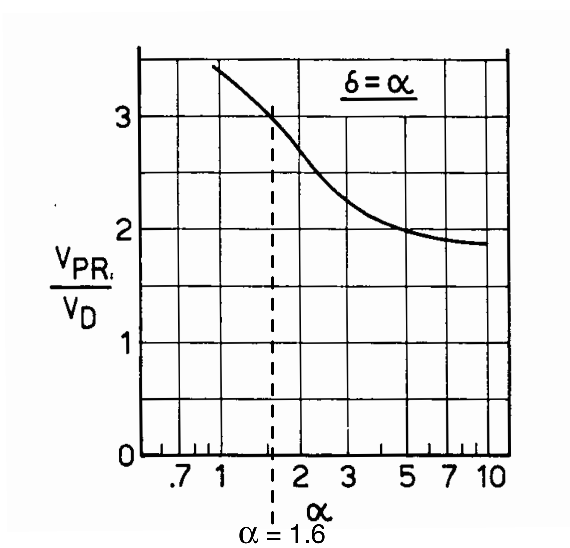
$$(0.6x)(x)(1.6x) = 0.96x^3$$
$$x = \sqrt[3]{\frac{2.87}{0.96}} = 1.42\,ft = 17.0"$$

l = (1.6)(1.42) = 2.27ft = 27.3"

w = 1.42ft = 17.0"

d = (0.6)(1.42) = 0.852ft = 10.2"

- Specify the Passive Radiator, get the volume displacement required:



$$V_{D(\max)} = x_{\max}\pi a^2 = (0.013)\pi(0.12)^2 = 588.1x10^{-6}\,m^2$$

$$\frac{V_{PR(\max)}}{V_{D(\max)}} = 2.95$$

$$V_{PR(\max)} = 2.95(V_{D(\max)}) = 1.73x10^{-3}\,m^2$$

$$x_{PR\max} = \frac{V_{PR(\max)}}{\pi a_{PR}^2}$$

So, we can specify a few options:

PR Diameter (in)	x_{max}
15	24mm
12	38mm

These are some very large excursions but there are passive radiators that can accomplish this, for example the TC Sounds VMP 12" PR which can be adjusted both in mass and excursion; the excursion goes up to 90 mm on that model!

We can sketch the frequency response observing that for our alpha value of 1.6 we will just start to get some peaking in the response.

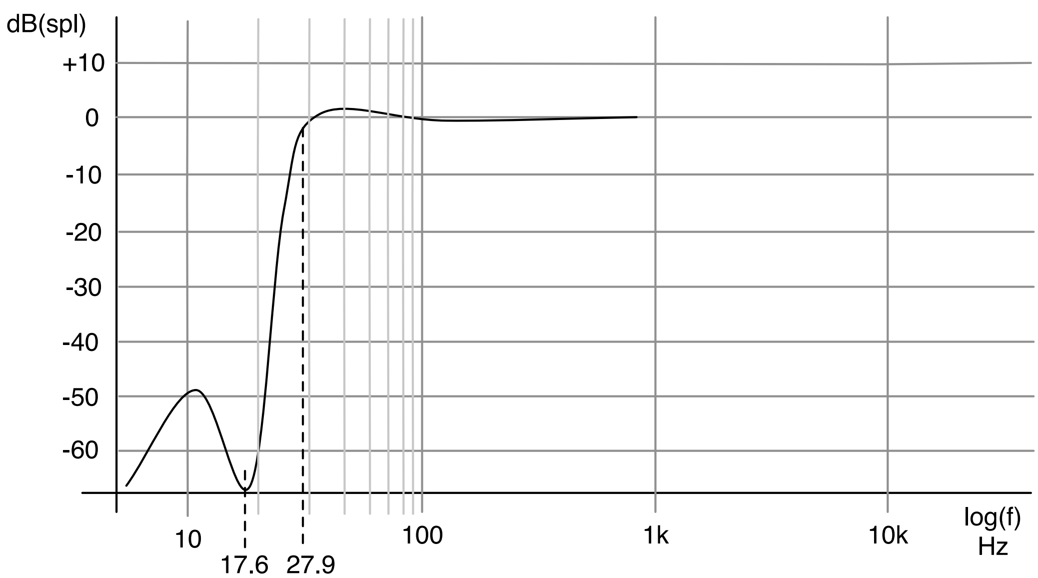


Figure 9.6: Predicted Frequency Response of our Passive Radiator Enclosure

The power and efficiency equations are the same as for the Vented Enclosure.

9.4 Bandpass (Subwoofer) Enclosures

A low frequency 4th Order Bandpass Enclosure can be designed as a hybrid combination of a closed box and vented box. The only opening to the outside is a port. Figure 9.X shows both the enclosure diagram and the equivalent circuit.

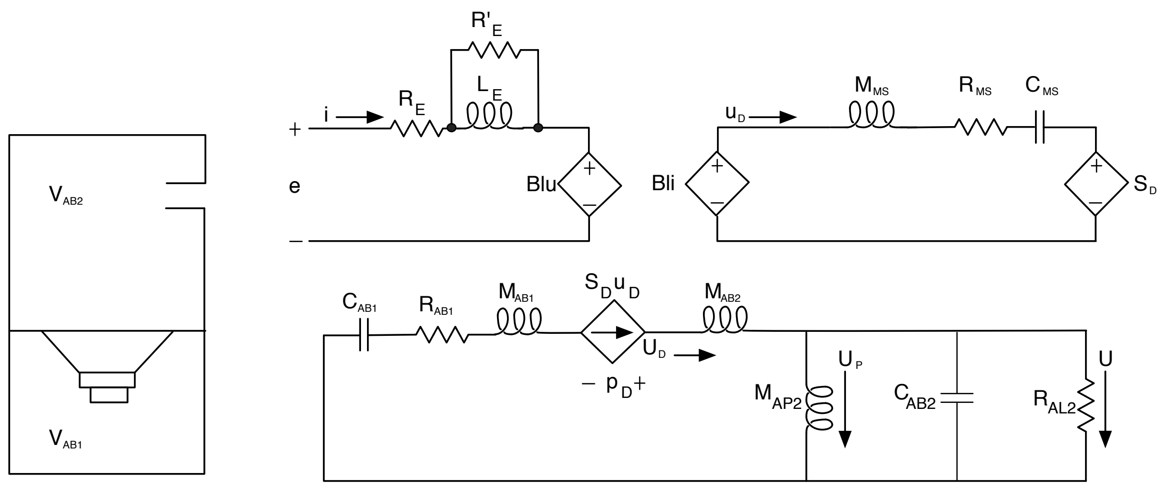


Figure 9.7: The 4th Order BP Enclosure has both a sealed and vented construction; the circuit reflects both parts.

The 4th order Band Pass response can range from overdamped to Chebychev; in the latter there will also be ripple in the passband. The enclosure is specified with a center frequency and bandwidth or two band edges, f_L and f_H.

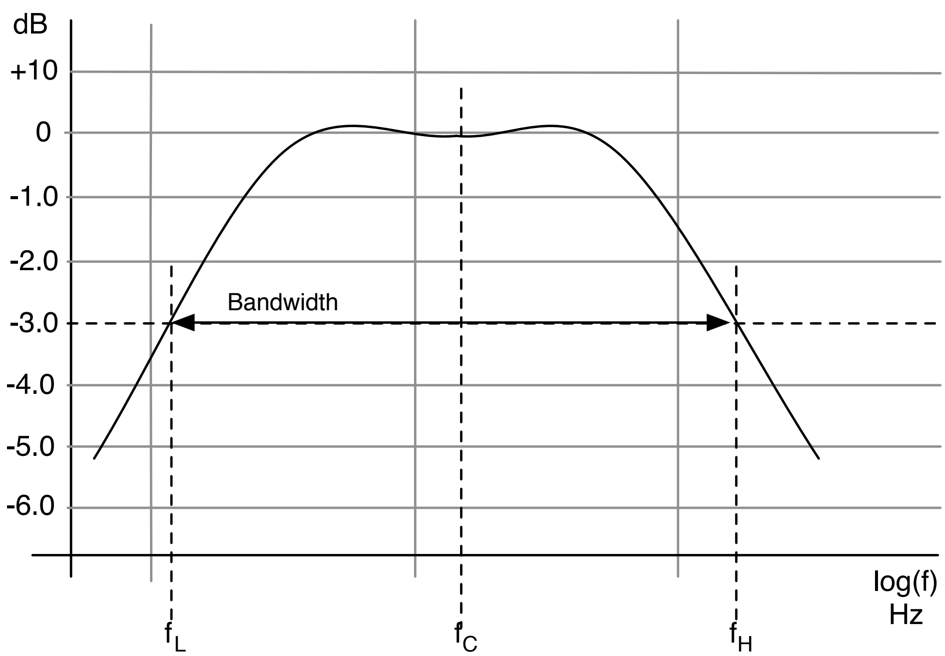


Figure 9.8: Typical 4th Order Bandpass Response; this one has ripple in the passband so it is Chebychev.

9.5 4th Order Bandpass Enclosure Design

The output of the design will be a closed box volume, vented box volume and port requirements. This will involve calculating two compliance ratios and box volumes. This design can be done in a few ways. Here are two of them:

M.Leach Method

Specify: f_L and f_H for the bandpass response.

Start with the Driver and get its Thiele Small Parameters:

$$f_S, Q_{ES}, V_{AS}$$

Use the rules of thumb for Q_{MC} for filled enclosures (Q_{MC} = 3.5/unfilled or 7.5/filled) and choose Q_L = 7 as usual for the vented half. Calculate:

$$f_0 = \sqrt{f_L f_H} \quad \alpha_1 = \left(\frac{f_0}{f_S}\right)^2 - 1 \quad Q_{EC} = Q_{ES}\sqrt{1+\alpha_1} \quad Q_{TC} = \frac{Q_{EC}Q_{MC}}{Q_{EC}+Q_{MC}}$$

$$A = \frac{f_H - f_L}{f_0}\frac{Q_L Q_{TC}}{Q_L + Q_{TC}} \quad Q_1 = \sqrt{A\left(\sqrt{2A^2+1} - A\right)}$$

$$B = \frac{f_0}{f_H - f_L}\sqrt{\left[1 - \frac{1}{2Q_1^2}\sqrt{\left(1-\frac{1}{2Q_1^2}\right)^2 + 1}\right]}$$

$$V_{AB1} = \frac{V_{AS}}{\alpha_1} \quad V_{AB1} = \frac{V_{AS}}{\alpha_2} \quad L_P = \left(\frac{c}{2\pi f_0}\right)^2\left(\frac{S_P}{V_{AB2}}\right) - 1.463\sqrt{\frac{S_P}{\pi}}$$

If $Q_1 > 0.707$ you will have ripple in the pass-band. The ripple amount will be:

$$dB_{ripple} = 20\log\left(\frac{Q_1^2}{\sqrt{Q_1^2 - 0.25}}\right)$$

Will's Tried & True Method - origin unknown

Use the tables on the following pages to select parameters. This design is capable of getting gain or attenuation in the passband (acoustic gain!). There are also different "angles" you can take when designing, for example starting with the low frequency value and working from there. You can design the specs first and then find a driver or start with a driver and specs.

Specify: f_L and f_H for the bandpass response.

Start with the Driver and get its Thiele Small Parameters:

$$f_S, Q_{TS}, V_{AS}$$

Calculate:

$$f_0 = \sqrt{f_L f_H}$$

$$Q_{BPF} = \frac{f_0 Q_{TS}}{f_S}$$

Pick a ripple value; go the table and check the gain for this Q_{BPF}. If it is not acceptable, start over with a different ripple table or driver or specs. In this ripple table, get the fL_factor and fH_factors, then check the movement of the original break frequencies:

$$f_L^{'} = (fL\_factor)\frac{f_S}{Q_{TS}}$$

$$f_H^{'} = (fH\_factor)\frac{f_S}{Q_{TS}}$$

If these frequencies are OK, you can continue otherwise re-design.

The ripple tables are labeled with "S" values. Get that value for t table and:

$$V_{AB1} = \frac{V_{AS}}{(Q_{BPF}/Q_{TS})^2 - 1} \qquad V_{AB2} = V_{AS}\left[2SQ_{TS}\right]^2$$

Finally, calculate the vent dimensions:

To calculate the minimum diameter of the port required to prevent port noises, you will also need to know the following:

Xmax = maximum linear displacement (mm)
Dia = Effective diameter of driver (cm)
Np = number of ports

Calculate the minimum port diameter from the following equations:

$$S_D = \pi\left(\frac{Dia}{100}\right)^{1/2}$$

$$V_D = \frac{S_D x_{\max}}{1000}$$

$$d_{\min}(cm) = \frac{100\left(20.3\frac{V_D^2}{f_B}\right)^{0.25}}{\sqrt{N_P}}$$

$$d_{\min}(in) = d_{\min}(cm)/2.54$$

M. Leach Method

- first choose a port radius, a:
- for multiple ports, combine the port cross sectional areas

$$a = \text{ port radius}$$

$$S_P = \pi a^2$$

$$L_P = \left(\frac{c}{2\pi f_0}\right)^2\left(\frac{S_P}{V_{AB2}}\right) - 1.463\sqrt{\frac{S_P}{\pi}}$$

T. Gravesen Method (metric)

- first choose a port diameter, d (cm):

$$L_P = \frac{N_P 23562.5 d^2}{f_0^2 V_{AB2}} - kd$$

$$d = \text{ port diameter (cm)}$$

$$N_P = \text{ number of ports}$$

$$\mathrm{V}_{AB2} = \text{ box volume (L)}$$

$$k = \text{ correction factor, 0.732 for normal vent}$$

Rectangular Vent:

To use a rectangular (slot) vent, find the diameter of a tube that has the same cross sectional area as the vent and use it in the equations. The slot correction factor will be different from the tube, but that can be tuned later.

$$d_{equiv} = 2\sqrt{(w)(h)}$$

$$w = \text{ width of slot}$$

$$h = \text{height of slot}$$

Ripple = 0 dB S = 0.7

QBP	fL factor	fH factor	Gain
0.4507	0.2167	0.9373	-8 dB
0.4774	0.2378	0.9584	-7 dB
0.5057	0.2606	0.9812	-6 dB
0.5356	0.2852	1.0058	-5 dB
0.5674	0.3118	1.0324	-4 dB
0.6010	0.3404	1.0610	-3 dB
0.6366	0.3712	1.0918	-2 dB
0.6743	0.4043	1.1248	-1 dB
0.7143	0.4397	1.1603	0 dB
0.7566	0.4777	1.1983	1 dB
0.8014	0.5184	1.2390	2 dB
0.8489	0.5619	1.2825	3 dB
0.8772	0.6084	1.3290	4 dB
0.9525	0.6581	1.3787	5 dB
1.0090	0.7111	1.4317	6 dB
1.0687	0.7675	1.4881	7 dB
1.1321	0.8277	1.5483	8 dB

Ripple = 0.35 dB S = 0.6

QBP	fL factor	fH factor	Gain
0.5258	0.2326	1.1886	-8 dB
0.5570	0.2560	1.2119	-7 dB
0.5900	0.2813	1.2373	-6 dB
0.6249	0.3088	1.2648	-5 dB
0.6619	0.3385	1.2945	-4 dB
0.7012	0.3706	1.3266	-3 dB
0.7427	0.4052	1.3612	-2 dB
0.7867	0.4425	1.3986	-1 dB
0.8333	0.4827	1.4387	0 dB
0.8827	0.5258	1.4818	1 dB
0.9350	0.5721	1.5281	2 dB
0.9904	0.6217	1.5778	3 dB
1.0491	0.6749	1.6309	4 dB
1.1113	0.7317	1.6877	5 dB
1.1771	0.7925	1.7485	6 dB
1.2469	0.8573	1.8134	7 dB
1.3207	0.9266	1.8826	8 dB

Ripple = 1.25 dB   S = 0.5

QBP	fL factor	fH factor	Gain
0.6310	0.2600	1.5312	-8 dB
0.6683	0.2867	1.5579	-7 dB
0.7079	0.3158	1.5870	-6 dB
0.7499	0.3474	1.6186	-5 dB
0.7943	0.3817	1.6528	-4 dB
0.8414	0.4189	1.6900	-3 dB
0.8913	0.4591	1.7302	-2 dB
0.9441	0.5025	1.7736	-1 dB
1.0000	0.5493	1.8204	0 dB
1.0593	0.5997	1.8709	1 dB
1.1220	0.6540	1.9251	2 dB
1.1885	0.7122	1.9833	3 dB
1.2589	0.7747	2.0458	4 dB
1.3335	0.8417	2.1128	5 dB
1.4125	0.9134	2.1845	6 dB
1.4962	0.9901	2.2612	7 dB
1.5849	1.0720	2.3431	8 dB

9.6 Transmission Line (TL) Enclosures

The Transmission Line (aka Waveguide) Enclosure is not new; it dates back to the 1930s when Stromberg and Carlson were trying to make a non-resonant enclosure. From Chapter 2 you remember that an infinitely long tube is modeled by a single resistor, therefore an infinitely long tube-enclosure would produce a non-resonant system. Another way to think about it is to consider that a standing wave set up in a tube will produce a reflected (backward) wavefront which will dampen the motion of the driver. The TL Enclosure seems to come and go in popularity; the Bose Wave Radio is the current popular TL design but these enclosures have also been used in sound reinforcement applications as well. There is also serious disagreement about the usefulness of the TL enclosure. It appears to be no more efficient than a vented enclosure.

Stromberg and Carlson used a tube that was 1/4 the wavelength (λ) of the resonant frequency of the driver. A 3/4 wavelength and 1/4 wavelength tube will both produce an anti-node at the open end which allows the wave to reflect back into the tube. By using a $1/4\lambda$ tube, the enclosure can be made to be smaller. Their design did not have any filling inside the enclosure. Their design worked and did dampen the resonant frequency of the driver producing a smoother bass response than an infinite baffle alone. Above the resonant frequency of the driver, the TL behaves like an infinite baffle enclosure.

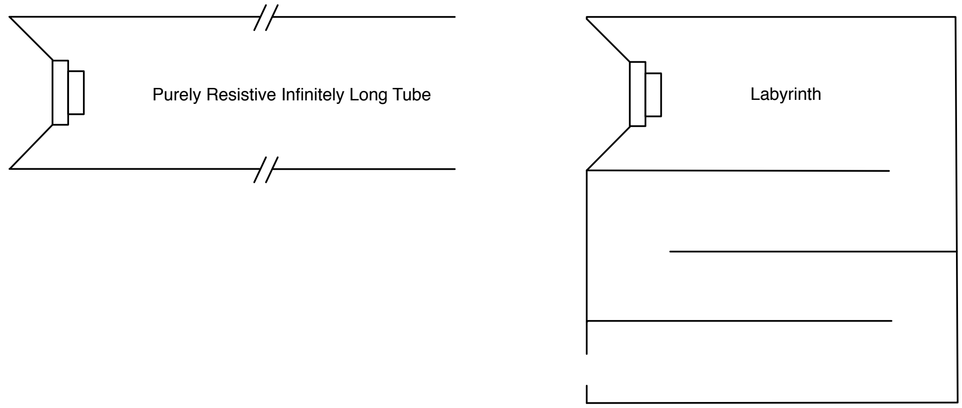


Figure 9.9: Two versions of the non-resonant TL enclosure; the Labyrinth or “folded” tube is the standard enclosure design today.

Later, Bailey experimented with TL designs in the 1960‘s and 1970’s. He added the concept of tapering the transmission line to get narrower and narrower as the distance from the driver increased. He also experimented with stuffing the transmission line with filling. The filling aided in dampening more resonances as well as adding mass loading. Fundamentally, the most important thing the filling does is slow down the speed of sound in the tube. This means that the tube can be made to be even shorter than a quarter wavelength. For long haired sheep’s wool, a 50% filling results in reduction of the speed of sound by 1/2 thus cutting the tube length in half again.

Collum notes that in theory, the transmission line may be open or closed at the end, though in practice manufacturers leave it open. This means that the tube acts like a delay. The propagation time through the tube will result in a vent-output wave that may be in phase or out of phase with the driver itself; at resonance the phase should be the same (as in the Bass Reflex design) providing the dampening load on the driver. He also notes that the mass of air moving in the tube effectively adds to the mass of the driver (this does not occur in conventional enclosures) and lowers the driver’s resonant frequency by a factor of sqrt(2). The mass loading of the filling material may also lower the resonant frequency.

Figure 9.10 compares the responses of the Bass Reflex QB3 and B4 curves with a TL plot. If they all have the same f_{-3} you can see that the TL enclosure has a more extended bass response below f_{-3} however the TL response shows a kind of stair-stepping down towards that point. These “steps” are a telltale component of the TL frequency response. The propagation delay nulls can be seen at the corners of the steps.

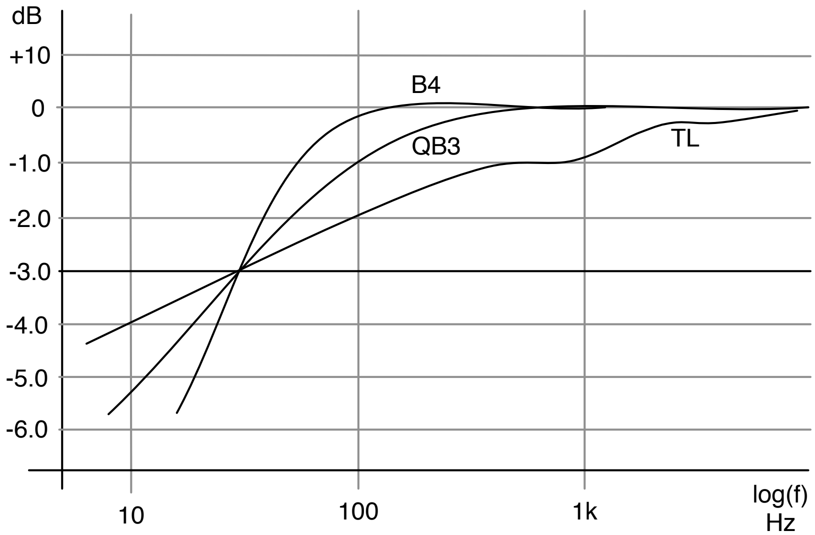


Figure 9.10: Comparing the TL and Bass Reflex responses; if they all share the same f_{-3} you can see that the TL enclosure has a more extended bass response below f_{-3}

9.7 Transmission Line Enclosure Design

The Transmission Line Enclosure design is unique in that you don't need a lot of tables and equations. You start with a driver and then design a tube that is 1/4 or 3/4 the wavelength of the driver's resonant frequency. Filling the tube is optional but many feel the benefits (more dampening and a shorter tube length) warrant it. The fill amount is 50% and the preferred material is long haired sheep's wool. To calculate the reduction in the speed of sound:

$$c' = \frac{c}{\sqrt{1 + P/\rho_0}}$$

$$P = \text{ the packing density of fill} = 8\text{kg/m}^3 \text{ for sheep's wool}$$

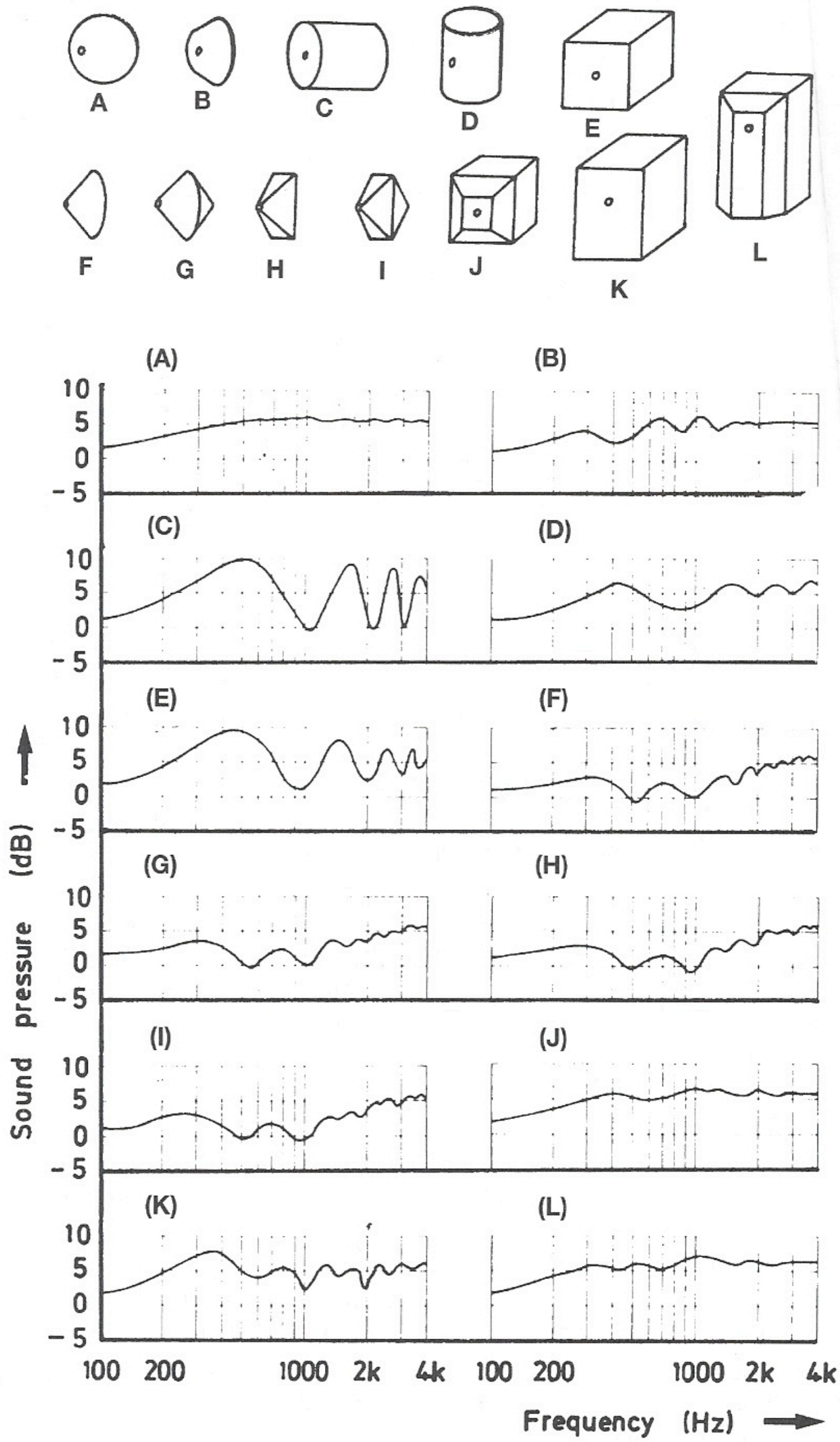
For the Eminence LAB12 speaker with a f_S = 22Hz, we would need the following tube length:

tube	1/4 Wave	3/4 Wave
unfilled	12.8 ft	38.4 ft
50% sheep wool fill	4.6 ft	13.8 ft

There are equations available that describe how to taper the tube and several reference designs for the folded labyrinth cabinet designs. PVC or other rigid tubes can also be used.

Tuning is usually done by ear with familiar source material. Another option is to apply the driver resonant frequency as an input and adjust the tube length until the dB(SPL) is at a maximum.

Diffraction and Cabinet Shape



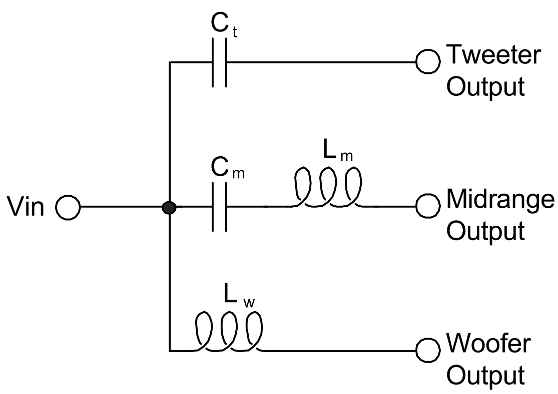
10 Pure LC Loudspeaker Crossover Filters

Probably the most common application for passive filters in audio is in loudspeaker crossover designs. Loudspeakers today are usually built using two or more drivers (speakers). Each driver is optimized to reproduce a certain band of frequencies. Two-way systems employ two drivers – a woofer for low frequency reproduction, and a tweeter for high frequencies. The audio signal is split between a pair of complementary low-pass and high-pass filter, called crossover filters, or together, simply a crossover. Three-way loudspeakers include a midrange driver in addition to the woofer and tweeter. A three-way crossover requires a band-pass filter to feed the midrange driver. The LC crossovers here differ only slightly from the passive filters we've seen so far – you should recognize the filter components. The points to note are:

- You assume the power amplifier driving the loudspeaker has a very low output impedance
- The loudspeaker's DC resistance (usually 4 or 8Ω) is used as the resistor (R) in the design
- You omit the loudspeaker (R) from the crossover schematics for clarity

10.1 2^{nd} Order 3-Way Design

The following design is probably the most common crossover found in audio. You must take care to purchase elements (L and C) with proper voltage and current ratings for the power amp that will be driving the system. Because power amps can put out large voltages and currents, typical small-signal elements may fail. If you are designing for a 2-way system, simply neglect the midrange components.



Design Equations

$$C_t = \frac{1}{2\pi f_t R_t}$$

$$C_m = \frac{1}{2\pi f_L R_m}$$

$$L_m = \frac{R_m}{2\pi f_H}$$

$$L_w = \frac{R_w}{2\pi f_w}$$

The design parameters are

f_t = tweeter crossover frequency
fw = woofer crossover frequency
f_L = bandpass low edge frequency
f_H = bandpass high edge frequency

R_t = DC resistance of tweeter
R_m = DC resistance of midrange driver
R_w = DC resistance of woofer

Example: Design a 2^{nd} order 3-Way Cross-over with the following specifications:

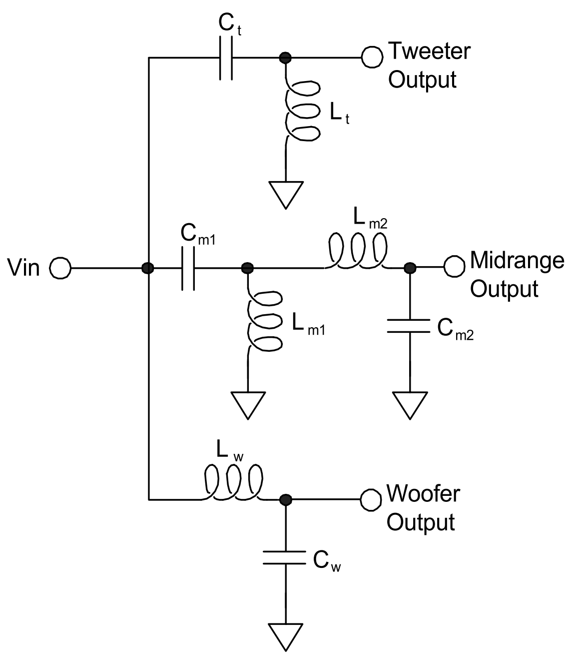
- Woofer, tweeter and midrange all have a DC Resistance of 8 ohms
- $f_t = f_H =$ 5 kHz
- $f_W = f_L =$ 350 Hz

Using the design equations and taking the nearest standard values, you get:

- C_t = 3.9 uF
- C_m = 56 uF
- L_m = 255 uH
- L_w = 3.63 mH

10.2 4th Order 3-Way Design

If steeper cutoff slopes are desired, this 4th order system may be employed. Note the doubling of the number of reactive components for each filter.



Design Equations

$$C_t = \frac{Q_t}{2\pi f_t R_t}$$

$$L_t = \frac{R_t}{2\pi f_t Q_t}$$

$$C_{m1} = \frac{Q_{m1}}{2\pi f_L R_m}$$

$$L_{m1} = \frac{R_m}{2\pi f_L Q_{m1}}$$

$$C_{m2} = \frac{Q_{m2}}{2\pi f_H R_m}$$

$$L_{m2} = \frac{R_m}{2\pi f_H Q_{m1}}$$

$$L_w = \frac{R_w}{2\pi f_w Q_w}$$

$$C_w = \frac{Q_w}{2\pi f_w R_w}$$

The design parameters are:

f_t = tweeter crossover frequency

f_w = w*oofer* crossover frequency

f_L = bandpass low edge frequency

f_H = bandpass high edge frequency

R_t = DC resistance of tweeter

R_m = DC resistance of midrange driver

R_w = DC resistance of woofer

Q_t = Q of HPF for tweeter

Q_w = Q of LPF for woofer

Q_{m1} = Q of low frequency edge of midrange bandpass response

Q_{m2} = Q of high frequency edge of midrange bandpass response

Example: Design a 4th order 3-Way Cross-over with the following specifications:

- Woofer, tweeter and midrange all have a DC Resistance of 8 ohms
- All Q values are 0.707 (Butterworth)
- $f_t = f_H =$ 8 kHz
- $f_W = f_L =$ 200 Hz

Using the design equations and taking the nearest standard values, you get:

- C_t = 1.76 uF L_t = 225uH
- C_{m1} = 70 uF L_{m1} = 9 mH
- C_{m2} = 1.76 uF L_{m2} = 225 uH
- C_w = 70 uF L_w = 9 mH

Example: After listening to your crossover, you decide to redesign it for a crispier high-end by making the Q for the tweeter circuit 2.5, and a punchier bass by increasing the Q of the woofer circuit to 10.

- Woofer, tweeter and midrange all have a DC Resistance of 8 ohms
- $Q_w = 10$, $Q_t = 2.5$, $Q_{mid} = 0.707$
- $f_t = f_H =$ 8 kHz
- $f_W = f_L =$ 200 Hz

Using the design equations and taking the nearest standard values, you get:

- C_t = 6.2 uF L_t = 63.6 uH
- C_{m1} = 70 uF L_{m1} = 9 mH
- C_{m2} = 1.76 uF L_{m2} = 225 uH
- C_w = 994 uF L_w = 636 uH

Direct-Radiator Loudspeaker System Analysis*

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The low-frequency performance of direct-radiator loudspeaker systems can be accurately specified and is quantitatively related to the basic parameters of the system components. These systems function at low frequencies as low-efficiency electroacoustic high-pass filters; the frequency-dependent behavior is described by rational polynomial functions whose coefficients contain basic component parameters. These basic parameters, which are simple to evaluate, determine the system low-frequency response, efficiency, and power ratings.

Editor's Note:

This is the first of a series of papers by R. H. Small which will have a long-term impact on direct-radiator loudspeaker theory. This paper is mainly concerned with terminology, definitions, and setting a thorough background for the following papers on specific kinds of loudspeaker systems.

The work on efficiency, power considerations, and large-signal effects is the most accurate that I know of. The appendix contains the only derivation I know of in print for Thiele's methods of driver-parameter measurement.

J. R. ASHLEY

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GLOSSARY OF SYMBOLS

B	magnetic flux density in driver air gap
c	velocity of sound in air ($=345$ m/s)
C_{AB}	acoustic compliance of air in enclosure
C_{AP}	acoustic compliance of passive radiator suspension
C_{AS}	acoustic compliance of driver suspension
C_{MS}	mechanical compliance of driver suspension ($=C_{AS}/S_D^2$)
C_{MES}	electrical capacitance due to driver mass ($=M_{AS}S_D^2/B^2l^2$)
e_g	open-circuit output voltage of source
f	natural frequency variable
f_{CT}	resonance frequency of driver in closed test box
f_S	resonance frequency of driver
$G(s)$	response function
k_x	system displacement constant
l	length of voice-coil conductor in magnetic field

L_{CES} electrical inductance due to driver compliance $(=C_{AS}B^2l^2/S_D^2)$

M_{ACT} acoustic mass of driver in closed test box including air load

M_{AP} acoustic mass of port or passive radiator including air load

M_{AS} acoustic mass of driver diaphragm assembly including air load

M_{MS} mechanical mass of driver diaphragm assembly including air load $(=M_{AS}S_D^2)$

P_A acoustic output power

P_{AR} displacement-limited acoustic power rating

P_E nominal electrical input power

P_{ER} displacement-limited electrical power rating

$P_{E(\max)}$ thermally limited maximum input power

Q ratio of reactance to resistance (series circuit) or resistance to reactance (parallel circuit)

Q_E Q of driver at f_S considering system electrical resistance $(R_g + R_E)$ only

Q_{ECT} Q of driver at f_{CT} considering electrical resistance R_E only

Q_{ES} Q of driver at f_S considering electrical resistance R_E only

Q_M Q of driver at f_S considering system nonelectrical resistances only

Q_{MCT} Q of driver at f_{CT} considering nonelectrical resistances only

Q_{MS} Q of driver at f_S considering driver nonelectrical resistances only

Q_T total Q of driver at f_S including all system resistances

R_{AB} acoustic resistance of enclosure losses due to internal energy absorption

R_{AL} acoustic resistance of enclosure losses due to leakage

R_{AP} acoustic resistance of port or passive radiator losses

R_{AS} acoustic resistance of driver suspension losses

R_{AT} acoustic resistance of total driver-circuit losses

R_E dc resistance of driver voice coil

R_{ES} electrical resistance due to driver suspension losses $(=B^2l^2/S_D^2R_{AS})$

R_g output resistance of source or amplifier

R_{MS} mechanical resistance of driver suspension losses $(= R_{AS}S_D^2)$

$\mathcal{R}_{AR}$ acoustic radiation resistance

s complex frequency variable $(=\sigma + j\omega)$

S_D effective projected surface area of driver diaphragm

T time constant $(=1/2\pi f)$

u linear velocity

U volume velocity

V_{AS} volume of air having same acoustic compliance as driver suspension $(=\rho_0c^2C_{AS})$

V_D peak displacement volume of driver diaphragm $(=S_Dx_{\max})$

x linear displacement

$x_{\max}$ peak displacement limit of driver diaphragm

$X(s)$ driver diaphragm displacement function

$Z_{VC}(s)$ voice-coil impedance function

η efficiency

η_0 reference efficiency

ρ_0 density of air $(=1.18 \text{ kg/m}^3)$

$\sigma_{x(P)}$ static displacement sensitivity of unenclosed driver expressed in meters per watt$^{1/2}$

ω radian frequency variable $(=2\pi f)$

INTRODUCTION: It is quite possible that the vagueness which infuses many discussions of loudspeakers has its roots in the chaotic terminology of the subject. The word "loudspeaker" itself long ago lost any specific meaning. Despite conflicting attempts by various nationalities to define it as a driver unit or as a complete system, the word retains value only as a general term and as an adjective. For the sake of clarity, this paper uses the common but more specific terms below.

A *source* is a device, usually an electronic power amplifier, which supplies electrical energy at a specified voltage or power level.

A loudspeaker *driver* is a transducer mechanism which converts electrical energy into mechanical and/or acoustical energy. The most common type of driver and the one dealt with in this paper is the moving-coil or electrodynamic driver consisting of a voice coil located in a permanently magnetized air gap and attached to a suspended diaphragm or "cone."

A *baffle* is a structure used to support a driver and to reduce or prevent cancellation of radiation from the front of the driver diaphragm by antiphase radiation from the rear.

An *enclosure* is a cabinet or box in which a driver is mounted for the purpose of radiating sound. The enclosure forms a closed geometrical surface except for the driver mounting aperture or other specified apertures.

A loudspeaker *system* is the combination of a driver (or drivers) with a structural radiation aid such as a horn, baffle, or enclosure which is used to convert electrical energy from a specified source into sound.

A *direct-radiator loudspeaker system* is a loudspeaker system which couples acoustical energy directly to the air from the driver diaphragm and/or simple enclosure apertures without the use of horns or other acoustical impedance-matching devices.

The *piston range* of a loudspeaker driver is that range of frequencies for which the wavelength of sound is longer than the driver diaphragm circumference. In this frequency range, a direct-radiator system using the driver in an enclosure will have an acoustic output which is essentially nondirectional.

Loudspeaker System Design

Direct-radiator loudspeaker systems have been in use for about half a century. During this time, much knowledge of the behavioral properties of various types of direct-radiator systems has been accumulated, but this knowledge is still uneven and incomplete. For example, closed-box systems are much better understood than vented-box systems, while quantitative design information for passive-radiator systems cannot be found in published form.

The design of a loudspeaker system is traditionally a trial-and-error process guided by experience: a likely driver is chosen and various enclosure designs are tried until the system performance is found to be satisfactory. In sharp contrast to this empirical design process is the synthesis of many other engineering systems. This begins with the desired system performance specifications and leads directly to specification of system components.

The latter approach requires the engineer to have precise knowledge of the relationships between system performance and component specifications. The method of analysis described in this paper is a means of obtaining this knowledge for the low-frequency performance of all types of direct-radiator loudspeaker systems; it is based on the high-pass-filter behavior of these systems.

Loudspeaker System Sensitivity and Efficiency

An ideal microphone converts sound pressure into voltage with equal sensitivity at all frequencies. Recording and reproducing systems are designed to process signal voltages representing sound pressure without distortion. To complete the sound reproduction process, an ideal loudspeaker system should convert voltage into sound pressure with equal sensitivity at all frequencies.

In practice, all loudspeaker systems have limited bandwidth. In the low-frequency region, they act as high-pass filters. The low-frequency design of a loudspeaker system may thus be regarded as the design of a high-pass filter [1], [2]. The principal difference is that the loudspeaker system designer has very limited control over the "circuit" configuration; his design freedom is limited to obtaining the best possible performance by manipulation of the system component values.

The frequency response of an electrical filter is normally described in terms of a dimensionless voltage or power ratio. Because a loudspeaker system is a transducer, its sensitivity versus frequency response is the ratio of two unlike quantities, sound pressure and voltage. However, the loudspeaker system response can also be defined in terms of a dimensionless power ratio which is proportional to the square of the above sensitivity ratio.

In the frequency range for which the system radiation is nondirectional, the free-field sound pressure at a fixed distance is proportional to the square root of the acoustic power radiated by the system [3, p. 189]. The electrical power delivered into a fixed resistance by the source is proportional to the square of the source output voltage. Thus the ratio of *the actual system acoustic output power* to *the electrical power delivered into a fixed resistance by the same source* represents exactly the square of the system sensitivity ratio (i.e., the system frequency response), except for a constant factor. If the fixed resistance is chosen to fairly represent the input impedance of the loudspeaker system, the value of the power ratio in the system passband is the nominal electroacoustic conversion efficiency of the system.

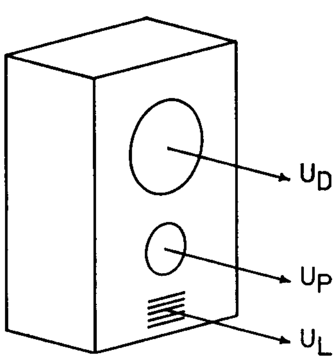

Fig. 1. Generalized direct-radiator loudspeaker system.

This method of defining loudspeaker efficiency is quite similar in principle to the power available efficiency definition used by Beranek [3, p. 190] in that both reveal the exact frequency response of the system. The principal advantage of the method used here is that the calculated passband efficiency of the system is independent of generator output resistance and realistically relates the acoustic power capability of the system to the electrical power rating of its source.

SMALL-SIGNAL PERFORMANCE RELATIONSHIPS

Acoustic Output Power

A generalized direct-radiator loudspeaker system [4, Fig. 1] is illustrated in Fig. 1. The system enclosure has apertures for a driver, a port (or passive radiator), and leakage. Electrical input to the driver produces air movement at the driver diaphragm, port, and leak; this air movement is shown in Fig. 1 as the acoustic volume velocities U_D, U_P, and U_L.

At very low frequencies, where the dimensions of and spacings between the enclosure apertures are much less than a wavelength, the system can be regarded as a combination of coincident simple sources [3, p. 93]. The acoustic output is thus nondirectional and is equivalent to that of a single simple source having a strength U_0 equal to the vector sum of the individual aperture volume velocities, i.e.,

$$U_0 = U_D + U_P + U_L. \tag{1}$$

The acoustic power radiated by the system is then

$$P_A = |U_0|^2 \mathcal{R}_{AR} \tag{2}$$

where

P_A acoustic output power
$\mathcal{R}_{AR}$ resistive part of radiation load on system.

Eq. (2) is generally valid to the upper limit of the driver piston range because the driver is normally the only significant radiator at frequencies high enough for the aperture spacings to become important.

In a recent paper [5], Allison and Berkovitz have demonstrated that the low-frequency load on a loudspeaker system in a typical listening room is essentially that for one side of a piston mounted in an infinite baffle. The resistive part of this radiation load [3, p. 216] is

$$\mathcal{R}_{AR} = \rho_0\omega^2/(2\pi c) \tag{3}$$

where

ρ_0 density of air
ω steady-state radian frequency
c velocity of sound in air.

Eq. (3) is valid only in the system piston range, but within this range the value of $\mathcal{R}_{AR}$ is independent of the size of the enclosure or its apertures.

Because mass cannot be created or stored at the en-

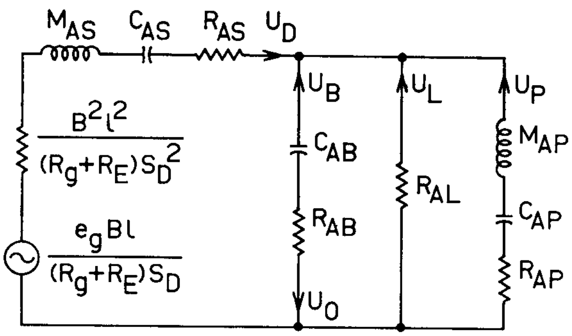

Fig. 2. Acoustical analogous circuit of generalized direct-radiator loudspeaker system.

closure boundaries, and because the sound pressure is normally much less than the atmosphere pressure, conservation of mass requires that

$$U_0 = -U_B \tag{4}$$

where U_B is the total volume velocity entering the enclosure. Eq. (4) holds even if the enclosure is internally divided. If the enclosure contains several cavities, then

$$U_B = U_{B1} + U_{B2} + U_{B3} + \cdots, \tag{5}$$

where each term on the right-hand side of Eq. (5) represents the net volume velocity entering each individual cavity.

Eqs. (1), (4), and (5) are general and hold for any number of cavities and apertures and any interconnection of these. They are vector equations which require that the relative phase of the various components be taken into account.

Although Eq. (4) is very simple, it is of key importance in the analysis of direct-radiator loudspeaker systems using an enclosure. In combination with Eq. (2), it reveals that the acoustic power radiated by the system is directly related to the volume velocity compressing and expanding air within the enclosure. This fact has been noted for bass-reflex enclosures by Beranek [3, p. 244], de Boer [1], and others; it is equally true for *all* direct-radiator system enclosures [4, eq. (72) ff].

Electrical Input Power

The nominal electrical input power to a loudspeaker system is defined here as the power delivered by the source into a resistor having the same value as the driver voice-coil resistance [2, eq. (10)]. Thus

$$P_E = \left[\frac{e_g}{R_g + R_E}\right]^2 R_E \tag{6}$$

where

- P_E nominal electrical input power
- e_g open-circuit output voltage of source
- R_g output resistance of source
- R_E dc resistance of driver voice coil.

The value of R_E is typically about 80% of the rated driver voice-coil impedance.

American [6], British [7], and international [8] standards make use of variously defined rating impedances in calculating the nominal input power to a loudspeaker driver. Because the calculated acoustic output power of the system depends on R_E and not on the fictitious rating impedance, the definition used here simplifies the expression for theoretical system efficiency derived below. This difference must be remembered if the computed piston-range reference efficiency of a system is to be compared with the efficiency measured according to the methods of one of the above standards.

EFFICIENCY

From Eqs. (2) and (6), the nominal power transfer ratio or efficiency η of a loudspeaker system is

$$\eta = \frac{P_A}{P_E} = |U_0|^2 R_{AR} \frac{(R_g+R_E)^2}{e_g^2 R_E}. \tag{7}$$

The evaluation of this efficiency expression for a given system requires a knowledge of the relationship between U_0 and e_g. This relationship is found by examining the acoustical circuit of the system.

The development of acoustical circuits is described in excellent detail by Olson [9] and Beranek [3, ch. 3]. Fig. 2 is the impedance-type acoustical analogous circuit for the generalized loudspeaker system of Fig. 1 [4, Fig. 15]. In Fig. 2,

- B magnetic flux density in driver air gap
- l length of voice-coil conductor in magnetic field of air gap
- S_D effective projected surface area of driver diaphragm
- M_{AS} acoustic mass of driver diaphragm assembly including voice coil and air load
- C_{AS} acoustic compliance of driver suspension
- R_{AS} acoustic resistance of driver suspension losses
- C_{AB} acoustic compliance of air in enclosure
- R_{AB} acoustic resistance of enclosure losses due to internal energy absorption
- R_{AL} acoustic resistance of enclosure losses due to leakage
- M_{AP} acoustic mass of port or passive radiator including air load
- C_{AP} acoustic compliance of passive radiator suspension
- R_{AP} acoustic resistance of port or passive radiator losses.

Starting from the circuit of Fig. 2, the acoustical analogous circuits of most common direct-radiator systems can be obtained by removing or short-circuiting appropriate elements. Note that for the analogy used in this circuit, voltages represent acoustic pressures and currents

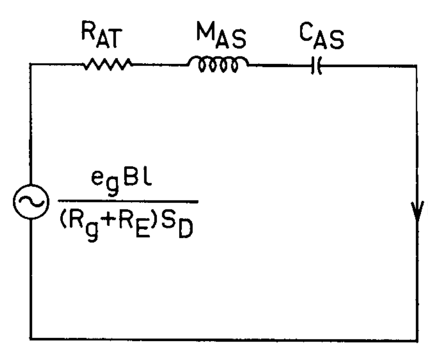

Fig. 3. Acoustical analogous circuit of infinite-baffle loudspeaker system.

represent volume velocities. The method of obtaining the system efficiency expression from analysis of the system acoustical circuit is illustrated below for the simple infinite-baffle system.

The acoustical analogous circuit of an infinite-baffle loudspeaker system is derived from the general circuit of Fig. 2 by removing the branches representing the passive radiator and enclosure leakage and short-circuiting the branch representing the interior of the enclosure to make the enclosure dissipation zero and the enclosure compliance infinite. The resulting circuit is shown in Fig. 3. A simplification has been made in this circuit by combining the remaining series resistances to form the total acoustic resistance

$$R_{AT} = R_{AS} + \frac{B^2 l^2}{(R_g + R_E) S_D^2}. \tag{8}$$

From circuit analysis of Fig. 3,

$$U_0 = \frac{e_g Bl}{(R_g + R_E) S_D s M_{AS}} \cdot G(s) \tag{9}$$

where

$$G(s) = \frac{s^2 C_{AS} M_{AS}}{s^2 C_{AS} M_{AS} + s C_{AS} R_{AT} + 1} \tag{10}$$

and s is the complex frequency variable.

For steady-state sinusoidal excitation $s = j\omega$, and Eqs. (5) and (9) may be combined with Eq. (7) to yield the infinite-baffle efficiency expression

$$\eta(j\omega) = \frac{\rho_0}{2\pi c} \frac{B^2 l^2}{R_E S_D^2 M_{AS}^2} |G(j\omega)|^2 \tag{11}$$

where $G(j\omega)$ is $G(s)$ from Eq. (10) with $s = j\omega$. Note that $G(j\omega)$ contains all the frequency-dependent terms of Eq. 11); the remainder of the expression contains only physical, numerical, and driver constants.

The last part of Eq. (11), i.e., the squared magnitude of $G(j\omega)$, is the infinite-baffle system frequency response expressed as a normalized power ratio. The normalized ratio of sound pressure to source voltage, i.e., the normalized sensitivity or sound pressure frequency response, is thus simply $|G(j\omega)|$; it can be seen from Eq. (10) that this is a second-order (12-dB per octave cutoff) high-pass filter function.

For any direct-radiator system using an enclosure, the expressions for total volume velocity and efficiency have the same form as Eqs. (9) and (11); only the function $G(s)$ is different for each system.

The system response function $G(s)$ contains complete information about the amplitude and phase versus frequency responses and the transient response of the system. $G(s)$ is always a high-pass filter function with a value of unity in the passband. Thus the constant part of Eq. (11) is the system passband efficiency.

ASSUMPTIONS AND APPROXIMATIONS

The acoustical analogous circuits of Figs. 2 and 3 are valid only for frequencies within the piston range of the driver; the circuit components are assumed to have values which are independent of frequency within this range.

Circuit elements which do not contribute enough impedance to affect the analysis are neglected. One of these elements is the radiation resistance. Although this resistance is responsible for the radiated power and is therefore included in Eq. (2), it is in fact quite small compared to the other impedances in the acoustical circuit [2, p. 489]. This is fortunate for purposes of analysis because the radiation resistance is not constant but varies with frequency squared. Also neglected is the driver voice-coil inductance which usually has negligible effects in the limited frequency range of this analysis.

The treatment of acoustical masses is simplified by adding together all masses appearing in series in the same branch of the analogous circuit. This means that physical and air-load masses are lumped together. While the resulting total mass is essentially constant with frequency, it may vary, in the case of the driver, with mounting location or mounting conditions. This must be remembered when dealing with the actual system and measuring its parameters.

SMALL-SIGNAL PARAMETERS

The response function and other describing equations of a loudspeaker system generally contain driver, enclosure, and source parameters. Knowledge of these relationships for a particular system is of practical use only if the parameter values are known or can be measured.

One key to the identification and measurement of the system parameters lies in the system electrical equivalent circuit. This is the dual of the system acoustical analogous circuit and may be derived from it; its formation is well explained in [9] and [3, ch. 3]. Once the circuit is determined, straightforward circuit analysis yields the relationship between the impedance measured at the voice-coil terminals of the actual system and the physical components which constitute the system. It is thus possible to determine the system parameters from measurement of the voice-coil circuit impedance.

Driver Parameters

The fundamental electromechanical driver parameters which control system small-signal performance are R_E, (Bl), S_D, C_{MS}, M_{MS}, and R_{MS}, where

C_{MS} mechanical compliance of driver suspension $(= C_{AS}/S_D^2)$

M_{MS} mechanical mass of driver diaphragm assembly including voice coil and air load $(= M_{AS} S_D^2)$

R_{MS} mechanical resistance of driver suspension losses $(= R_{AS} S_D^2)$.

These parameters are fundamental because each can be set independently of the others, and each has some effect on the system small-signal performance.

For purposes of analysis and design, it is advantageous to describe the driver in terms of the four basic parameters used by Thiele [2] which are related to those above but are easier to measure and to work with. These are as follows.

f_S resonance frequency of moving system of driver, defined by Eq. (12) and usually specified for driver in air with no baffle (f_{SA}) or on a specified baffle (f_{SB})

V_{AS} acoustic compliance of driver, expressed as an equivalent volume of air according to Eq. (15)

Q_{MS} ratio of driver electrical equivalent frictional resistance to reflected motional reactance at f_S, defined by Eq. (13).

Q_{ES} ratio of voice-coil dc resistance to reflected motional reactance at f_S, defined by Eq. (14).

The parameters Q_{MS} and Q_{ES} correspond to Thiele's Q_a and Q_e. They have been given the extra subscript S to make it clear that they apply to the driver alone and to prevent confusion with the *system* parameters Q_M and Q_E, corresponding to Thiele's Q_a and Q_e (total), defined at the end of this section.

Driver Electrical Equivalent Circuit

The electrical equivalent circuit of a driver in air or mounted on an infinite baffle is shown in Fig. 4. In this circuit,

C_{MES} electrical capacitance due to driver mass $(= M_{AS}S_D^2/B^2l^2)$

L_{CES} electrical inductance due to driver compliance $(=C_{AS}B^2l^2/S_D^2)$

R_{ES} electrical resistance due to driver suspension losses $(=B^2l^2/S_D^2R_{AS})$.

The circuit of Fig. 4 is the dual of Fig. 3. An important difference is that the real voice-coil terminals are available in Fig. 4.

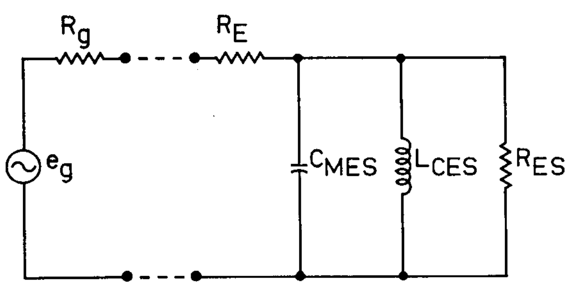

Fig. 4. Electrical equivalent circuit of moving-coil electrodynamic driver.

In Fig. 4, the driver reactances form a resonant circuit which has a resonance frequency $\omega_S = 2\pi f_S$, or a characteristic time constant T_S, given by

$$T_S^2 = 1/\omega_S^2 = C_{MES}L_{CES} = C_{AS}M_{AS}. \qquad (12)$$

The Q of the driver resonant circuit with R_{ES} acting alone is

$$Q_{MS} = \omega_S C_{MES} R_{ES} = 1/(\omega_S C_{AS} R_{AS}). \qquad (13)$$

Similarly, the Q with R_E acting alone, i.e., with $R_g = 0$, is

$$Q_{ES} = \omega_S C_{MES} R_E = \omega_S R_E M_{AS} S_D^2/(B^2l^2). \qquad (14)$$

The parameter V_{AS} is a volume of air having the same acoustic compliance as the driver suspension. Thus [3, p. 129]

$$V_{AS} = \rho_0 c^2 C_{AS}. \qquad (15)$$

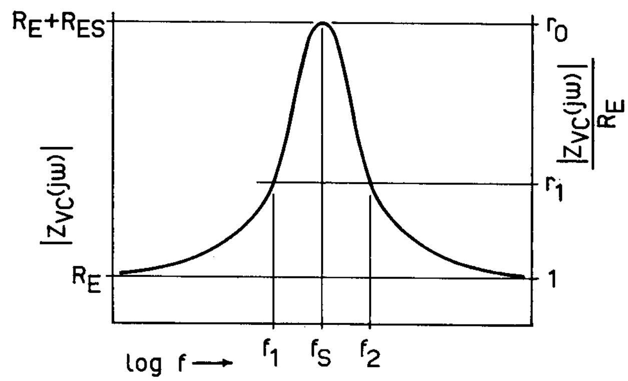

Fig. 5. Driver voice-coil impedance magnitude.

Driver Voice-Coil Impedance Function

The impedance of the circuit to the right of the voice-coil terminals in Fig. 4 is

$$Z_{VC}(s) = R_E + R_{ES}\left[\frac{sT_S/Q_{MS}}{s^2T_S^2 + sT_S/Q_{MS} + 1}\right]. \qquad (16)$$

The steady-state magnitude $|Z_{VC}(j\omega)|$ of Eq. (16) is plotted in Fig. 5; this has the form of a resonance curve which is displaced upward by an amount R_E.

Measurement of Driver Parameters

If the voice-coil impedance of an actual driver is plotted against frequency with the driver in air or on a simple test baffle, the resulting plot will have the same shape as Fig. 5. The driver resonance frequency f_S is easily located where the measured impedance is a maximum. If the ratio of the maximum voice-coil impedance to the dc resistance R_E is defined as r_0, and the two frequencies $f_1 < f_S$ and $f_2 > f_S$ are found where the impedance magnitude is $\sqrt{r_0}R_E$, then as shown in the Appendix,

$$Q_{MS} = \frac{f_S\sqrt{r_0}}{f_2 - f_1} \qquad (17)$$

and

$$Q_{ES} = \frac{Q_{MS}}{r_0 - 1}. \qquad (18)$$

To obtain the value of V_{AS}, a known compliance is added to the moving system by mounting the driver in a small unlined test box which is closed except for the driver aperture. The above driver parameters are then remeasured and values obtained for the new resonance frequency f_{CT} and the electrical Q, Q_{ECT}. Then, as shown in the Appendix,

$$V_{AS} = V_T\left[\frac{f_{CT}Q_{ECT}}{f_S Q_{ES}} - 1\right] \qquad (19)$$

where V_T is the net internal volume of the test box.

Source Parameters

The amplifier specifications that affect the small-signal performance of a loudspeaker system are frequency response and output resistance.

The frequency response of a good audio amplifier is usually wider and flatter than that of the loudspeaker system, and thus the frequency response function obtained from the system efficiency expression effectively describes the overall low-frequency response from the amplifier input terminals. The overall response may be modified or adjusted if desired by the addition to the amplifier of supplementary electrical filters [2].

The amplifier output resistance R_g is in series with the driver voice-coil resistance R_E and therefore affects the system behavior by influencing the total Q in the driver branch. Most modern amplifiers are designed to have a high damping factor, which means that R_g is made small compared to any expected value of R_E. This condition is usually assumed in the design of general-purpose loudspeaker systems, and the driver parameters are adjusted to give the required total Q.

If an amplifier and loudspeaker system are designed as a unit, extra design freedom may be gained by adjusting R_g to provide the desired total Q. Using suitable feedback techniques, R_g may be made positive, zero, or negative.

Measurement of Amplifier Source Resistance

The value of R_g may be found by driving the amplifier with a sinusoidal signal and measuring the amplifier output voltage under conditions of no load and rated load. If the no-load output voltage is e_0, the loaded output voltage is e_L, and the load resistance is R_L, then

$$R_g = R_L \frac{e_0 - e_L}{e_L}. \qquad (20)$$

If there is no measurable difference between e_0 and e_L, R_g may be considered zero as far as its effect on total Q is concerned. Accurate measurement is not required in this case, as it is the *total* resistance $(R_g + R_E)$ that is important.

Amplifier specifications often give the value of R_g (or the damping factor for rated load) measured at 1 kHz. For purposes of calculating system Q at low frequencies, the value measured at 50 Hz is more meaningful.

Enclosure Parameters

The enclosure parameters vary in number according to the type of system. Referring to Fig. 2, all of the vertical branches on the right of the figure contain enclosure components.

The most important property of the enclosure is its physical volume V_B which determines the compliance C_{AB}. If the component M_{AP} is present in the system, with or without C_{AP}, the enclosure will exhibit a resonance frequency f_B (or time constant T_B). If C_{AP} is present, an additional resonance frequency f_P (or time constant T_P) is introduced. The enclosure or aperture losses may be accounted for by defining Q for the various branches at specified frequencies (f_B or f_P).

Measurement of Enclosure Parameters

In general, the change in the driver voice-coil impedance which occurs when the driver is placed in the enclosure permits identification of the enclosure parameters. Because the relationships are different for every type of enclosure, they are not presented here but will be included in later papers describing each type of system.

Composite System Parameters

In the analysis of direct-radiator loudspeaker systems, certain combinations of the component parameters occur naturally, and consistently, in the system-describing functions. One of these is the ratio of driver compliance to enclosure compliance C_{AS}/C_{AB}. This parameter, the *system compliance ratio*, is of fundamental importance to direct-radiator systems using an enclosure. It appears in the analyses published by Beranek [3, ch. 8] and Thiele [2], and in the equivalent stiffness ratio form S_A/S_S used by Novak [10]. The importance of this parameter to system performance justifies giving it a simplified symbol; in later papers the symbol α introduced by Benson [4, eq. (91)] will be used.

In tuned-enclosure systems, the frequency ratio f_B/f_S occurs naturally in the analysis. This is the *system tuning ratio*; Novak [10] has given it the symbol h.

In every type of system, the driver parameter Q_{ES} is altered by the presence of the source parameter R_g to form a system parameter

$$Q_E = Q_{ES} \frac{R_g + R_E}{R_E}. \qquad (21)$$

The effective value of $(R_g + R_E)$ includes any significant resistance present in connecting leads and crossover inductors.

Similarly, the driver parameter Q_{MS} is modified if the system acoustical analogous circuit has an acoustic resistance in series with R_{AS}. The new system parameter Q_M is usually found by measurement.

The total Q of the driver branch of the system is then given by a composite system parameter

$$Q_T = \frac{Q_E Q_M}{Q_E + Q_M}. \qquad (22)$$

FREQUENCY RESPONSE

Response Function

The response function $G(s)$ of a loudspeaker system may be obtained from the complete efficiency expression as illustrated earlier or by a simpler general method which provides only the response function. In Fig. 6 the acoustical analogous circuit of Fig. 2 is reduced to only four essential components:

p_g acoustic driving pressure given by

$$p_g = \frac{e_g Bl}{(R_g + R_E) S_D} \qquad (23)$$

Z_{AS} impedance of driver branch, normally given by

$$Z_{AS}(s) = R_{AT} + sM_{AS} + \frac{1}{sC_{AS}} \qquad (24)$$

Z_{AB} impedance of branch representing enclosure interior, normally given by

$$Z_{AB}(s) = R_{AB} + \frac{1}{sC_{AB}} \qquad (25)$$

Z_{AA} impedance of all enclosure apertures (except that for the driver) which contribute to total output volume velocity. Note that U_A in Fig.

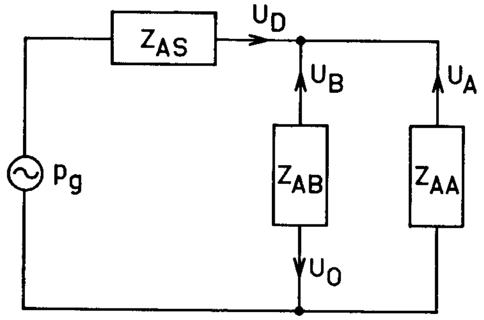

Fig. 6. Simplified acoustical analogous circuit corresponding to Fig. 2.

6 is equal to the sum of U_L and U_P in Fig. 2. Z_{AA} is determined by the specific enclosure design.

The response function is then in all cases

$$G(s) = sM_{AS}\frac{U_0}{p_g} = \frac{sM_{AS}}{Z_{AB} + Z_{AS} + Z_{AB}Z_{AS}/Z_{AA}}. \quad (26)$$

Simplifying the Response Function

The response function obtained from the system acoustical analogous circuit is always a normalized high-pass filter function which is in the form of the ratio of two polynomials in s. The polynomial coefficients contain various combinations of the acoustical masses, compliances, and resistances contained in the system.

The response function is easier to interpret if the acoustical quantities in the coefficients are replaced by the simpler system parameters described in the previous section. Because the coefficients must have dimensions of time only, it is always possible to redefine them in terms of system time constants (or resonance frequencies) together with such dimensionless quantities as Q, compliance ratios, mass ratios, and resistance ratios. These variables are easier for the electrical engineer to interpret than the unfamiliar acoustical quantities.

For the infinite-baffle system analyzed earlier, the response function $G(s)$ is given by Eq. (10). This expression is simplified by substituting

$$T_S^2 = C_{AS}M_{AS} \quad (12)$$

and

$$Q_T = 1/(\omega_S C_{AS} R_{AT}) \quad (27)$$

where Q_T is the total Q (at f_S) of the driver connected to the source. This is the same parameter defined for the general case in Eq. (22). Then

$$G(s) = \frac{s^2T_S^2}{s^2T_S^2 + sT_S/Q_T + 1}. \quad (28)$$

Using the Response Function

Once the system response function is known, the response of any specific system design can be determined if the system parameters are known or are measured so that the corresponding response function coefficients can be calculated. This process is useful in determining the response of existing or proposed systems but gives little insight into the means of improving such systems.

A more useful approach is to explore the behavior of the system response function to determine which coefficient values (i.e., parameter values) produce the most desirable response characteristics. This sounds like a formidable and time-consuming task suitable for computer application, but fortunately the response shapes of greatest interest to the loudspeaker system designer, e.g., those providing flat response in the passband, have already been studied extensively by filter designers.

Because loudspeaker systems have minimum-phase behavior at low frequencies, the amplitude, phase, delay, and transient responses are all related and cannot be specified independently. The most common criterion for optimum response in audio systems is flatness of the amplitude response over a maximum bandwidth, but there may be cases where the designer requires an optimized transient response or delay characteristic. Whatever criterion is used, it is translated into a set of optimum polynomial coefficients so that the system parameter values can be specified or adjusted accordingly.

The adjustment of loudspeaker system response is clearly analogous to the alignment of conventional types of filters. This is particularly apparent where the adjustment goal is the achievement of a predetermined response condition, rather than trial-and-error optimization.

Consider again the infinite-baffle system which has the response function given by Eq. (28). The general form of this class of response function as used by filter designers is

$$G(s) = \frac{s^2T_0^2}{s^2T_0^2 + a_1sT_0 + 1} \quad (29)$$

where

T_0 nominal filter time constant
a_1 damping, or shape, coefficient.

The behavior of Eq. (29) is well known and thus reveals the behavior of the infinite-baffle system when $T_S = T_0$ and $Q_T = 1/a_1$. Using standard curves for Eq. (29), the steady-state magnitude $|G(j\omega)|$ of Eq. (28) is plotted in Fig. 7 for several values of Q_T. The curve for $Q_T = 0.50$ corresponds to the condition for critical damping of the resonant circuit. The curve for $Q_T =$ 0.71 is a maximally flat (Butterworth) alignment which has no amplitude peaking. The curves for Q_T = 1.0, 1.4, and 2.0 have amplitude peaks of approximately 1 dB, 3½ dB, and 6 dB, respectively, but provide extensions of half-power bandwidth as compared to the maximally flat alignment.

For this simple system, the design engineer can choose the response shape he desires and specify the system parameters accordingly; he can also see at a glance the effects of parameter tolerances.

REFERENCE EFFICIENCY

The first part of the efficiency expression (11) for a loudspeaker system contains only physical constants and driver parameters, while the last part, the system response function squared, is always unity for the portion of the piston range above system cutoff. Thus the first part of the expression is the passband or reference efficiency of the system. This reference efficiency, designated η_0, is given by

$$\eta_0 = \frac{\rho_0}{2\pi c}\cdot\frac{B^2l^2}{R_E S_D^2 M_{AS}^2}. \quad (30)$$

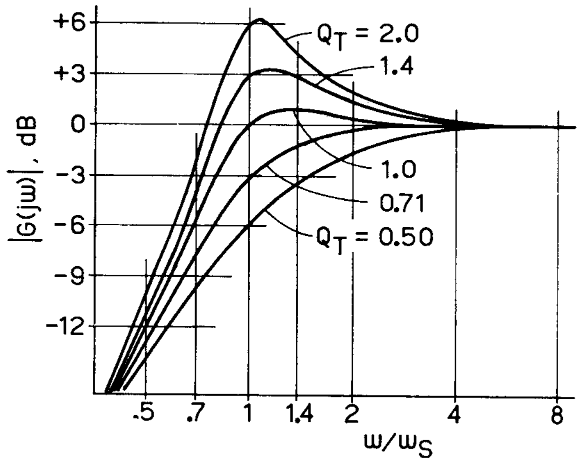

Fig. 7. Normalized frequency response of infinite baffle loudspeaker system.

In terms of the fundamental electromechanical driver parameters, this is

$$\eta_0 = \frac{\rho_0}{2\pi c} \cdot \frac{B^2 l^2}{R_E} \cdot \frac{S_D{}^2}{M_{MS}{}^2}. \tag{31}$$

It must be remembered that M_{AS} and M_{MS} include relevant air-load masses and any deliberate mass loading imposed by the enclosure.

Combining Eqs. (12), (14), and (15) with Eq. (30), the expression for reference efficiency becomes

$$\eta_0 = \frac{4\pi^2}{c^3} \cdot \frac{f_S{}^3 V_{AS}}{Q_{ES}}. \tag{32}$$

The reference efficiency of the system can thus be calculated from the basic driver parameters discussed in Section "Small-Signal Parameters." This result is surprising at first, because these parameters can be determined from simple electrical measurements. This means that the system piston-range electroacoustic efficiency can be found without any direct mechanical, magnetic, or acoustical measurements.

Note that Eq. (32) yields an efficiency twice as large as [2, eq. (76)]. This is because Thiele's expression is derived for the radiation load of a 4π-sr free field, while Eq. (32) assumes the radiation load of a 2π-sr free field. The latter is used here because it is more nearly representative of the radiation load presented to a loudspeaker system by a typical listening room [5].

The physical constants in Eq. (32) have a value of 9.6×10^{-7} in the International System, and this value may be used to compute efficiency if f_S is expressed in hertz and V_{AS} is expressed in cubic meters. However, the value of V_{AS} for most drivers is more conveniently expressed in liters (one liter $= 10^{-3}$ cubic meters). Thus for V_{AS} in liters,

$$\eta_0 = 9.6 \times 10^{-10} \frac{f_S{}^3 V_{AS}}{Q_{ES}}. \tag{33}$$

Alternatively, if V_{AS} is expressed in cubic feet,

$$\eta_0 = 2.7 \times 10^{-8} \frac{f_S{}^3 V_{AS}}{Q_{ES}}. \tag{34}$$

The calculated value of efficiency may be converted into decibels ($10 \log_{10}\eta_0$) or percent ($100\ \eta_0$). The reference efficiency of direct-radiator systems is quite low, typically of the order of one percent.

The resonance frequency of a loudspeaker driver is usually measured with the driver mounted on a standard test baffle having an area of a few square meters [7, sec. 3b], [8, sec. 4.4.1]. Alternatively, some manufacturers prefer to use an effectively infinite baffle, or no baffle at all. Because most drivers are ultimately used in enclosures, the system designer is most interested in the resonance frequency, Q and reference efficiency for an air-load mass equivalent to that of an enclosure; this condition is most nearly approached by a finite "standard" baffle.

If deliberate mass loading of the driver is employed in the system, e.g., placing a restricted aperture in front of the driver, the system reference efficiency will be less than the basic efficiency of the driver. The system efficiency can still be found from Eq. (32) if the values of f_S and Q_{ES} are measured under mass-loaded conditions. The efficiency reduction will be proportional to the square of the mass increase, as shown by Eq. (30).

LARGE-SIGNAL PERFORMANCE

Power Ratings and Large-Signal Parameters

Loudspeaker standards such as [6]–[8] provide only a general guide for the establishment of loudspeaker (driver) power ratings: the input power rating should be such that an amplifier of equivalent undistorted output power rating can be used with the loudspeaker without causing damage or excessive distortion.

At moderately high frequencies, where little diaphragm displacement is required of the driver, the power handling capability of a loudspeaker system is limited by the ability of the driver voice coil to dissipate heat. This leads to a thermally limited absolute maximum input power rating for the *driver*, regardless of the system design. This input power rating is designated $P_{E(max)}$.

At low frequencies much more diaphragm displacement is required of the driver, and it is necessary to establish an input power rating which ensures that the diaphragm is not driven beyond a specified displacement limit. This displacement-limited input power rating is often less than $P_{E(max)}$. Because diaphragm displacement is a function of enclosure design, the displacement-limited power rating is a property of the *system*, not the driver, although it depends on the driver displacement limit.

The displacement limit of a particular driver may be determined by any of a number of criteria. Among these are

1) prevention of suspension damage,
2) limitation of frequency-modulation distortion [11],
3) limitation of nonlinear (harmonic and amplitude-modulation) distortion [12].

For the purpose of this paper it is assumed that a peak displacement limit can be established; this limit is designated x_{max}.

The fundamental large-signal parameter of a driver at low frequencies is then

$$V_D = S_D x_{max}. \tag{35}$$

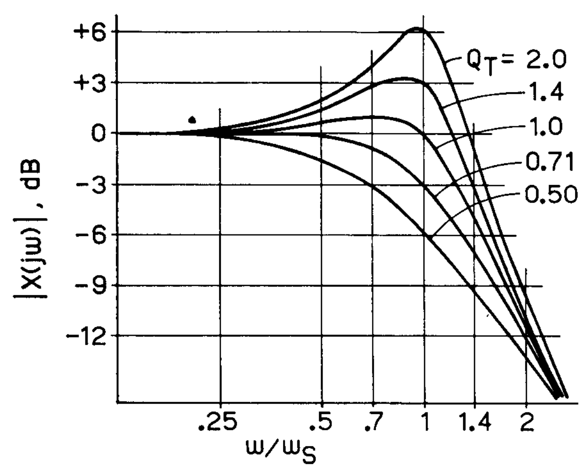

Fig. 8. Normalized diaphragm displacement of driver mounted on infinite baffle.

This parameter, the *diaphragm peak displacement volume*, is the volume of air displaced by the driver diaphragm in moving from rest to its peak displacement limit. It describes the volume displacement limitation and therefore the volume velocity versus frequency limitation of the driver. The practical usefulness of this parameter is illustrated in the following section.

Thus, in addition to the driver small-signal parameters discussed earlier, the system designer must know (or specify) the large-signal parameters $P_{E(\max)}$ and V_D.

Diaphragm Displacement

The small-signal diaphragm displacement of a loudspeaker system driver is determined from the system acoustical analogous circuit. The circuit is first analyzed to obtain the diaphragm volume velocity U_D. Division by S_D then gives the diaphragm velocity u_D, and a further division by s (i.e., integration) yields the diaphragm displacement x_D. The diaphragm displacement expression is always of the form

$$x_D = P_E^{1/2}\,\sigma_{x(P)}\,k_x\,X(s) \qquad (36)$$

where

P_E nominal input power defined by Eq. (6)

$\sigma_{x(P)}$ static (dc) displacement sensitivity of unenclosed driver, expressed in meters per watt$^{1/2}$ and given by

$$\sigma_{x(P)} = \left[\frac{C_{MS}^2B^2l^2}{R_E}\right]^{1/2} = \left[\frac{V_{AS}}{2\pi\rho_0c^2f_SQ_{ES}S_D^2}\right]^{1/2} \qquad (37)$$

k_x system displacement constant of unity or less

$X(s)$ normalized system displacement function.

$X(s)$ is always a *low-pass* filter function which has a value of unity at zero frequency.

For a particular system, the product of the displacement constant k_x and the displacement function $X(s)$ is evaluated by either of two methods. In the first method, the displacement expression (36) is established as described above and divided by $P_E^{1/2}\sigma_{x(P)}$ using Eqs. (6) and (37). In the second method, the acoustical analogous circuit is analyzed for the admittance seen by the generator, and this quantity is divided by sC_{AS}; referring to Fig. 6, this means that in all cases

$$k_xX(s) = \frac{1}{sC_{AS}}\cdot\frac{1+Z_{AB}/Z_{AA}}{Z_{AB}+Z_{AS}+Z_{AB}Z_{AS}/Z_{AA}}. \qquad (38)$$

The resulting expression is then split into a constant factor k_x and a frequency-dependent factor $X(s)$ normalized to unity at zero frequency.

For the infinite-baffle system, circuit analysis of Fig. 3 reveals that the displacement constant is unity and the displacement function is

$$X(s) = \frac{1}{s^2T_S^2+sT_S/Q_T+1}. \qquad (39)$$

The steady-state magnitude $|X(j\omega)|$ of this function is plotted against normalized frequency in Fig. 8. For this simple system, the curves are exact mirror images of those of Fig. 7.

DISPLACEMENT-LIMITED POWER RATINGS

Electrical Power Rating

A useful indication of the sinusoidal steady-state displacement-limited electrical input power capacity of a loudspeaker system is obtained by assuming linear diaphragm displacement for large input signals and limiting the peak value of x_D in Eq. (36) to $x_{\max}$. Thus

$$P_{ER} = \frac{1}{2}\left[\frac{x_{\max}}{\sigma_{x(P)}k_x|X(j\omega)|_{\max}}\right]^2 \qquad (40)$$

where

P_{ER} displacement-limited electrical input power rating in watts

$|X(j\omega)|_{\max}$ maximum magnitude attained by system displacement function, i.e., its value at the frequency of maximum diaphragm displacement.

Substituting Eqs. (35) and (37) into Eq. (40),

$$P_{ER} = \pi\rho_0c^2\frac{f_SQ_{ES}V_D^2}{V_{AS}k_x^2|X(j\omega)|_{\max}^2}. \qquad (41)$$

Acoustic Power Rating

The displacement-limited electrical power rating of a loudspeaker system places a limitation on the continuous power rating of the amplifier to be used with the system. This power rating, together with the reference efficiency of the system, then determines the maximum continuous acoustic power that can be radiated in the flat (upper) region of the system passband. Thus, using Eqs. (32) and (41), the steady-state displacement-limited acoustic power rating P_{AR} of the loudspeaker system is

$$P_{AR} = \frac{4\pi^3\rho_0}{c}\cdot\frac{f_S^4V_D^2}{k_x^2|X(j\omega)|_{\max}^2}. \qquad (42)$$

This rating may easily be converted into a sound pressure level rating for standardized radiation and measurement conditions, e.g., [8, sec. 3.16]. The factor $4\pi^3\rho_0/c$ has the value 0.42 for SI units, i.e., for f_S in Hz and V_D in m^3.

Power Ratings of Infinite-Baffle System

The displacement-limited acoustic power rating of a driver mounted on an infinite baffle is found by setting $k_x = 1$ in Eq. (42). Thus,

$$P_{AR(IB)} = \frac{4\pi^3\rho_0}{c} \cdot \frac{f_S^4 V_D^2}{|X(j\omega)|_{max}^2}. \quad (43)$$

For a given value of V_D, the acoustic power rating is a strong function of the driver resonance frequency. It is also sensitive to Q_T through $|X(j\omega)|_{max}$ (see Fig. 8), but is maximized for $Q_T \leq 0.71$.

As an example, consider an infinite-baffle system having a resonance frequency of 50 Hz and a second-order Butterworth response. If the driver is a 12-inch unit (effective radius 0.12 m) capable of $\pm$ 4 mm peak displacement, then $V_D = 0.18$ dm^3, and the acoustic power rating is $P_{AR} = 0.086$ watt. This is equivalent to a sound pressure level rating of 101.5 dB at a distance of 1 meter [3, p. 14].

Setting k_x equal to unity in Eq. (41), the displacement-limited electrical power rating of the infinite-baffle system is

$$P_{ER(IB)} = \pi\rho_0 c^2 \frac{f_S Q_{ES} V_D^2}{V_{AS}|X(j\omega)|_{max}^2}. \quad (44)$$

This equation demonstrates quantitatively the well-known fact that a woofer designed for acoustic-suspension use (i.e., with very low resonance and high compliance) has a low (input) power handling capacity, compared to that of a conventional woofer, if it is operated in air or on an infinite baffle.

The electrical power rating of the system in the above numerical example depends on the value of driver compliance. If the total moving mass of the driver has a typical value of 30 grams, the driver compliance, from Eq. (12), must be $V_{AS} = 0.1$ m^3. Ignoring mechanical losses and taking $Q_{ES} = Q_T = 0.71$, the electrical power rating from Eq. (44) is then $P_{ER} = 5$ watts. Comparing P_{AR} with P_{ER}, or using Eq. (33), the reference efficiency of the driver is $\eta_0 = 1.7\%$.

Note that the same ratings also apply to an infinite-baffle system using an 8-inch driver (effective radius 0.08 m) capable of $\pm$ 9-mm peak displacement (so that $V_D = 0.18$ dm^3) and having the same resonance frequency, acoustic compliance, and Q.

Assumptions and Corrections

The accuracy of the calculated displacement-limited power ratings depends on the assumptions that the diaphragm displacement is linear up to x_{max} and that the source power bandwidth extends down to the frequency of maximum displacement. Both assumptions may lead to conservative ratings.

For example, the infinite-baffle system described above reaches maximum displacement only at very low frequencies. This system might typically be driven by an amplifier with a low-frequency power bandwidth (-3 dB) of 30 Hz. If the plot of $|X(j\omega)|$ (with constant voltage drive) for $Q_T = 0.71$ in Fig. 8 is multiplied by the normalized power output curve of this amplifier, the resulting maximum value of $|X(j\omega)|$ falls from unity to about 0.7. A more realistic set of power ratings for this loudspeaker system would thus be $P_{ER} = 10$ watts and $P_{AR} = 0.17$ watt.

Similarly, if x_{max} is defined at a displacement beyond the linear range of the driver, then the actual input power required to reach this peak displacement will be higher than the calculated value. A correction factor can easily be computed from the actual displacement versus input characteristic of the driver.

CONCLUSION

The low-frequency response, efficiency, and power ratings of a direct-radiator loudspeaker system are determined by the parameters of the system components. These relationships are reciprocal; specification of the system performance places definite requirements on the component parameters. The most important system component is the driver, which is completely described only when a sufficient number of small-signal and large-signal parameters are specified.

An interesting result of the analysis in this paper is that the driver diaphragm area S_D does not appear explicitly in the small-signal response, small-signal efficiency, or displacement-limited power ratings of a loudspeaker system. This means that it is theoretically possible to design drivers of different diameter with identical values of the parameters f_S, Q_{MS}, Q_{ES}, V_{AS}, and V_D. Used in identical enclosures, these drivers must give identical small-signal performance and displacement-limited power capacity. The principal differences are that the larger driver will cost more but require less diaphragm displacement and thus produce less modulation distortion for a given acoustic output [11], [12].

Although the electrodynamic moving-coil driver has been manufactured throughout the world for decades, hardly a single manufacturer provides complete low-frequency parameter information with his products, or has ever been asked to do so. In the future, trial-and-error design of loudspeaker systems using available drivers will increasingly be replaced by system synthesis based on final performance specifications and resulting in specific driver parameter requirements. Driver manufacturers must be ready to meet demands of this kind and to provide complete parameter information with their products.

The parameters used to describe driver behavior in this paper are not the only consistent set that can be used. However, they do have the advantage of being easy to measure and to comprehend, and, as later papers will show, they are well suited for use in the analysis and design of complete systems.

APPENDIX

DRIVER PARAMETER MEASUREMENTS

Driver Q

From Eqs. (13) and (14),

$$\frac{Q_{MS}}{Q_{ES}} = \frac{R_{ES}}{R_E}. \quad (45)$$

The ratio of voice-coil maximum impedance to dc resistance, from Fig. 5, is therefore

$$r_0 = \frac{R_{ES} + R_E}{R_E} = 1 + \frac{Q_{MS}}{Q_{ES}} \quad (46)$$

from which

$$Q_{ES} = \frac{Q_{MS}}{r_0 - 1}. \quad (18)$$

Also, the total driver Q with a zero-impedance source ($R_g = 0$) is given by

$$Q_{TS} = \frac{Q_{MS} Q_{ES}}{Q_{MS} + Q_{ES}} = \frac{Q_{MS}}{r_0}. \quad (47)$$

Eq. (16) now becomes

$$Z_{VC}(s) = R_E \frac{r_0 + Q_{MS}(sT_S + 1/sT_S)}{1 + Q_{MS}(sT_S + 1/sT_S)} \quad (48)$$

and

$$|Z_{VC}(j\omega)|^2 = R_E^2 \frac{r_0^2 + Q_{MS}^2(\omega/\omega_S - \omega_S/\omega)^2}{1 + Q_{MS}^2(\omega/\omega_S - \omega_S/\omega)^2}. \quad (49)$$

At any two frequencies $\omega_1 < \omega_2$ such that $\omega_1\omega_2 = \omega_S^2$, it can be shown using (49) that the impedance magnitudes will be equal. Let this magnitude be defined by

$$|Z_{VC}(j\omega_1)| = |Z_{VC}(j\omega_2)| = r_1 R_E. \quad (50)$$

Then

$$|Z_{VC}(j\omega_{1,2})|^2 = r_1^2 R_E^2 = R_E^2 \frac{r_0^2 + Q_{MS}^2[(\omega_2 - \omega_1)/\omega_S]^2}{1 + Q_{MS}^2[(\omega_2 - \omega_1)/\omega_S]^2} \quad (51)$$

and therefore

$$Q_{MS} = \frac{\omega_S}{\omega_2 - \omega_1} \sqrt{\frac{r_0^2 - r_1^2}{r_1^2 - 1}}. \quad (52)$$

If $r_1 = \sqrt{r_0}$, Eq. (52) reduces to

$$Q_{MS} = \frac{f_S \sqrt{r_0}}{f_2 - f_1}. \quad (17)$$

Choosing $r_1 = \sqrt{r_0}$ not only makes the calculation simple but provides good measurement accuracy because f_1 and f_2 are reasonably well separated and are located in regions of high slope on the impedance curve.

As shown above, the frequencies f_1 and f_2 where the the measured voice-coil impedance magnitude is $\sqrt{r_0} R_E$ should satisfy the condition

$$\sqrt{f_1 f_2} = f_S. \quad (53)$$

For most real drivers this is not precisely so because the fundamental driver parameters, particularly compliance and mechanical resistance, vary slightly with frequency or diaphragm excursion. Also, the voice-coil inductance, if large, will skew the curve slightly. However, for most well-designed drivers, the result computed from (53) is within about 1 Hz of the measured value. Eq. (53) is thus a useful check to catch measurement errors or to identify drivers which cannot be represented accurately by a set of constant-value parameters.

Driver Compliance

A simple unlined test enclosure at atmospheric pressure has an acoustic compliance C_{AB} related to its net internal volume V_T by [3, p. 129]

$$C_{AB} = V_T/\rho_0 c^2. \quad (54)$$

A driver having total acoustical mass M_{AS} and compliance C_{AS} has a self-resonance defined by

$$T_S^2 = 1/\omega_S^2 = M_{AS} C_{AS}. \quad (12)$$

When this driver is mounted in the closed test box, a new resonance will be measured which is given by

$$T_{CT}^2 = 1/\omega_{CT}^2 = M_{ACT} \frac{C_{AB} C_{AS}}{C_{AB} + C_{AS}} \quad (55)$$

where M_{ACT} is the new total moving mass resulting from any change in the value of the diaphragm air load mass. Then

$$\frac{\omega_{CT}^2}{\omega_S^2} = \frac{M_{AS}}{M_{ACT}} \left[1 + \frac{C_{AS}}{C_{AB}} \right]. \quad (56)$$

From Eq. (14),

$$Q_{ES} = \omega_S R_E M_{AS} S_D^2/(B^2 l^2). \quad (57)$$

Similarly,

$$Q_{ECT} = \omega_{CT} R_E M_{ACT} S_D^2/(B^2 l^2). \quad (58)$$

Therefore,

$$\frac{M_{AS}}{M_{ACT}} = \frac{\omega_{CT} Q_{ES}}{\omega_S Q_{ECT}} \quad (59)$$

and combining Eqs. (56) and (59),

$$1 + \frac{C_{AS}}{C_{AB}} = \frac{\omega_{CT} Q_{ECT}}{\omega_S Q_{ES}}. \quad (60)$$

From Eqs. (15) and (54),

$$\frac{C_{AS}}{C_{AB}} = \frac{V_{AS}}{V_T} \quad (61)$$

and therefore

$$\frac{V_{AS}}{V_T} = \frac{\omega_{CT} Q_{ECT}}{\omega_S Q_{ES}} - 1 \quad (62)$$

or

$$V_{AS} = V_T \left[\frac{f_{CT} Q_{ECT}}{f_S Q_{ES}} - 1 \right]. \quad (19)$$

The initial driver measurements (f_S and Q_{ES}) may be made with a baffle of any size or with no baffle. It is advisable, however, especially with low-resonance drivers, that the driver have its axis horizontal for both sets of measurements to avoid excessive static diaphragm displacement due to gravity.

Energy absorption in the test enclosure walls affects only the measured value of Q_{MCT} and thus has no effect on the compliance calculation. However, absorbing material placed inside the enclosure can affect the value of C_{AB} and should therefore not be used.

It is particularly important to avoid leaks in the test enclosure because these can also change the effective value of C_{AB} and seriously reduce the accuracy of the measurement. The test enclosure must be constructed carefully, and the driver under test must be checked for a tight seal at the mounting gasket. Some drivers have a built-in leakage path around the voice coil, others through a porous edge-suspension material. Measurements on these drivers must be used with caution. To test for leakage, apply an input signal of about 10 Hz at moderate level and listen carefully all around the enclosure and driver for "breathing" indicative of a leak.

Measurement Technique

Loudspeaker impedance measurements are commonly taken with either constant-voltage [3, p. 503] or constant-current [10, p. 13] drive. If the driver is perfectly linear or the measuring level is low enough, the two methods should give the same result. The constant-voltage method has the advantage of more nearly duplicating the usual operating conditions of the driver.

Accurate measurement of small-signal parameters requires a signal level that is small enough for all voltage and current waveforms to be undistorted sinusoids. Use an oscilloscope to observe waveforms and adjust the signal level accordingly. It is often necessary, particularly with unloaded high-compliance drivers, to measure parameters at an input level of 0.1 watt or less.

Measure the driver voice-coil resistance accurately with a dc bridge. A dummy resistance of the same value can then be made up and used as a calibrating load on the equipment for measuring impedance.

Do not trust the frequency scale of audio-sweep type beat frequency oscillators. For maximum accuracy, take frequency readings with a frequency or period counter or from the scale of a stable, accurately calibrated sinewave generator.

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Numerous authors have contributed through their published works to the basic ideas which are developed here. I am particularly indebted to A. N. Thiele for having originated both the filter-oriented approach to analysis and the simple methods of parameter measurement which are described here, and to J. E. Benson for originating the simple generalized loudspeaker system concept, for contributing many hours to the discussion of terminology and symbols, and for carefully checking the equations and computations in the manuscript.

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Note: Mr. Small's biography appeared in the January/February issue.

Closed-Box Loudspeaker Systems
Part I: Analysis

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The closed-box loudspeaker system is effectively a second-order (12 dB/octave cutoff) high-pass filter. Its low-frequency response is controlled by two fundamental system parameters: resonance frequency and total damping. Further analysis reveals that the system electroacoustic reference efficiency is quantitatively related to system resonance frequency, the portion of total damping contributed by electromagnetic coupling, and total system compliance; for air-suspension systems, efficiency therefore effectively depends on frequency response and enclosure size. System acoustic power capacity is found to be fundamentally dependent on frequency response and the volume of air that can be displaced by the driver diaphragm; it may also be limited by enclosure size. Measurement of voice-coil impedance and other mechanical properties provides basic parameter data from which the important low-frequency performance capabilities of a system may be evaluated.

GLOSSARY OF SYMBOLS

B	magnetic flux density in driver air gap
c	velocity of sound in air ($=345$ m/s)
C_{AB}	acoustic compliance of air in enclosure
C_{AS}	acoustic compliance of driver suspension
C_{AT}	total acoustic compliance of driver and enclosure
C_{MEC}	electrical capacitance representing moving mass of system ($=M_{AC}S_D^2/B^2l^2$)
e_g	open-circuit output voltage of source (Thevenin's equivalent generator for amplifier output port)
f	natural frequency variable
f_C	resonance frequency of closed-box system
f_{CT}	resonance frequency of driver in closed, unfilled, unlined test enclosure
f_S	resonance frequency of unenclosed driver
$G(s)$	response function
k_x	displacement constant
k_P	power rating constant
k_η	efficiency constant
l	length of voice-coil conductor in magnetic gap
L_{CET}	electrical inductance representing total system compliance ($=C_{AT}B^2l^2/S_D^2$)
M_{AC}	acoustic mass of driver in enclosure including air load
M_{AS}	acoustic mass of driver diaphragm assembly including air load
P_{AR}	displacement-limited acoustic power rating
P_{ER}	displacement-limited electrical power rating
$P_{E(max)}$	thermally-limited maximum input power
Q	ratio of reactance to resistance (series circuit) or resistance to reactance (parallel circuit)
Q_{EC}	Q of system at f_C considering electrical resistance R_E only

Q_{ES} — Q of driver at f_S considering electrical resistance R_E only
Q_{MC} — Q of system at f_C considering system non-electrical resistances only
Q_{MS} — Q of driver at f_S considering driver non-electrical resistances only
Q_{TC} — total Q of system at f_C including all system resistances
Q_{TCO} — value of Q_{TC} with $R_g = 0$
Q_{TS} — total Q of driver at f_S considering all driver resistances
R_{AB} — acoustic resistance of enclosure losses caused by internal energy absorption
R_{AS} — acoustic resistance of driver suspension losses
R_E — dc resistance of driver voice coil
R_{ES} — electrical resistance representing driver suspension losses ($=B^2l^2/S_D{}^2R_{AS}$)
R_g — output resistance of source (Thevenin's equivalent resistance for amplifier output port)
s — complex frequency variable ($=\sigma + j\omega$)
S_D — effective surface area of driver diaphragm
T — time constant ($=1/2\pi f$)
U_O — system output volume velocity
V_{AB} — volume of air having same acoustic compliance as air in enclosure ($=\rho_0 c^2 C_{AB}$)
V_{AS} — volume of air having same acoustic compliance as driver suspension ($=\rho_0 c^2 C_{AS}$)
V_{AT} — total system compliance expressed as equivalent volume of air ($=\rho_0 c^2 C_{AT}$)
V_B — net internal volume of enclosure
V_D — peak displacement volume of driver diaphragm ($=S_D x_{max}$)
x_{max} — peak linear displacement of driver diaphragm
$X(s)$ — displacement function
$Z_{VC}(s)$ — voice-coil impedance function
α — compliance ratio ($=C_{AS}/C_{AB}$)
γ_B — ratio of specific heat at constant pressure to that at constant volume for air in enclosure
η_0 — reference efficiency
ρ_0 — density of air ($=1.18$ kg/m^3)
ω — radian frequency variable ($=2\pi f$)

1. INTRODUCTION

Historical Background

The theoretical prototype of the closed-box loudspeaker system is a driver mounted in an enclosure large enough to act as an infinite baffle [1, Chap. 7]. This type of system was used quite commonly until the middle of this century.

The concept of the modern air-suspension loudspeaker system was established in a U.S. patent application of 1944 by Olson and Preston [2], [3], but the system was not widely introduced until high-fidelity sound reproduction became popular in the 1950's.

A compact air-suspension loudspeaker system for high-fidelity reproduction was described by Villchur [4] in 1954. Several more papers [5], [6], [7] set out the basic principle of operation but caused a spirited public controversy [8], [9], [10]. Unfortunately, some of the confusion established at the time still remains, particularly with regard to the purpose and effect of materials used to fill the enclosure interior. A recent attempt to dispell this confusion [11] seems to have reduced the level of controversy, and the fundamental validity of the air-suspension approach has been amply proved by its proliferation.

Technical Background

Closed-box loudspeaker systems are the simplest of all loudspeaker systems using an enclosure, both in construction and in analysis. In essence, they consist of an enclosure or box which is completely closed and airtight except for a single aperture in which the driver is mounted.

The low-frequency output of a direct-radiator loudspeaker system is completely described by the acoustic volume velocity crossing the enclosure boundaries [12]. For the closed-box system, this volume velocity is entirely the result of motion of the driver cone, and the analysis is relatively simple.

Traditional closed-box systems are made large so that the acoustic compliance of the enclosed air is greater than that of the driver suspension. The resonance frequency of the driver in the enclosure, i.e., of the system, is thus determined essentially by the driver compliance and moving mass.

The air-suspension principle reverses the relative importance of the air and driver compliances. The driver compliance is made very large so that the resonance frequency of the system is controlled by the much smaller compliance of the air in the enclosure in combination with the driver moving mass. The significance of this difference goes beyond the smaller enclosure size or any related performance improvements; it demonstrates forcibly that the loudspeaker driver and its enclosure cannot be designed and manufactured independently of each other but must be treated as an inseparable *system*.

In this paper, closed-box systems are examined using the approach described in [12]. The analysis is limited to the low-frequency region where the driver acts as a piston (i.e., the wavelength of sound is longer than the driver diaphragm circumference) and the enclosure is active in controlling the system behavior.

The results of the analysis show that the important low-frequency performance characteristics of closed-box systems of both conventional and air-suspension type are directly related to a small number of basic and easily-measured system parameters.

The analytical relationships impose definite quantitative limits on both small-signal and large-signal performance of a system but, at the same time, show how these limits may be approached by careful system adjust-

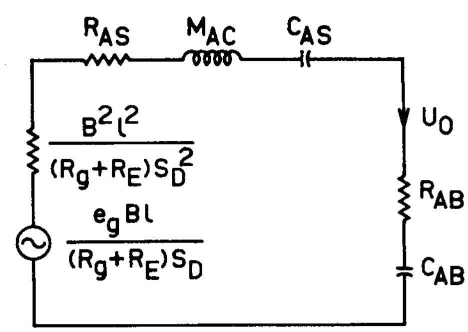

Fig. 1. Acoustical analogous circuit of closed-box loudspeaker system (impedance analogy).

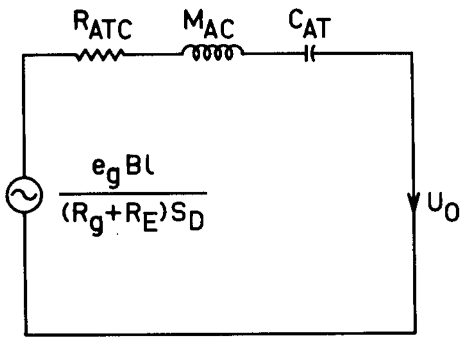

Fig. 2. Simplified acoustical analogous circuit of closed-box loudspeaker system.

ment. The same relationships lead directly to methods of synthesis (system design) which are free of trial-and-error procedures and to simple methods for evaluating and specifying system performance at low frequencies.

2. BASIC ANALYSIS

The impedance-type acoustical analogous circuit of the closed-box system is well known and is presented in Fig. 1. In this circuit, the symbols are defined as follows.

- B Magnetic flux density in driver air gap.
- l Length of voice-coil conductor in magnetic field of air gap.
- e_g Open-circuit output voltage of source.
- R_g Output resistance of source.
- R_E Dc resistance of driver voice coil.
- S_D Effective projected surface area of driver diaphragm.
- R_{AS} Acoustic resistance of driver suspension losses.
- M_{AC} Acoustic mass of driver diaphragm assembly including voice coil and air load.
- C_{AS} Acoustic compliance of driver suspension.
- R_{AB} Acoustic resistance of enclosure losses caused by internal energy absorption.
- C_{AB} Acoustic compliance of air in enclosure.
- U_O Output volume velocity of system.

By combining series elements of like type, this circuit can be simplified to that of Fig. 2. The total system acoustic compliance C_{AT} is given by

$$C_{AT} = C_{AB}C_{AS}/(C_{AB} + C_{AS}), \quad (1)$$

and the total system resistance, R_{ATC}, is given by

$$R_{ATC} = R_{AB} + R_{AS} + \frac{B^2l^2}{(R_g + R_E)S_D^2}. \quad (2)$$

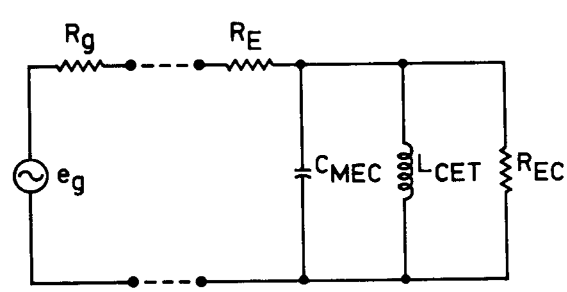

Fig. 3. Simplified electrical equivalent circuit of closed-box loudspeaker system.

The electrical equivalent circuit of the closed-box system is formed by taking the dual of the acoustic circuit of Fig. 1 and converting each element to its electrical equivalent [1, Chapter 3]. Simplification of this circuit by combining elements of like type results in the simplified electrical equivalent circuit of Fig. 3. This circuit is arranged so that the actual voice-coil terminals are available. In Fig. 3, the symbols are given by

$$C_{MEC} = M_{AC}S_D^2/B^2l^2, \quad (3)$$

$$L_{CET} = C_{AT}B^2l^2/S_D^2, \quad (4)$$

$$R_{EC} = \frac{B^2l^2}{(R_{AB} + R_{AS})S_D^2}. \quad (5)$$

The circuits presented above are valid only for frequencies within the driver piston range; the circuit elements are assumed to have values which are independent of frequency within this range. As discussed in [12], the effects of the voice-coil inductance and the resistance of the radiation load are neglected.

To simplify the analysis of the system and the interpretation of its describing functions, the following system parameters are defined.

ω_C ($=2\pi f_C$) Resonance frequency of system, given by

$$1/\omega_C^2 = T_C^2 = C_{AT}M_{AC} = C_{MEC}L_{CET}. \quad (6)$$

Q_{MC} Q of system at f_C considering non-electrical resistances only, given by

$$Q_{MC} = \omega_C C_{MEC} R_{EC}. \quad (7)$$

Q_{EC} Q of system at f_C considering electrical resistance R_E only, given by

$$Q_{EC} = \omega_C C_{MEC} R_E. \quad (8)$$

Q_{TCO} Total Q of system at f_C when driven by source resistance of $R_g = 0$, given by

$$Q_{TCO} = Q_{EC}Q_{MC}/(Q_{EC} + Q_{MC}). \quad (9)$$

Q_{TC} Total Q of system at f_C including all system resistances, given by

$$Q_{TC} = 1/(\omega_C C_{AT} R_{ATC}). \quad (10)$$

α System compliance ratio, given by

$$\alpha = C_{AS}/C_{AB}. \quad (11)$$

If the system driver is mounted on a baffle which provides the same total air-load mass as the system enclosure, the driver parameters defined in [12, eqs. (12), (13) and (14)] become

$$T_S^2 = 1/\omega_S^2 = C_{AS}M_{AC}, \quad (12)$$

$$Q_{MS} = \omega_S C_{MEC} R_{ES}, \quad (13)$$

$$Q_{ES} = \omega_S C_{MEC} R_E, \quad (14)$$

where $R_{ES} = B^2l^2/S_D^2R_{AS}$ is an electrical resistance representing the driver suspension losses. The driver compliance equivalent volume is unaffected by air-load masses and is in every case [12, eq. (15)]

$$V_{AS} = \rho_o c^2 C_{AS}, \quad (15)$$

where ρ_o is the density of air (1.18 kg/m³) and c is the

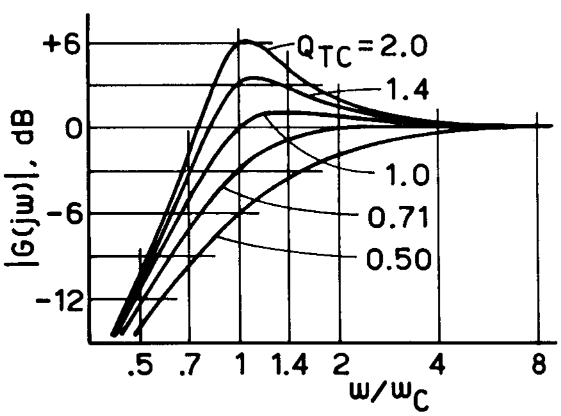

Fig. 4. Normalized amplitude vs normalized frequency response of closed-box loudspeaker system for several values of total system Q.

velocity of sound in air (345 m/s). In this paper, the general driver parameters f_S (or T_S), Q_{MS} and Q_{ES} will be understood to have the above values unless otherwise specified.

Comparing (1), (6), (8), (11), (12) and (14), the following important relationships between the system and driver parameters are evident:

$$C_{AS}/C_{AT} = \alpha + 1, \qquad (16)$$

$$f_C/f_S = T_S/T_C = (\alpha + 1)^{1/2}, \qquad (17)$$

$$Q_{EC}/Q_{ES} = (\alpha + 1)^{1/2}. \qquad (18)$$

Following the method of [12], analysis of the circuits of Figs. 2 and 3 and substitution of the parameters defined above yields the system response function

$$G(s) = \frac{s^2 T_C^2}{s^2 T_C^2 + sT_C/Q_{TC} + 1}, \qquad (19)$$

the diaphragm displacement function

$$X(s) = \frac{1}{s^2 T_C^2 + sT_C/Q_{TC} + 1}, \qquad (20)$$

the displacement constant

$$k_x = 1/(\alpha + 1), \qquad (21)$$

and the voice-coil impedance function

$$Z_{VC}(s) = R_E + R_{EC}\frac{sT_C/Q_{MC}}{s^2 T_C^2 + sT_C/Q_{MC} + 1}, \qquad (22)$$

where $s = \sigma + j\omega$ is the complex frequency variable.

3. RESPONSE

Frequency Response

The response function of the closed-box system is given by (19). This is a second-order (12 dB/octave cutoff) high-pass filter function; it contains information about the low-frequency amplitude, phase, delay and transient response characteristics of the closed-box system [13]. Because the system is minimum-phase, these characteristics are interrelated; adjustment of one determines the others. In audio systems, the flatness and extent of the steady-state amplitude-vs-frequency response—or simply frequency response—is usually considered to be of greatest importance.

The frequency response $|G(j\omega)|$ of the closed-box system is examined in the appendix. Several typical response curves are illustrated in Fig. 4 with the frequency scale normalized to ω_C. The curve for $Q_{TC} = 0.50$ is a second-order critically-damped alignment; that for $Q_{TC} = 0.71$ (i.e., $1/\sqrt{2}$) is a second-order Butterworth ($B2$) maximally-flat alignment. Higher values of Q_{TC} lead to a peak in the response, accompanied by a relative extension of bandwidth which initially is greater than the relative response peak. For large values of Q_{TC}, however, the response peak continues to increase without any significant extension of bandwidth. Technically, these responses for Q_{TC} greater than $1/\sqrt{2}$ are second-order Chebyshev ($C2$) equal-ripple alignments.

Whatever response shape may be considered optimum, Fig. 4 indicates the value of Q_{TC} required to achieve this alignment and the variation in response shape that will result if Q_{TC} is altered, i.e., misaligned, from the required value. For intermediate values of Q_{TC} not included in Fig. 4, Fig. 5 gives normalized values of the response peak magnitude $|G(j\omega)|_{max}$, the normalized frequency $f_{G\,max}/f_C$ at which this peak occurs, and the normalized cutoff (half-power) frequency f_3/f_C for which the response is 3 dB below passband level. The analytical expressions for the quantities plotted in Fig. 5 are given in the appendix.

Transient Response

The response of the closed-box system to a step input is plotted in Fig. 6 for several values of Q_{TC}; the time scale is normalized to the periodic time of the system resonance frequency. For values of Q_{TC} greater than 0.50, the response is oscillatory with increasing values of Q_{TC} contributing increasing amplitude and decay time [13].

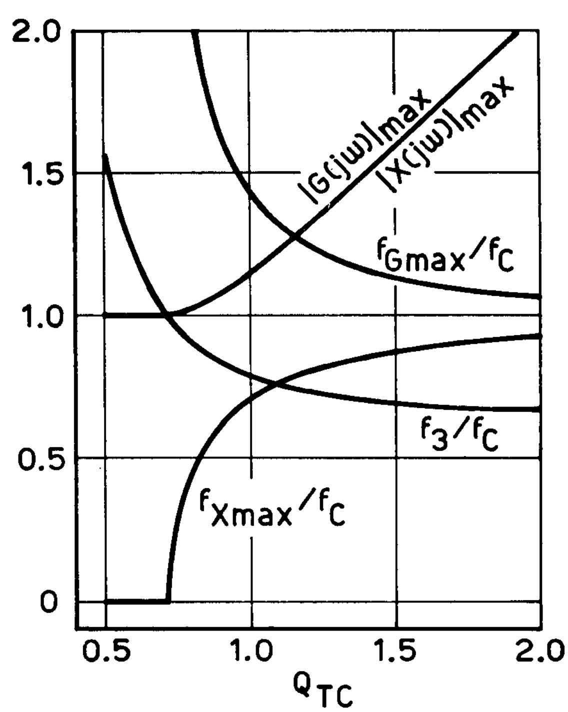

Fig. 5. Normalized cutoff frequency, and normalized frequency and magnitude of response and displacement peaks, as a function of total Q for the closed-box loudspeaker system.

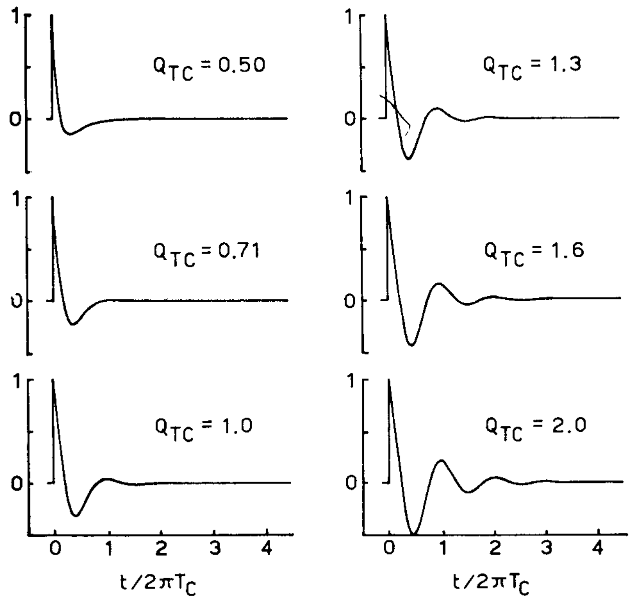

Fig. 6. Normalized step response of the closed-box loudspeaker system.

4. EFFICIENCY

Reference Efficiency

The closed-box system efficiency in the passband region, or system reference efficiency, is the reference efficiency of the driver operating with the particular value of air-load mass provided by the system enclosure. From [12, eq. (32)], this is

$$\eta_o = \frac{4\pi^2}{c^3} \cdot \frac{f_S{}^3 V_{AS}}{Q_{ES}}, \tag{23}$$

where f_S, Q_{ES} and V_{AS} have the values given in (12), (14) and (15). This expression may be rewritten in terms of the system parameters defined in section 2. Using (16), (17) and (18),

$$\eta_o = \frac{4\pi^2}{c^3} \cdot \frac{f_C{}^3 V_{AT}}{Q_{EC}}, \tag{24}$$

where

$$V_{AT} = \rho_o c^2 C_{AT} \tag{25}$$

is a volume of air having the same total acoustic compliance as the driver suspension and enclosure acting together. For SI units, the value of $4\pi^2/c^3$ is 9.64×10^{-7}.

Efficiency Factors

Equation (24) may be written

$$\eta_o = k_\eta f_3{}^3 V_B, \tag{26}$$

where

- f_3 is the cutoff (half-power or -3 dB) frequency of the system,
- V_B is the net internal volume of the system enclosure,
- k_η is an efficiency constant given by

$$k_\eta = \frac{4\pi^2}{c^3} \cdot \frac{f_C{}^3}{f_3{}^3} \cdot \frac{V_{AT}}{V_B} \cdot \frac{1}{Q_{EC}}. \tag{27}$$

The efficiency constant k_η may be separated into three factors: $k_{\eta(Q)}$ related to system losses, $k_{\eta(C)}$ related to system compliances, and $k_{\eta(G)}$ related to the system response. Thus

$$k_\eta = k_{\eta(Q)}\, k_{\eta(C)}\, k_{\eta(G)}, \tag{28}$$

where

$$k_{\eta(Q)} = Q_{TC}/Q_{EC}, \tag{29}$$

$$k_{\eta(C)} = V_{AT}/V_B, \tag{30}$$

$$k_{\eta(G)} = \frac{4\pi^2}{c^3} \cdot \frac{1}{(f_3/f_C)^3 Q_{TC}}. \tag{31}$$

Loss Factor

Modern amplifiers are designed to have a very low output-port (Thevenin) impedance so that, for practical purposes, $R_g = 0$. The value of Q_{TC} for any system used with such an amplifier is then equal to Q_{TCO} as given by (9). Equation (29) then reduces to

$$k_{\eta(Q)} = Q_{TCO}/Q_{EC} = 1 - (Q_{TCO}/Q_{MC}). \tag{32}$$

This expression has a limiting value of unity, but will approach this value only when mechanical losses in the system are negligible (Q_{MC} infinite) and all required damping is therefore provided by electromagnetic coupling ($Q_{EC} = Q_{TCO}$).

The value of $k_{\eta(Q)}$ for typical closed-box systems varies from about 0.5 to 0.9. Low values usually result from the deliberate use of mechanical or acoustical dissipation, either to ensure adequate damping of diaphragm or suspension resonances at higher frequencies, or to conserve magnetic material and therefore cost.

Compliance Factor

Equation (30) may be expanded to

$$k_{\eta(C)} = \frac{C_{AT}}{C_{AB}} \cdot \frac{V_{AB}}{V_B}, \tag{33}$$

where

$$V_{AB} = \rho_o c^2 C_{AB} \tag{34}$$

is a volume of air having an acoustic compliance equal to C_{AB}.

There is an important difference between V_B, the net internal volume of the enclosure, and V_{AB}, a volume of air which represents the acoustic compliance of the enclosure. If the enclosure contains only air under adiabatic conditions, i.e., no lining or filling materials, then V_{AB} is equal to V_B. But if the enclosure does contain such materials, V_{AB} is larger than V_B. The increase in V_{AB} is inversely proportional to the change in the value of γ, the ratio of specific heat at constant pressure to that at constant volume for the air in the enclosure. This has a value of 1.4 for the empty enclosure and decreases toward unity if the enclosure is filled with a low-density material of high specific heat [1, p. 220]. Equation (33) may then be simplified to

$$k_{\eta(C)} = \frac{\alpha}{\alpha + 1} \cdot \frac{1.4}{\gamma_B}, \tag{35}$$

where γ_B is the value of γ applicable to the enclosure.

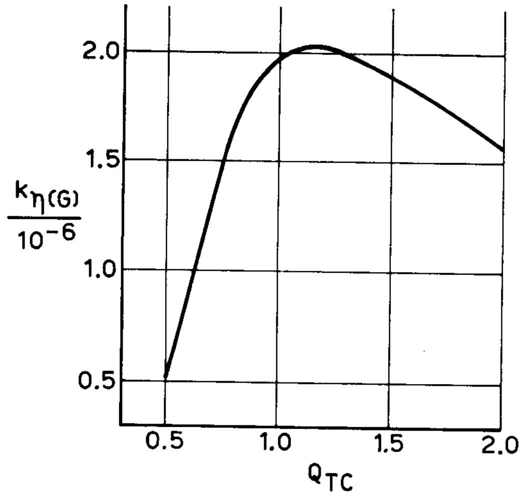

Fig. 7. Response factor $k_{\eta(G)}$ as a function of total Q for the closed-box loudspeaker system.

For "empty" enclosures, (35) has a limiting value of unity for $a >> 1$. Air-suspension systems usually have a values between 3 and 10.

If the enclosure is filled, the $1.4/\gamma_B$ term exceeds unity, but two interactions occur. First, because the filling material increases C_{AB}, the value of a is lower than for the empty enclosure. Second, the addition of the material increases energy absorption within the enclosure, decreasing Q_{MC} and therefore reducing the value of $k_{\eta(Q)}$ in (32).

With proper selection of the amount, kind, and location of filling material, the net product of $k_{\eta(Q)}$ and $k_{\eta(C)}$ increases compared to the empty enclosure condition, but the increase is seldom more than about 15%. Haphazard addition of unselected materials may even reduce the product of these factors. Although theoretically possible, it is extremely unusual in practice for this product

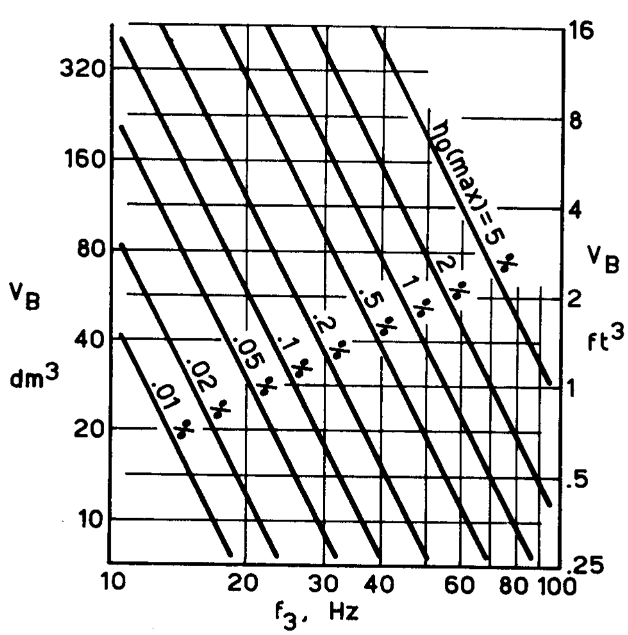

Fig. 8. The relationship of maximum reference efficiency to cutoff frequency and enclosure volume for the closed-box loudspeaker system.

to exceed unity. The effects of filling materials are discussed further in section 7.

Response Factor

The value of $k_{\eta(G)}$ in (31) depends only on Q_{TC} because (f_3/f_C) is a function of Q_{TC} as shown in Fig. 5 and (75) of the appendix. Fig. 7 is a plot of $k_{\eta(G)}$ *vs* Q_{TC}. Just above $Q_{TC} = 1.1$, $k_{\eta(G)}$ has a maximum value of 2.0×10^{-6}. This value of Q_{TC} corresponds to a $C2$ alignment with a ripple or passband peak of 1.9 dB. Compared to the $B2$ alignment having the same bandwidth, this alignment is 1.8 dB more efficient.

Maximum Reference Efficiency, Bandwidth, and Enclosure Volume

Selecting the value of $k_{\eta(G)}$ for the maximum-efficiency $C2$ alignment, and taking unity as the maximum attainable value of $k_{\eta(Q)}k_{\eta(C)}$, the maximum reference efficiency $\eta_{o(max)}$ that could be expected from an idealized closed-box system for specified values of f_3 and V_B is, from (26) and (28),

$$\eta_{o(max)} = 2.0 \times 10^{-6} f_3^3 V_B, \qquad (36)$$

where f_3 is in Hz and V_B is in m^3. This relationship is illustrated in Fig. 8, with V_B (given here in cubic decimeters—1 dm^3 = 1 liter = 10^{-3} m^3) plotted against f_3 for various values of $\eta_{o(max)}$ expressed in percent.

Figure 8 represents the physical efficiency-bandwidth-volume limitation of closed-box system design. Any system having given values of f_3 and V_B must always have an actual reference efficiency lower than the value of $\eta_{o(max)}$ given by Fig. 8. Similarly, a system of specified efficiency and volume must have a cutoff frequency higher than that indicated by Fig. 8, etc. These basic relationships have been known on a qualitative basis for years (see, e.g., [11]). An independently derived presentation of the important quantitative limitation was given recently by Finegan [14].

There are two known methods of circumventing the physical limitation imposed by (36) or Fig. 8. One is the stabilized negative-spring principle [15] which enables V_{AT} to be made much larger than V_B but requires additional design complexity. The other is the use of amplifier assistance which extends response with the aid of equalizing networks or special feedback techniques [16]. The second method requires additional amplifier power in the region of extended response and a driver capable of dissipating the extra power.

The actual reference efficiency of any practical system may be evaluated directly from (24) if the values of f_C, Q_{EC} and V_{AT} are known or are measured. For air-suspension systems, especially those using filling materials, V_{AT} is often very nearly equal to V_B.

Efficiency-Bandwidth-Volume Exchange

The relationship between reference efficiency, bandwidth, and enclosure volume indicated by (26) and illustrated for maximum-efficiency conditions in Fig. 8 implies that these system specifications can be exchanged one for another if the factors determining k_η remain constant. Thus if the system is made larger, the parameters may be adjusted to give greater efficiency or extended bandwidth. Similarly, if the cutoff frequency is

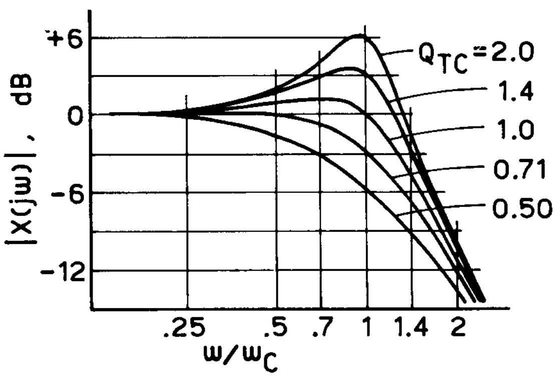

Fig. 9. Normalized diaphragm displacement of closed-box system driver as a function of normalized frequency for several values of total system Q.

raised, the parameters may be adjusted to give higher efficiency or a smaller enclosure.

If the value of k_η is increased, by reducing mechanical losses, by adding filling material, by increasing α, or by changing the response shape, the benefit may be taken in the form of smaller size, or higher efficiency, or extended bandwidth, or a combination of these. Each choice requires a specific adjustment of the enclosure or driver parameters.

5. DISPLACEMENT-LIMITED POWER RATINGS

Displacement Function

The closed-box system displacement function given by (20) is a second-order low-pass filter function. The properties of this function are examined in the appendix.

The normalized diaphragm displacement magnitude $|X(j\omega)|$ is plotted in Fig. 9 with frequency normalized to ω_C for several values of Q_{TC}. The curves are exact mirror images of those of Fig. 4. For intermediate values of Q_{TC}, Fig. 5 gives normalized values of the displacement peak magnitude $|X(j\omega)|$ and the normalized frequency $f_{X\max}/f_C$ at which this peak occurs. Analytical expressions for these quantities are given in the appendix.

Acoustic Power Rating

Assuming linear large-signal diaphragm displacement, the steady-state displacement-limited acoustic power rating P_{AR} of a loudspeaker system, from [12, eq. (42)], is

$$P_{AR} = \frac{4\pi^3\rho_o}{c} \cdot \frac{f_S{}^4 V_D{}^2}{k_x{}^2 |X(j\omega)|_{\max}{}^2}, \tag{37}$$

where V_D is the peak displacement volume of the driver diaphragm, given by

$$V_D = S_D x_{\max}, \tag{38}$$

and $x_{\max}$ is the peak linear displacement of the driver diaphragm, usually set by the amount of voice-coil overhang. Substituting (17) and (21) into (37), the steady-state displacement-limited acoustic power rating of the closed-box system becomes

$$P_{AR(CB)} = \frac{4\pi^3\rho_o}{c} \cdot \frac{f_C{}^4 V_D{}^2}{|X(j\omega)|_{\max}{}^2}. \tag{39}$$

For SI units, the constant $4\pi^3\rho_o/c$ is equal to 0.424.

Power Output, Bandwidth, and Displacement Volume

Equation (39) may be rewritten as

$$P_{AR(CB)} = k_P f_3{}^4 V_D{}^2, \tag{40}$$

where k_P is a power rating constant given by

$$k_P = \frac{4\pi^3\rho_o}{c} \cdot \frac{1}{(f_3/f_C)^4 |X(j\omega)|_{\max}{}^2}. \tag{41}$$

The acoustic power rating of a system having a specified cutoff frequency f_3 and a driver displacement volume V_D is thus a function of k_P; and k_P is solely a function of Q_{TC} as shown by (75) and (78) of the appendix.

The variation of k_P with Q_{TC} is plotted in Fig. 10. A maximum value occurs for Q_{TC} very close to 1.1. This is practically the same 1.9 dB ripple $C2$ alignment that gives maximum efficiency. For this condition, (40) becomes

$$P_{AR(CB)\max} = 0.85 f_3{}^4 V_D{}^2, \tag{42}$$

where P_{AR} is in watts for f_3 in Hz and V_D in m^3.

Equation (42) is illustrated in Fig. 11. P_{AR} is expressed in both watts (left scale) and equivalent SPL at one meter [1, p. 14] for 2π steradian free-field radiation conditions (right scale); this is plotted as a function of f_3 for various values of V_D. The SPL at one meter given on the right-hand scale is a rough indication of the level produced in the reverberant field of an average listening room for a radiated acoustic power given by the left-hand scale [1, p. 318].

Figure 11 represents the physical large-signal limitation of closed-box system design. It may be used to determine the optimum performance tradeoffs (P_{AR} *vs* f_3) for a given diaphragm and voice-coil design or to find the minimum value of V_D which is required to meet a given specification of f_3 and P_{AR}. The techniques noted earlier which may be used to overcome the small-signal limitation of Fig. 8 do not affect the large-signal limitation imposed by Fig. 11.

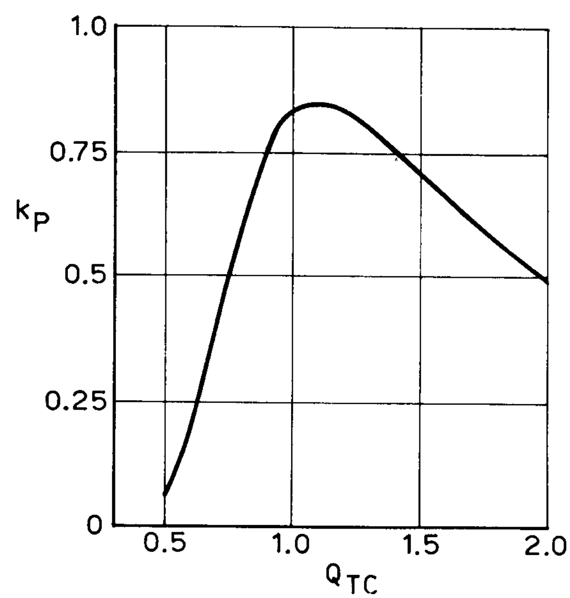

Fig. 10. Power rating constant k_P as a function of total Q for the closed-box loudspeaker system.

Power Output, Bandwidth, and Enclosure Volume

The displacement-limited power rating relationships given above exhibit no dependence on enclosure volume. For fixed response, it is the diaphragm displacement volume V_D that controls the system power rating. However, V_D cannot normally be made more that a few percent of V_B; beyond this point, increases in V_D result in unavoidable non-linear distortion, regardless of driver linearity, caused by non-linear compression of the air in the enclosure [3], [10]. If V_D is limited to a fixed fraction of V_B, the fraction depending on the amount of distortion considered acceptable, then Fig. 11 may be relabeled to show the minimum enclosure volume required to provide a given combination of f_3 and P_{AR} for the specified distortion level, as well as the required V_D.

Program Bandwidth

Figure 10 indicates that k_P and hence the system steady-state acoustic power rating decreases for values of Q_{TC} below 1.1 if f_3 and V_D are held constant. However, it is clear from Fig. 5 that the frequency of maximum diaphragm displacement, $f_{X\max}$, is below f_3 for $Q_{TC} < 1.1$, and that as Q_{TC} decreases, $f_{X\max}$ moves further and further below f_3. This suggests that the steady-state rating becomes increasingly conservative, as Q_{TC} decreases, for loudspeaker systems operated with program material having little energy content below f_3. The effect of restricted power bandwidth in most amplifiers further reduces the likelihood of reaching rated displacement at $f_{X\max}$ for these alignments [12, section 7].

For closed-box loudspeaker systems used for high-fidelity music reproduction and having a cutoff frequency of about 40 Hz or less, or operated on speech only and having a cutoff frequency of about 100 Hz or less, an approximate *program* power rating is that given by (42) or Fig. 11 for any value of Q_{TC} up to 1.1. Above this value, $f_{X\max}$ is within the system passband and the program rating is effectively the same as the steady-state rating.

Electrical Power Rating

The displacement-limited electrical and acoustic power ratings of a loudspeaker system are related by the system reference efficiency [12, section 7]. Thus, if the acoustic power rating and reference efficiency of a system are known, the corresponding electrical rating may be calculated as the ratio of these.

For the closed-box system, (24) and (39) give the electrical power rating P_{ER} as

$$P_{ER(CB)} = \pi \rho_o c^2 \frac{f_C Q_{EC}}{V_{AT}} \cdot \frac{V_D^2}{|X(j\omega)|_{\max}^2}. \qquad (43)$$

The dependence of this rating on the important system constants is more easily observed from the form obtained by dividing (40) by (26):

$$P_{ER} = \frac{k_P}{k_\eta} f_3 \frac{V_D^2}{V_B}. \qquad (44)$$

It is particularly important to realize that for a given acoustic power capacity, the displacement-limited electrical power rating is inversely proportional to efficiency.

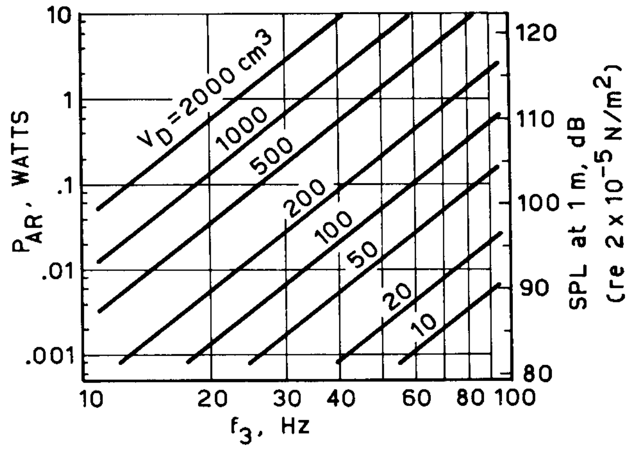

Fig. 11. The relationship of rated acoustic output power to cutoff frequency and driver displacement volume for a closed-box loudspeaker system aligned to obtain maximum rated power.

Also, displacement non-linearity for large signals tends to increase P_{ER} over the theoretical linear value. Thus a high input power rating is not necessarily a virtue; it may only indicate a low value of k_η or a high distortion limit.

The overall electrical power rating which a manufacturer assigns to a loudspeaker system must take into account both the displacement-limited power capacity of the system, P_{ER}, and the thermally-limited power capacity of the driver, $P_{E(\max)}$, together with the spectral and statistical properties of the type of program material for which the rating will apply. The statistical properties of the signal are important in determining whether P_{ER} or $P_{E(\max)}$ will limit the overall power rating, because the overall rating sets the maximum safe continuous-power rating of the amplifier to be used. For reliability and low distortion, the overall rating must never exceed P_{ER}; but it may be allowed to exceed $P_{E(\max)}$ in proportion to the peak-to-average power ratio of the intended program material.

The resulting system rating is important when selecting a loudspeaker system to operate with a given amplifier and vice-versa. But it must be remembered that the electrical rating gives no clue to the acoustic power capacity unless the reference efficiency is known.

6. PARAMETER MEASUREMENT

It has been shown that the important small-signal and large-signal performance characteristics of a closed-box loudspeaker system depend on a few basic parameters. The ability to measure these basic parameters is thus a useful tool, both for evaluating the performance of an existing loudspeaker system and for checking the results of a new system design which is intended to meet specific performance criteria.

Small-Signal Parameters: f_C, Q_{MC}, Q_{EC}, Q_{TCO}, α, V_{AT}

The voice-coil impedance function of the closed-box system is given by (22). The steady-state magnitude $|Z_{VC}(j\omega)|$ of this function is plotted against normalized frequency in Fig. 12.

The measured impedance curve of a closed-box sys-

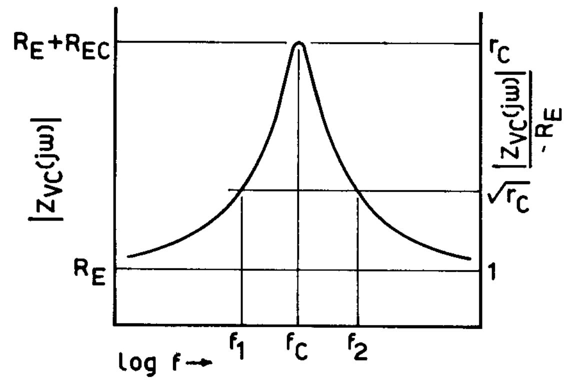

Fig. 12. Magnitude of closed-box loudspeaker system voice-coil impedance as a function of frequency.

tem conforms closely to the shape of Fig. 12. This impedance curve permits identification of the first four parameters as follows:

1) Measure the dc voice-coil resistance R_E.
2) Find the frequency f_C at which the impedance has maximum magnitude and zero phase, i.e., is resistive. Let the ratio of maximum impedance magnitude to R_E be defined as r_C.
3) Find the two frequencies $f_1 < f_C$ and $f_2 > f_C$ for which the impedance magnitude is equal to $R_E \sqrt{r_C}$.
4) Then, as in [12, appendix],

$$Q_{\mathrm{MC}} = \frac{f_C \sqrt{r_C}}{f_2 - f_1}, \tag{45}$$

$$Q_{\mathrm{EC}} = Q_{\mathrm{MC}}/(r_C - 1), \tag{46}$$

$$Q_{\mathrm{TCO}} = Q_{\mathrm{MC}}/r_C. \tag{47}$$

To obtain the value of α for the system, remove the driver from the enclosure and measure the driver parameters f_S, Q_{MS} and Q_{ES} (with or without a baffle) as described in [12]; the method is the same as that given above for the system. The compliance ratio is then [12, appendix]

$$\alpha = \frac{f_C Q_{\mathrm{EC}}}{f_S Q_{\mathrm{ES}}} - 1. \tag{48}$$

Drivers with large voice-coil inductance or systems having a large crossover inductance may exhibit some difference between the frequency of maximum impedance magnitude and the frequency of zero phase. If the inductance cannot be bypassed or equalized for measurement purposes [17, section 14], it is better to take f_C as the frequency of maximum impedance magnitude, regardless of phase. It must be expected, however, that some measurement accuracy will be lost in these circumstances.

V_{AT} is evaluated with the help of (1), (11), (15), (25) and (34):

$$V_{\mathrm{AT}} = V_{\mathrm{AB}} V_{\mathrm{AS}}/(V_{\mathrm{AB}} + V_{\mathrm{AS}}) = \frac{\alpha}{\alpha + 1} V_{\mathrm{AB}}. \tag{49}$$

For unfilled enclosures, $V_{\mathrm{AB}} = V_B$ and the value of V_{AT} may be computed directly using the measured value of α. If the system enclosure is normally filled, an extra set of measurements is required. The filling material is removed from the enclosure, or the driver is transferred to a similar but unfilled test enclosure. For this combination, the resonance frequency f_{CT} and the corresponding Q values Q_{MCT} and Q_{ECT} are measured by the above method. Then, as shown in [12, appendix],

$$V_{\mathrm{AS}} = V_B \left[\frac{f_{\mathrm{CT}} Q_{\mathrm{ECT}}}{f_S Q_{\mathrm{ES}}} - 1 \right], \tag{50}$$

where V_B is the net internal volume of the unfilled enclosure used (the system enclosure or test enclosure). Using (11), (15) and (34), V_{AB} for the filled system enclosure is then given by

$$V_{\mathrm{AB}} = V_{\mathrm{AS}}/\alpha. \tag{51}$$

This value of V_{AB} may now be used to evaluate V_{AT} using (49).

Large-Signal Parameters: $P_{E(\max)}$ and V_D

The measurement of driver thermal power capacity is best left to manufacturers, who are familiar with the required techniques [18, section 5.7] and are usually quite happy to supply the information on request. Some estimate of thermal power capacity may often be obtained from knowledge of voice-coil diameter and length, the materials used, and the intended use of the driver [19].

The driver displacement volume V_D is the product of S_D and $x_{\max}$. It is usually sufficient to evaluate S_D by estimating the effective diaphragm diameter. Some manufacturers specify the "throw" of a driver, which is usually the peak-to-peak linear displacement, i.e., $2x_{\max}$. If this information is not available, the value of $x_{\max}$ may be estimated by observing the amount of voice-coil overhang outside the magnetic gap. For a more rigorous evaluation, where the necessary test equipment is available, operate the driver in air with sine-wave input at its resonance frequency and measure the peak displacement for which the radiated sound pressure attains about 10% total harmonic distortion.

7. ENCLOSURE FILLING

It is stated in section 4 that the addition of an appropriate filling material to the enclosure of an air-suspension system raises the value of the efficiency constant k_η. The use and value of such materials have been the subject of much controversy and study [4], [8], [9], [10], [11], [20].

There is no serious disagreement about the value of such materials for damping standing waves within the enclosure at frequencies in the upper piston range and higher. The controversy centers on the value of the materials at low frequencies. A more complete description of the effects of these materials will help to assess their value to various users.

Compliance Increase

If the filling material is chosen for low density but high specific heat, the conditions of air compression within the enclosure are altered from adiabatic to isothermal, or partly so [1, p. 220]. This increases the effective acoustic compliance of the enclosure, which is equivalent to increasing the size of the unfilled enclosure. The maximum theoretical increase in compliance is 40%, but using practical materials the actual increase is probably never more than about 25%.

Mass Loading

Often, the addition of filling material increases the total effective moving mass of the system. This has been carefully documented by Avedon [10]. The mechanism is not entirely clear and may involve either motion of the filling material itself or constriction of air passages near the rear of the diaphragm, thus "mass-loading" the driver. Depending on the initial diaphragm mass and the conditions of filling, the mass increase may vary from negligible proportions to as much as 20%.

Damping

Air moving inside a filled enclosure encounters frictional resistance and loses energy. Thus the component R_{AB} of Fig. 1 increases when the enclosure is filled. The resulting increase in the total system mechanical losses ($R_{AB} + R_{AS}$) can be substantial, especially if the filling material is relatively dense and is allowed to be quite close to the driver where the air particle velocity and displacement are highest. While unfilled systems have typical Q_{MC} values of about 5–10 (largely the result of driver suspension losses), filled systems generally have Q_{MC} values in the range of 2–5.

Value to the Designer

If a loudspeaker system is being designed from scratch, the effect of filling material on compliance is a definite advantage. It means that the enclosure size can be reduced or the efficiency improved or the response extended. Any mass increase which accompanies the compliance increase is simply taken into account in designing the driver so that the total moving mass is just the amount desired. The losses contributed by the material are a disadvantage in terms of their effect on $k_{\eta(Q)}$, but this is a small price to pay for the overall increase in k_η which results from the greater compliance. In fact, if efficiency is not a problem, the effect of increased frictional losses may be seen to relax the magnet requirements a little, thus saving cost.

Where a loudspeaker system is being designed around a given driver, the compliance increase contributed by the material is still an advantage because it permits the enclosure to be made smaller for a particular (achievable) response. The effect of increased mass is to reduce the driver reference efficiency by the square of the mass increase; this may or may not be desirable. The increased mass will also cause the value of Q_{EC} to be higher for a given value of f_C. This will be opposed by the effect of the material losses on Q_{MC}.

Often it is hoped that the addition of large amounts of filling material to a system will contribute enough additional damping to compensate for inadequate magnetic coupling in the driver. To the extent that the material increases compliance more than it does mass, Q_{EC} will indeed fall a little. And while Q_{MC} may be substantially decreased, the total reduction in Q_{TC} is seldom enough to rescue a badly underdamped driver as illustrated in [20]. If such a driver must be used, the application of acoustic damping directly to the driver as described in [21] is both more effective and more economical than attempting to overfill the enclosure.

Measuring the Effects of Filling Materials

The contribution of filling materials to a given system can be determined by careful measurement of the system parameters with and without the material in place. The added-weight measurement method used by Avedon [10] can be very accurate but is suited only to laboratory conditions. Alternatively, the type of measurements described in section 6 may be used:

1) With the driver in air or on a test baffle, measure f_S, Q_{MS}, Q_{ES}.
2) With the driver in the unfilled enclosure, measure f_{CT}, Q_{MCT}, Q_{ECT}.
3) With the driver in the filled enclosure, measure f_C, Q_{MC}, Q_{EC}.
4) Then, using the method of [12, appendix], the ratio of total moving mass with filling to that without filling is

$$M_{AC}/M_{ACT} = f_{CT}Q_{EC}/f_C Q_{ECT}, \tag{52}$$

and the enclosure compliance increase caused by filling is

$$V_{AB}/V_B = \frac{(f_{CT}Q_{ECT}/f_S Q_{ES}) - 1}{(f_C Q_{EC}/f_S Q_{ES}) - 1}. \tag{53}$$

5) The net effect of the material on total system damping may be found by computing Q_{TCO} for the filled system from (9) or (47) and comparing this to the corresponding $Q_{TCTO} = Q_{MCT}Q_{ECT}/(Q_{MCT} + Q_{ECT})$ for the unfilled system. These values represent the total Q (Q_{TC}) for each system when driven by an amplifier of negligible source resistance.

The usual result is that the filling material increases both compliance and mass but decreases total Q. The decrease in total Q may be a little or a lot, depending on the initial value and on the material chosen and its location in the enclosure.

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Closed-Box Loudspeaker Systems
Part II: Synthesis

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Part I of this paper provides a basic low-frequency analysis of the closed-box loudspeaker system with emphasis on small-signal and large-signal behavior, basic performance limitations, and the determination of important system parameters from voice-coil impedance measurements. Part II discusses some important implications of the findings of Part I and introduces the subject of system synthesis: the complete design of loudspeaker systems to meet specific performance goals. Given a set of physically-realizable system performance specifications, the analytical results of Part I enable the system designer to calculate directly the required specifications of the system components.

Editor's Note: Part I of Closed-Box Loudspeaker Systems appeared in the December 1972 issue of the Journal.

8. DISCUSSION

Driver Size

It has long been an accepted principle that a large bass driver is better than a small one. While this attitude seems to be justified by experience, it has recently been called into question [22]. The analysis in this paper demonstrates that driver size alone does not determine or limit system performance in areas of small-signal response, efficiency, or displacement-limited power capacity.

A large driver inevitably costs more than a small driver having identical small-signal and large-signal parameters of the kind discussed here. However, it is physically easier to obtain a large value of V_D and hence a high acoustic power capacity from a large driver, and the modulation distortion [23] produced by a large driver will be less than that of a small driver delivering the same acoustic output power.

Thus a large driver has no inherent advantage over a small one so far as small-signal response and efficiency are concerned. It may in fact have a cost disadvantage. But where high acoustic output at low distortion is required, the large driver has a definite advantage.

Enclosure Size

It is clear from section 4 that an air-suspension system having a high compliance ratio can duplicate the performance of a larger conventional closed-box system having a low compliance ratio. However, once the compliance ratio is made larger than about 4, there is no way to gain a significant reduction in enclosure size without affecting system performance.

A small air-suspension system, when compared to a large air-suspension system, must have a higher cutoff frequency, or lower efficiency, or both. As has been claimed many times, it is possible to design a small system to have the same *response* as a large system. But if both are non-wasteful air-suspension designs, then as shown by (26) or Fig. 8 the efficiency of the small system must be lower than that of the large system in direct proportion to size.

It is often possible to provide the same maximum acoustic output as well as the same response from the small system, but the lower efficiency of this system will dictate a higher input power rating and therefore a driver voice coil capable of dissipating more heat. Also, it is easily shown that for these conditions the driver of the small system will require a larger magnet (e.g., a heavier diaphragm of the same size may be driven through the same displacement, or a smaller diaphragm of the same mass may be driven through a larger displacement). Thus for this condition the driver for the small system must be more expensive than that for the large system.

It may be concluded that the pressure to design more and more compact high-quality loudspeaker systems leads directly to systems of reduced efficiency and, in most cases, reduced acoustic power capacity. If acoustic power capacity is not sacrificed, these compact systems require expensive drivers and must be used with powerful amplifiers.

Performance Specifications

Of all the components used in audio recording and reproduction, loudspeaker systems have the least complete and least informative performance specifications. In the low-frequency range at least, this need not be so.

If a specified voltage is applied to a direct-radiator loudspeaker system, the output of the system at low frequencies may be expressed in terms of an acoustic volume velocity which is *substantially independent of the acoustic load* [12], [24]. The "response" of a loudspeaker system expressed in this way is meaningless to most loudspeaker users, but a specification of the acoustic power or distant sound pressure delivered into a standard free-field load by this volume velocity is both meaningful and useful.

While the sound pressure delivered to a room is different from that delivered to a free field, the difference clearly is a property of the room, not of the loudspeaker system. If the room performance is very poor, it can be corrected acoustically or, in some cases, equalized electronically. This is in no way a deterrent to accurate specification of the basic loudspeaker system response by using a standard free-field load. In fact, the findings of Allison and Berkovitz [25] indicate that a 2π sr free-field load is a very reasonable approximation to a typical room load.

Such a standard-load approach has of course been used for years in loudspeaker measurement standards [18], [26], [27]. If it were applied more universally, it would provide a very useful—and presently unavailable—quantitative means of comparing loudspeaker systems. It is a particularly attractive method for specifying the low-frequency response of a system, because the nominal free-field low-frequency response and reference efficiency can be obtained quite easily from the basic parameters of the system.

A few manufacturers already supply these basic parameters or the directly-related free-field response and efficiency data. The practice deserves encouragement.

Typical System Performance

A sampling of closed-box systems of British, American and European origin was tested in late 1969 by measuring the system small-signal parameters as described in section 6. The frequency response and efficiency were then obtained from the relationships of sections 3 and 4.

Resonance frequencies (f_C) varied from 40 Hz to 90 Hz. Total Q (Q_{TCO}) varied from 0.4 to 2.0. Reference efficiencies (η_o) varied from 0.28% to 1.0%. While there was no general pattern of parameter combinations, all but a few of the systems fell into one of two categories:

1) Cutoff frequency (f_3) below 50 Hz with little or no peaking (Q_{TCO} up to 1.1). Size generally larger than 40 dm^3 (1.4 ft^3).
2) Cutoff frequency above 50 Hz with definite peaking (Q_{TCO} between 1.4 and 2.0). Size smaller than 60 dm^3 (2 ft^3)

One explanation for this situation was spontaneously provided (and demonstrated) by a salesman who sold American systems in both categories. Only category 1 systems would reproduce low organ and orchestral fundamentals, while category 2 systems had demonstrably stronger bass on popular music. Sales thus tended to be determined by the musical tastes of the customer. There is marketing sense in this, and economic sense as well, because the same driver which has category 1 performance in a large enclosure has category 2 performance—with a higher acoustic power capacity—in a small enclosure.

9. SYSTEM SYNTHESIS

System-Driver Relationships

The majority of closed-box systems operate with amplifiers having negligible output resistance, have a total moving mass no greater than that of the driver on a baffle, and obtain most of their total damping from electromagnetic coupling and mechanical losses in the driver. For these conditions, (7), (9), (13), (17) and (18) may be used to derive

$$\frac{Q_{TCO}}{Q_{TS}} \approx \frac{Q_{EC}}{Q_{ES}} = \frac{f_C}{f_S} = (\alpha + 1)^{1/2}, \qquad (54)$$

and thus

$$f_C/Q_{TCO} \approx f_S/Q_{TS}, \qquad (55)$$

where Q_{TS} is the total Q of the driver at f_S for zero source resistance [12, eq. (47)], i.e.,

$$Q_{TS} = Q_{ES}Q_{MS}/(Q_{ES} + Q_{MS}). \qquad (56)$$

These equations show that for any enclosure-driver combination (i.e., value of α) the system resonance frequency and Q will be in the same ratio as those of the driver, but individually raised by a factor $(\alpha + 1)^{1/2}$. This increase is plotted as a function of α in Fig. 13.

This approximate relationship and the basic response, efficiency and power capacity relationships derived earlier are used below to develop system design procedures for two important cases: that of a fixed driver design, and that of only the final system specifications given.

Design with a Given Driver

One difficulty of trying to design an enclosure to "fit" a given driver is that the driver may be completely unsuitable in the first place. A convenient test of suitability for closed-box system drivers is provided by (51) and (54); the driver parameters must be known, or measured.

Equation (54) insists that the driver resonance frequency must always be lower than that of the system. If the designer wishes to avoid an enclosure which is wastefully large, i.e., he desires an air-suspension system, then α must be at least 3 and the driver resonance frequency must be no more than half the maximum tolerable system resonance frequency.

Similarly, Q_{TS} must be lower than the highest acceptable value of Q_{TCO}, and by approximately the same factor which relates f_S to the desired or highest acceptable value of f_C.

Finally, from (51), the value of V_{AS} must be at least several times larger than the enclosure size desired.

If the driver parameters appear satisfactory, the design of the system is carried out by selecting the most desirable combination of f_C and Q_{TCO} which satisfies (55) and then calculating α from (17). The required enclosure size (net internal volume) is then, from (51),

$$V_B = V_{AS}/\alpha, \quad (57)$$

or somewhat smaller if the enclosure is filled.

The reference efficiency is calculated from (23), and the acoustic power rating from (39) or (42). The electrical power rating is then, from section 5,

$$P_{ER} = P_{AR}/\eta_o. \quad (58)$$

Example of Design with a Given Driver

Using a standard baffle and unlined test enclosure, a European-made 12-inch woofer sold for air-suspension use is found to have the following small-signal parameters:

$f_S = 19$ Hz
$Q_{MS} = 3.7$
$Q_{ES} = 0.35$
$V_{AS} = 540$ dm^3 (19 ft^3).

Using (56) and (23),

$Q_{TS} = 0.32$
$\eta_o = 1.02\%$.

The manufacturer's power rating is 25 W, and the peak linear displacement is estimated to be 6 mm (¼ in). The effective diaphragm radius is estimated to be 0.12 m, giving $S_D = 4.5 \times 10^{-2}$ m^2 and $V_D = 2.7 \times 10^{-4}$ m^3 or 270 cm^3.

The values of f_S, Q_{TS} and V_{AS} for this driver appear to be quite favorable. The values of f_C, Q_{TCO} and f_3 to be expected from various suitable values of α are given in Table 1 together with the corresponding enclosure compliance V_{AB} (volume of an unfilled enclosure).

The $\alpha = 4$ alignment gives almost exactly a $B2$ response for an unfilled enclosure volume of 135 dm^3 or 4.8 ft^3. This would be quite suitable for a floor-standing system. The $\alpha = 9$ alignment gives excellent performance in a volume of only 60 dm^3 (2.1 ft^3). The $\alpha = 12$ alignment could probably be achieved in a 40 dm^3 (1.4 ft^3) enclosure with filling. Q_{TCO} would then be lower than shown, probably about unity, giving a cutoff frequency of about 53 Hz. This would be quite adequate "bookshelf" performance.

Taking the larger $B2$-aligned system, the displacement-limited acoustic power rating for program material, from (42), is

$$P_{AR} = 0.19 \text{ W},$$

and the corresponding electrical power rating is

$$P_{ER} = 19 \text{ W}.$$

This is well within the power rating given by the manufacturer, so the system can safely be operated with an amplifier having a continuous power rating of 20 W.

The "bookshelf" design, because of its higher value of f_3, has displacement-limited ratings of about 0.5 W acoustical and 50 W electrical. This is much higher than the manufacturer's rating. In the absence of the actual value of $P_{E(max)}$ on which the manufacturer's rating is based, it is probably best to limit the amplifier power to 25 W. The system can then produce an acoustic output of 0.25 W.

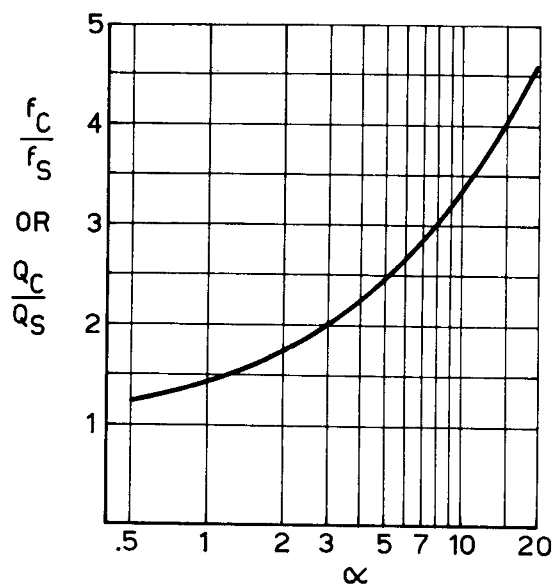

Fig. 13. Ratio of closed-box system resonance frequency and Q to driver resonance frequency and Q as a function of the system compliance ratio α.

Design from Specifications

Most engineering products are designed to suit specific requirements. Quite commonly, the "requirements" for a particular product contain conflicting factors, and the

Table 1. Expected Performance of the Given Driver

α	f_C, Hz	Q_{TCO}	f_3, Hz	V_{AB}, dm^3
4	42.5	0.72	42	135
6	50.3	0.85	44	90
9	60.0	1.01	47	60
12	68.6	1.15	50	45

engineer is called upon to assess the requirements and to adjust them to a condition of physical and economic realizability. Fig. 8, for example, frustrates the desires of many marketing managers who would be delighted to offer a one cubic foot (28 dm^3) air-suspension system giving flat response to 20 Hz at high efficiency.

The desired response of a closed-box loudspeaker system may be based on amplitude, phase, delay or transient considerations [13], but can always be reduced to a specification of f_C and Q_{TC}. Once the response is specified, either the enclosure volume V_B or the reference efficiency η_o may be specified independently; the other will then be determined or restricted to a minimum or maximum value. Finally, the power capacity may be specified in terms of either P_{ER} or P_{AR}. If both P_{ER} and P_{AR} must be fixed independently, this will detemine η_o and thus restrict V_B as above.

A typical set of design specifications might start with values of f_C, Q_{TC}, V_B and P_{AR}, together with a rating impedance which fixes R_E. Unless a special amplifier is to be used, it can be assumed that $Q_{TC} = Q_{TCO}$. Note that V_B effectively specifies the enclosure; the design problem is then to specify the driver.

The design process begins by assigning realistic values to Q_{MC} and α. The value of Q_{MC} has only a relatively minor effect on system performance through $k_{\eta(Q)}$. As noted in section 7, typical values are 2–5 for systems using filling material and 5–10 for unfilled systems. If no better guide to the expected value of Q_{MC} is available, assume $Q_{MC} = 5$. The required value of Q_{EC} for the system is then calculated from (9).

If maximum efficiency consistent with the initial specifications is desired, then the air-suspension principle must be used. This requires that α be at least 3 or 4, but its value will otherwise have only a small effect on system performance through $k_{\eta(C)}$ and may be chosen to have any value consistent with physical realizability of the driver. If α is chosen too large, the driver will be found to require unrealistically high compliance which, if realizable at all, may lead to poor mechanical stability of the suspension. A suitable choice of α is usually in the range of 3–10.

Next, the value of V_{AB} is established. This is equal to V_B for unfilled systems, but is increased by the factor $1.4/\gamma_B$ (typically 1.15 to 1.2) if the enclosure is filled.

The required driver small-signal parameters are then, from (17) and (18),

$$f_S = f_C/(\alpha+1)^{1/2}, \tag{59}$$

$$Q_{ES} = Q_{EC}/(\alpha+1)^{1/2}, \tag{60}$$

and

$$V_{AS} = \alpha V_{AB}. \tag{51}$$

V_{AT} is determined from (49). The reference efficiency to be expected from the completed system is calculated from (24). Alternatively, $k_{\eta(Q)}$, $k_{\eta(C)}$ and $k_{\eta(G)}$ may be evaluated separately and η_o determined from (26). The system electrical power rating P_{ER} is then calculated from (58). A comparable or lower value is assigned to $P_{E(max)}$, depending on the peak-to-average power ratio of the program material with which the system will be used.

The required value of V_D is calculated directly from (39) using Fig. 5 or (78) to determine $|X(j\omega)|_{max}$, or from (42), as appropriate. This value must be no larger than a few percent of V_B.

The driver is now specified by its most important parameters f_S, Q_{ES}, V_{AS}, V_D and $P_{E(max)}$ as well as its voice-coil resistance R_E which is typically 80% of the desired rating impedance. The system designer is faced with the problem of obtaining a driver which has the required parameters. If he has a driver factory available, he may have the required driver fabricated as described in the next section. If he does not possess this luxury, he must find a driver from among those available on the market.

At present, very few of the loudspeaker drivers offered for sale are provided with complete parameter information, either in the form above or any other. While this situation will no doubt improve with time, particularly as increasing demands are made on manufacturers to provide such information, today's system designer must obtain samples where possible and measure the parameters as described in [12]. The small-signal parameters should be measured with the driver mounted on a standard test baffle having an area of one or two square meters, e.g., [18, section 4.4.1], so that the diaphragm air load is approximately that which will apply to the driver in the system enclosure.

Example of System Design from Specifications

A closed-box air-suspension loudspeaker system to be used with a high-damping-factor amplifier is to be designed to meet the following specifications:

f_3	40 Hz
Response	$B2$
V_B	2 ft^3 (56.6 dm^3)
P_{AR}	0.25 W program peaks; expected peak/average ratio 5 dB.

The enclosure is to be lined, but not filled. It is assumed that the enclosure and driver losses will correspond to $Q_{MC} = 5$ and that it will be physically possible to obtain a compliance ratio of $\alpha = 5$.

The first two specifications translate directly into

$$f_C = 40 \text{ Hz}$$

and

$$Q_{TC} = Q_{TCO} = 0.707.$$

For $Q_{MC} = 5$, (9) gives

$$Q_{EC} = 0.824.$$

For $\alpha = 5$, $(\alpha+1)^{1/2} = \sqrt{6} = 2.45$, so from (59) and (60),

$$f_S = 16.3 \text{ Hz}$$

and

$$Q_{ES} = 0.336.$$

Also, for the unfilled enclosure, (51) gives

$$V_{AS} = 10 \text{ ft}^3 \text{ (283 dm}^3\text{)}.$$

Then, from (49),

$$V_{AT} = 1.67 \text{ ft}^3 \text{ (47.2 dm}^3\text{)}.$$

From (29), (30) and (31),

$$k_{\eta(Q)} = 0.858,$$
$$k_{\eta(C)} = 0.833,$$
$$k_{\eta(G)} = 1.36 \times 10^{-6}.$$

Thus

$$k_\eta = 0.97 \times 10^{-6}$$

and from (26),

$$\eta_o = 0.00351 \text{ or } 0.35\%.$$

The reference efficiency can also be calculated directly from (24) because f_C, V_{AT} and Q_{EC} are known.

The displacement-limited electrical power rating, from (58), is

$$P_{ER} = 71.5 \text{ W}.$$

An amplifier of this power rating must be used to obtain the specified acoustic output. For the expected peak/average power ratio, the thermal rating $P_{E(max)}$ of the driver must be at least 22.5 W.

Using (42) for the program power rating,

$$V_D = 3.4 \times 10^{-4} \text{ m}^3 \text{ or } 340 \text{ cm}^3.$$

This is only 0.6% of V_B, so linearity of the air compliance is no problem.

10. DRIVER DESIGN

General Method

The process of system design leads to specification of the required driver in terms of basic parameters. These parameters are used to carry out the physical design of the driver.

First, V_D must be divided into acceptable values of S_D and x_{max}. The choice of S_D may have to be a compromise among cost, distortion, and available mounting area.

The required mechanical compliance of the diaphragm suspension is then

$$C_{MS} = C_{AS}/S_D^2 = V_{AS}/(\rho_o c^2 S_D^2), \qquad (61)$$

and the required total mechanical moving mass is

$$M_{MS} = 1/[(2\pi f_S)^2 C_{MS}]. \qquad (62)$$

This total moving mass includes any mass added by filling material, as well as the air loads M_{M1} and M_{MB} on front and rear of the diaphragm. The latter can be evaluated from [1, pp. 216-217]. The mechanical mass of the diaphragm and voice-coil assembly is then

$$M_{MD} = M_{MS} - (M_{M1} + M_{MB}), \qquad (63)$$

less any allowance for mass added by filling material.

The magnet and voice coil must provide electromagnetic damping given by

$$B^2l^2/R_E = 2\pi f_S M_{MS}/Q_{ES}, \qquad (64)$$

or, for the value of R_E specified, a Bl product given by

$$Bl = (2\pi f_S R_E M_{MS}/Q_{ES})^{1/2}. \qquad (65)$$

This Bl product, together with the mechanical compliance, must be maintained with good linearity for a diaphragm displacement of $\pm x_{max}$. This effectively means that the voice-coil overhang outside the gap must be about x_{max} at each end. Also, the voice coil must be capable of dissipating as heat, without damage, an electrical input power $P_{E(max)}$. This design problem is familiar to driver manufacturers.

The driver parameter Q_{MS} usually plays a minor role in system performance, but it cannot be neglected entirely. The value of Q_{MS} in practical designs is often affected by decisions related to performance at higher frequencies. Where the driver diaphragm is required to be free of strong resonance modes at high frequencies, the outer edge suspension is usually designed to reflect a minimum of the vibrational energy travelling outward from the voice coil through the diaphragm material. This means that energy is dissipated in the suspension, and a low value of Q_{MS} results. The intended use of the driver or the constructional methods preferred by the manufacturer thus determines the approximate value of Q_{MS}. In a completed closed-box system, the value of Q_{MS} and the enclosure and filling material losses determine Q_{MC} and therefore the value of $k_{\eta(Q)}$ for the system.

Drivers for Air-Suspension Systems

It was stated earlier that the compliance ratio of an air-suspension system is not very important so long as it is greater than about 3 or 4. This means that the exact values of driver compliance, resonance frequency and Q are not of critical importance. It is in fact the moving mass M_{MS} and the electromagnetic damping B^2l^2/R_E that are of greatest importance. These can be calculated directly from the system parameters alone. Substituting (16), (17) and (18) into (61), (62) and (64), or using (3), (6), (8) and (25),

$$M_{MS} = S_D^2 M_{AC} = \rho_o c^2 S_D^2/(4\pi^2 f_C^2 V_{AT}), \qquad (66)$$

and

$$B^2l^2/R_E = 2\pi f_C M_{MS}/Q_{EC}. \qquad (67)$$

The exact value of mechanical compliance is not critically important so long as it is high enough to give approximately the desired compliance ratio. This is an advantage of the air-suspension design principle, because mechanical compliance is one of the more difficult driver parameters to control in production.

Example of Driver Design

The driver required for the example in the previous section has the following parameter specifications:

$$f_S = 16.3 \text{ Hz}$$
$$Q_{ES} = 0.336$$
$$V_{AS} = 283 \text{ dm}^3$$
$$V_D = 340 \text{ cm}^3$$
$$P_{E(max)} = 22.5 \text{ W}$$

The driver size will probably have to be at least 12 inches to meet the specifications of V_D and $P_{E(max)}$. This is checked by assuming a typical diaphragm radius of 0.12 m for the 12-inch driver, giving

$$S_D = 4.5 \times 10^{-2} \text{ m}^2.$$

For the required displacement volume of 340 cm³, the peak linear displacement must be

$$x_{max} = V_D/S_D = 7.5 \times 10^{-3} \text{ m} = 7.5 \text{ mm (0.3 in)}.$$

The total "throw" required is then 15 mm (0.6 in) which is realizable in a 12-inch driver. By comparison, the same displacement volume requires a throw of 22 mm (0.9 in) for a 10-inch driver, or 9.6 mm (0.38 in) for a 15-inch driver.

Continuing with the 12-inch design,

$$S_D^2 = 2.0 \times 10^{-3}\ \text{m}^4.$$

The required mechanical compliance and mass are then, from (61) and (62),

$$C_{MS} = 9.9 \times 10^{-4}\ \text{m/N},$$
$$M_{MS} = 97\ \text{g}.$$

M_{MS} is the total moving mass including air loads. Assuming that the front air load is equivalent to that for an infinite baffle and that the driver diaphragm occupies one-third of the area of the front of the enclosure, the mass of the voice coil and diaphragm alone is

$$M_{MD} = M_{MS} - (3.14a^3 + 0.65\pi\rho_o a^3) = 87\ \text{g}.$$

The magnetic damping must be, from (64),

$$B^2l^2/R_E = 30\ \text{N} \cdot \text{s/m (MKS mechanical ohms)}.$$

For an "8Ω" rating impedance, R_E is typically about 6.5 Ω. The required Bl product for the driver is then

$$Bl = 14\ \text{T} \cdot \text{m}$$

which must be maintained with good linearity over the voice-coil throw of 15 mm (0.6 in). The voice coil must also be able to dissipate 22.5 W nominal input power [12, eq. (6)] without damage.

Further examples of driver synthesis based on system small-signal requirements are contained in [28]; the method used is based on the same approach taken above but is arranged for automatic processing by time-shared digital computer. (The Thiele basic efficiency [17] used in this reference is based on a 4π sr free-field load and gives one-half the value of the reference efficiency used here.)

11. DESIGN VERIFICATION

The suitability of a prototype driver designed in accordance with the above methods may be checked by measuring the driver parameters as described in [12].[1] For an air-suspension driver, it is not necessary that f_S, Q_{ES}, and V_{AS} have exactly the specified values. What is important is that the quantities $f_S^2 V_{AS}$ and f_S/Q_{ES}, which together indicate the effective moving mass and electromagnetic coupling, should check with the same combinations of the specified parameters. Then, if V_{AS} is large enough to give a satisfactory value of α for the system, the driver design is satisfactory.

Similarly, the completed system may be checked by measuring its parameters as described in section 6 and comparing these to the initial specifications.[1] The actual system performance may also be verified by measurement in an anechoic environment or by an indirect method [24].

[1] A recent paper by Benson contains an improved method of Q measurement which compensates for errors introduced by large voice-coil inductance [32, Appendix 2]. The compensation is achieved by replacing f_C in eq. (45) of Part I of this paper— and f_S in [12, eq. (17)]—with the expression $\sqrt{f_1 f_2}$. The measured values of f_C and f_S are unchanged, and no other equations are affected.

12. CONCLUSION

The quantitative relationships presented in this paper make possible the low-frequency design of closed-box systems by direct synthesis from specifications and clearly show whether it is physically possible to realize a desired set of specifications. They should be useful to loudspeaker system designers who wish to obtain the best possible combination of small-signal and large-signal performance within the constraints imposed by a particular design problem.

These relationships should also be useful to driver manufacturers, because they indicate the range of basic driver parameters needed for modern closed-box system design and the extent to which costly magnetic material must be allocated to satisfy both the small-signal and large-signal requirements of the system.

Because the low-frequency performance of a completed system depends on a small number of easily-measured system parameters, it is always possible to specify—and verify—the low-frequency small-signal performance for standard free-field conditions. This information is of much greater value to users of loudspeakers than frequency limits quoted without decibel tolerances and without specification of the acoustic environment.

It is sincerely hoped that the quantitative relationships and physical limitations presented here—and in later papers for other types of direct-radiator systems—will not only be useful to system designers but will also contribute eventually to more uniform, realistic and accurate product specifications.

13. ACKNOWLEDGMENTS

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14. APPENDIX—SECOND-ORDER FILTER FUNCTIONS

General Expressions

Tables of filter functions normally give only the details of a low-pass prototype function. The corresponding high-pass or band-pass forms are obtained by suitable transformations. The general form of a prototype low-pass second-order filter function, $G_L(s)$, normalized to unity in the passband, is

$$G_L(s) = \frac{1}{s^2T_0^2 + a_1 sT_0 + 1}, \tag{68}$$

where T_0 is the nominal filter time constant, and the coefficient a_1 determines the actual filter characteristic. The corresponding high-pass filter function, $G_H(s)$, which preserves the same nominal time constant, is obtained by the transformation

$$G_H(sT_O) = G_L(1/sT_O). \tag{69}$$

This gives the general high-pass expression

$$G_H(s) = \frac{s^2T_O^2}{s^2T_O^2 + a_1sT_O + 1}. \tag{70}$$

Equations (68) and (70) have exactly the same form as (20) and (19) for the displacement and response functions of the closed-box system. The two sets of equations are equivalent for

$$T_O = T_C \text{ and } a_1 = 1/Q_{TC}. \tag{71}$$

Study of the steady-state magnitude-*vs*-frequency behavior of filter functions for sinusoidal excitation is facilitated by using the magnitude-squared forms

$$|G_L(j\omega)|^2 = \frac{1}{\omega^4T_O^4 + A_1\omega^2T_O^2 + 1} \tag{72}$$

and

$$|G_H(j\omega)|^2 = \frac{\omega^4T_O^4}{\omega^4T_O^4 + A_1\omega^2T_O^2 + 1}, \tag{73}$$

where

$$A_1 = a_1^2 - 2. \tag{74}$$

Cutoff Frequency

The half-power frequency $\omega_3 = 2\pi f_3$ of the high-pass function is obtained by setting (73) equal to ½ and solving for ω. Using (71) and (74), the normalized half-power frequency of the closed-box system is given by

$$f_3/f_C = \left[\frac{(1/Q_{TC}^2 - 2) + \sqrt{(1/Q_{TC}^2 - 2)^2 + 4}}{2}\right]^{1/2}. \tag{75}$$

Frequencies of Maximum Amplitude

The frequency of maximum amplitude of either frequency response or diaphragm displacement is found by taking the derivative of (72) or (73) with respect to frequency and setting this equal to zero. This yields for the normalized frequency of maximum response

$$f_{G\max}/f_C = \frac{1}{[1 - 1/(2Q_{TC}^2)]^{1/2}} \tag{76}$$

for $Q_{TC} > 1/\sqrt{2}$. For $Q_{TC} \leq 1/\sqrt{2}$, $f_{G\max}/f_C$ is infinite.

The normalized frequency of maximum diaphragm displacement is

$$f_{X\max}/f_C = [1 - 1/(2Q_{TC}^2)]^{1/2} \tag{77}$$

for $Q_{TC} > 1/\sqrt{2}$. For $Q_{TC} \leq 1/\sqrt{2}$, $f_{X\max}/f_C$ is zero.

Amplitude Maxima

Substituting the above values of frequency into the expressions for $|G(j\omega)|^2$ and $|X(j\omega)|^2$ corresponding to (72) and (73), the amplitude maxima are found to be

$$|G(j\omega)|_{\max} = |X(j\omega)|_{\max} = \left[\frac{Q_{TC}^4}{Q_{TC}^2 - 0.25}\right]^{1/2} \tag{78}$$

for $Q_{TC} > 1/\sqrt{2}$, and unity otherwise.

Types of Responses

The range of system alignments which may be obtained by varying Q_{TC} are thoroughly described in [13]. Particular alignments of interest, with brief characteristics, are:

Butterworth maximally-flat-amplitude response (*B*2) [13], [29]

$Q_{TC} = 1/\sqrt{2} = 0.707$, $f_3/f_C = 1.000$

Bessel maximally-flat-delay response (*BL*2) [13], [29], [30]

$Q_{TC} = 1/\sqrt{3} = 0.577$, $f_3/f_C = 1.272$

"Critically-damped" response [13]

$Q_{TC} = 0.500$, $f_3/f_C = 1.554$

Chebyshev equal-ripple response (*C*2) [13], [31]

$Q_{TC} > 1/\sqrt{2}$, other properties given by (75)-(78). A very popular alignment of this type is

$Q_{TC} = 1.000$, $f_3/f_C = 0.786$,
$|G(j\omega)|_{\max} = |X(j\omega)|_{\max} = 1.155$ or 1.25 dB.

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Note: Dr. Small's biography appeared in the December 1972 issue of the Journal.

Vented-Box Loudspeaker Systems
Part I: Small-Signal Analysis

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The low-frequency performance of a vented-box loudspeaker system is directly related to a small number of easily measured system parameters. This system is a fourth-order (24-dB per octave cutoff) high-pass filter which can be adjusted to have a wide variety of response characteristics. Enclosure losses have a significant effect on system performance and should be taken into account when assessing or adjusting vented-box systems. The efficiency of a vented-box loudspeaker system is shown to be quantitatively related to system frequency response, internal losses, and enclosure size.

LIST OF IMPORTANT SYMBOLS

f_B	Resonance frequency of vented enclosure
f_H	Frequency of upper voice-coil impedance peak
f_L	Frequency of lower voice-coil impedance peak
f_M	Frequency of minimum voice-coil impedance between f_L and f_H
f_S	Resonance frequency of driver
f_{SB}	Resonance frequency of driver mounted in enclosure
f_3	Half-power (-3 dB) frequency of loudspeaker system response
$G(s)$	Response function
h	System tuning ratio, $= f_B/f_S$
k_P	Power rating constant
k_η	Efficiency constant
P_{AR}	Displacement-limited acoustic power rating
P_{ER}	Displacement-limited electrical power rating
$P_{E(\max)}$	Thermally limited maximum input power
Q_A	Enclosure Q at f_B resulting from absorption losses
Q_B	Total enclosure Q at f_B resulting from all enclosure and vent losses
Q_L	Enclosure Q at f_B resulting from leakage losses
Q_P	Enclosure Q at f_B resulting from vent frictional losses
Q_{ES}	Driver Q at f_S considering electrical resistance R_E only
Q_{MS}	Driver Q at f_S considering driver nonelectrical losses only
Q_{TS}	Total driver Q at f_S resulting from all driver resistances
Q_T	Total driver Q at f_S resulting from all system resistances
R_E	Dc resistance of driver voice coil
V_{AS}	Volume of air having same acoustic compliance as driver suspension
V_B	Net internal volume of enclosure
V_D	Peak displacement volume of driver diaphragm
$x_{\max}$	Peak linear displacement of driver diaphragm
$X(s)$	Displacement function
α	System compliance ratio, $= V_{AS}/V_B$
η_0	Reference efficiency.

1. INTRODUCTION

Historical Background

The concept of the vented loudspeaker enclosure was introduced by Thuras in a U.S. patent application of 1930 [1]. The principle of operation of the system is described in considerable detail in this document which recognizes the interaction of diaphragm and vent radiation, presents several possible methods of construction, and includes a polynomial expression for the frequency-dependent behavior.

In 1952 Locanthi [2] provided the first means of calculating the exact magnitude of diaphragm–vent interaction and introduced the use of electrical analog networks to study the performance of vented-box systems.

In 1954 Beranek [3, ch. 8] derived a polynomial expression for the response of a vented-box system which was much simpler than Thuras' expression. Beranek ignored diaphragm–vent interaction and gave results for the relative response at three discrete frequencies, taking into account the system losses and including the exact effects of the variation with frequency of the radiation load resistance.

The first successful attempt to penetrate both the analysis and design of the vented-box system was published by van Leeuwen in 1956 [4]. This paper examines diaphragm–vent interaction and the effects of both parallel and series resistance in the vent. The analysis gives polynomial expressions for the frequency response and indicates the system poles and their relationship to the system transient response. Van Leeuwen studied the voice-coil impedance and determined accurate methods of calculating the driver and system parameters (and their nonlinearities) from measurement of this impedance. Also, he presented system design methods for obtaining a response characteristic of the equal-ripple (Chebyshev) type and illustrated the use of analog circuits to study the voice-coil impedance and the steady-state and transient response of the system. Unfortunately, this paper was published only in Dutch and was not widely read.

In 1959 de Boer [5], incorporating the diaphragm–vent interaction analysis of Lyon [6], showed clearly that the problem of vented-box system design was a problem of high-pass filter synthesis. Working independently, Novak [7] published in the same year an analysis which provided a simplified transfer function, methods for determining the driver and system parameters from voice-coil impedance measurements, and a clear indication of the amount of driver damping required for flat response.

A year later, Keibs [8] published a penetrating analysis which provided specific quantitative design criteria for the conditions of maximally flat amplitude response and optimum (as defined) transient response.

In 1961 two papers published almost simultaneously but independently brought the understanding of vented-box systems in English-language publications up to and beyond the level attained by van Leeuwen. First de Boer, who had in fact read van Leeuwen's paper, extended his own earlier approach using network-synthesis techniques to provide a much more lucid result. De Boer's paper [9] provides design solutions for both Butterworth and Chebyshev responses. While de Boer's analytical approach can only be described as elegant, the paper is mainly theoretical and does not provide any detailed guide to physical realization.

Later in 1961, Thiele [10], working with the simplified model established by Novak [7], published an analysis which included exhaustive treatment of the practical matters of realization. It is interesting that Thiele's paper, written completely independently of de Boer's, follows almost exactly the analysis–approximation–synthesis procedure outlined by de Boer in his introduction. Thiele's paper provides a much wider range of "optimum" responses than any previous paper, treats the amplifier as an integral part of the system, and provides simple and accurate methods of determining both driver and system parameters through measurement of the voice-coil impedance. It is probably fair to say that Thiele's paper was the first to provide an essentially complete, comprehensive, and practical understanding of vented-box systems on a quantitative level.

While both de Boer and Thiele published in English, neither paper appears to have been widely read (or understood) at the time of publication. Only after 10 years has Thiele's paper been recognized as a classic and republished for a wider audience.

In 1969 Nomura [11] pointed out that enclosure losses often contribute substantial response errors. Nomura's paper provides design solutions for Chebyshev, "degenerated" Chebyshev, and Butterworth responses which include the effects of absorption losses in the enclosure.

A very recent paper by Benson [32] contains the most complete small-signal treatment of vented-box systems yet available and covers several interesting topics not discussed here. A number of footnotes have been added to the text of this paper to make reference to the improved understanding or techniques developed by Benson or to indicate areas in which further information may be gained from his paper.

Technical Background

The vented-box loudspeaker system is a direct-radiator system using an enclosure which has two apertures. One aperture accommodates a driver. The other, called a vent or port, allows air to move in and out of the enclosure in response to the pressure variations within the enclosure.

The vent may be formed as a simple aperture in the enclosure wall or as a tunnel or duct which extends inward from the aperture. In either case, the behavior of the air in the vent is reactive, i.e., it acts as an inertial mass. At low frequencies, the motion of air in the vent contributes substantially to the total volume velocity crossing the enclosure boundaries and therefore to the system output [12].

The analysis of vented-box systems in this paper is essentially an extension of Thiele's approach [10]; it follows the organization of [12] which is in fact a generalized description of Thiele's methods. The principal extensions to Thiele's work include treatment of efficiency–response relationships and large-signal behavior, evaluation of diaphragm–vent interaction, assessment of the magnitude and effects of normal enclosure losses, and calculation of alignment data for systems having such losses. The treatment of enclosure losses is different from that of Nomura [11] because the absorption losses considered by Nomura are found to contribute only a portion of the losses present in practical enclosures.

Some of the analytical results presented in this paper are either obtained or illustrated with the help of an analog circuit simulator similar to that used by Locanthi [2]. Such a simulator is an invaluable aid in the analysis and design of loudspeaker systems because it provides rapid assessment of both time-domain and frequencydomain performance. It is particularly useful in investigating the effects of losses, component tolerances, system misalignment, etc., on response, diaphragm excursion, and voice-coil impedance. It provides results in a fraction of the time that would be required using normal computational methods.

The analytical relationships developed in this paper show that the important performance characteristics of vented-box systems are directly related to a number of basic and easily measured system parameters. Both the assessment and the specification of performance at low frequencies for such systems are therefore relatively simple tasks.

In Parts I and II it is shown that these analytical relationships impose definite quantitative limitations on both small-signal and large-signal performance of vented-box systems and indicate the extent to which the important performance characteristics may be traded off against one another.

In Part III these relationships lead to a method of synthesis (system design) which is free of trial-and-error procedures. This method starts with the desired performance characteristics, checks these for realizability, and results in complete specification of the required system components.

The appendices of the paper are included in Part IV.

2. BASIC ANALYSIS

The impedance-type acoustical analogous circuit of a vented-box loudspeaker system is presented in Fig. 1. This circuit is derived from the generalized circuit of [12, Fig. 2] by short-circuiting the port compliance element. In Fig. 1, the symbols are defined as follows:

e_g	Open-circuit output voltage of source or amplifier
B	Magnetic flux density in driver air gap
l	Length of voice-coil conductor in magnetic field of air gap
S_D	Effective projected surface area of driver diaphragm
R_g	Output resistance of source or amplifier
R_E	Dc resistance of driver voice coil
C_{AS}	Acoustic compliance of driver suspension
M_{AS}	Acoustic mass of driver diaphragm assembly including voice coil and air load
R_{AS}	Acoustic resistance of driver suspension losses
C_{AB}	Acoustic compliance of air in enclosure
R_{AB}	Acoustic resistance of enclosure losses caused by internal energy absorption
R_{AL}	Acoustic resistance of enclosure losses caused by leakage
M_{AP}	Acoustic mass of port or vent including air load
R_{AP}	Acoustic resistance of port or vent losses
U_D	Volume velocity of driver diaphragm
U_P	Volume velocity of port or vent
U_L	Volume velocity of enclosure leakage
U_B	Volume velocity entering enclosure
U_0	Total volume velocity leaving enclosure boundaries.

This circuit may be simplified to that of Fig. 2 by combining the series resistances in the driver branch to form a single acoustic resistance R_{AT}, where

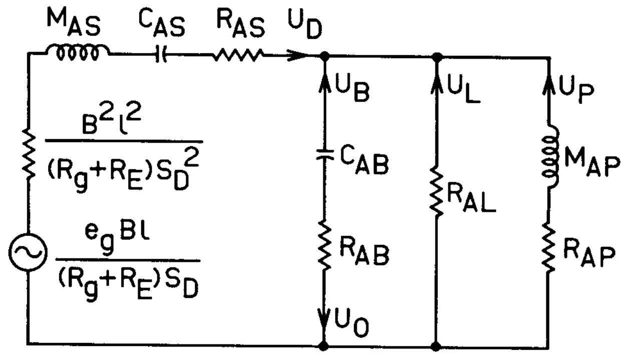

Fig. 1. Acoustical analogous circuit of vented-box loudspeaker system.

$$R_{AT} = R_{AS} + \frac{B^2l^2}{(R_g + R_E)S_D^2} \qquad (1)$$

and by defining

$$p_g = \frac{e_gBl}{(R_g + R_E)S_D} \qquad (2)$$

as the value of the Thevenin acoustic pressure generator at the left of the circuit. Finally, R_{AB} and R_{AP} are neglected because, as described in the next section, their effects can normally be accounted for by a suitable adjustment to the value of R_{AL}.

By comparison, the circuit used by Novak [7] and Thiele [10] is obtained from that of Fig. 2 by removing the resistance R_{AL}.

The electrical equivalent circuit of the vented-box system is formed by taking the dual of Fig. 1 and converting all impedance elements to their electrical equivalents by the relationship

$$Z_E = B^2l^2/(Z_AS_D^2) \qquad (3)$$

where Z_A is the impedance of an element in the impedance-type acoustical analogous circuit and Z_E is the impedance of the corresponding element in the electrical equivalent circuit. A simplified electrical equivalent circuit corresponding to Fig. 2 is shown in Fig. 3. In this circuit,

C_{MES}	Corresponds to driver mass M_{AS}
L_{CES}	Corresponds to driver suspension compliance C_{AS}
R_{ES}	Corresponds to driver suspension resistance R_{AS}
L_{CEB}	Corresponds to enclosure compliance C_{AB}
R_{EL}	Corresponds to enclosure leakage resistance R_{AL}
C_{MEP}	Corresponds to vent mass M_{AP}.

The circuits presented above are valid only for frequencies within the piston range of the system driver; the element values are assumed to be independent of frequency within this range.

As discussed in [12], the effects of the voice-coil inductance and the resistance of the radiation load are neglected. The effect of external acoustic interaction between driver diaphragm and vent [2], [6] has also been neglected. The reasons for this are given later in the paper.

The analysis of the system and the interpretation of its describing functions are simplified by defining a num-

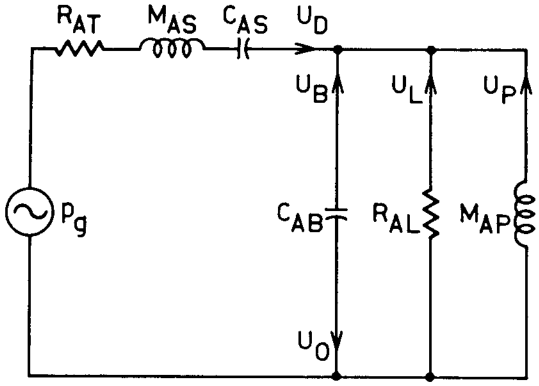

Fig. 2. Simplified acoustical analogous circuit of vented-box loudspeaker system.

ber of component and system parameters. For the enclosure, these are

$$T_B^2 = 1/\omega_B^2 = C_{AB}M_{AP} = C_{MEP}L_{CEB} \quad (4)$$

$$Q_L = \omega_B C_{AB} R_{AL} = 1/(\omega_B C_{MEP} R_{EL}). \quad (5)$$

From Figs. 2 and 3 it can be seen that $\omega_B = 2\pi f_B$ is the resonance frequency of the enclosure–vent circuit, and that Q_L represents the Q of this resonant circuit at ω_B resulting from the leakage losses.

Similarly, the system driver is described by the driver parameters introduced in [12]. These are

$$T_S^2 = 1/\omega_S^2 = C_{AS}M_{AS} = C_{MES}L_{CES} \quad (6)$$

$$Q_{MS} = \omega_S C_{MES} R_{ES} = 1/(\omega_S C_{AS} R_{AS}) \quad (7)$$

$$Q_{ES} = \omega_S C_{MES} R_E = \omega_S R_E M_{AS} S_D^2/(B^2 l^2) \quad (8)$$

$$V_{AS} = \rho_0 c^2 C_{AS}. \quad (9)$$

In Eq. (9) ρ_0 is the density of air (1.18 kg/m³) and c is the velocity of sound in air (345 m/s). In this paper it is assumed that the values of the first three parameters apply to the driver when the diaphragm air-load mass is that for the driver mounted in the system enclosure [3, pp. 216-217].

The interaction of the source, driver, and enclosure give rise to further system parameters. These are the system compliance ratio α, given by

$$\alpha = C_{AS}/C_{AB} = L_{CES}/L_{CEB} \quad (10)$$

the system tuning ratio h, given by

$$h = f_B/f_S = \omega_B/\omega_S = T_S/T_B \quad (11)$$

and the total Q of the driver connected to the source Q_T, given by

$$Q_T = 1/(\omega_S C_{AS} R_{AT}). \quad (12)$$

Following the method of [12], analysis of the circuits of Figs. 2 and 3 and substitution of the parameters defined above yields the system response function

$$G(s) = \frac{s^4 T_B^2 T_S^2}{s^4 T_B^2 T_S^2 + s^3(T_B^2 T_S/Q_T + T_B T_S^2/Q_L) + s^2[(\alpha+1)T_B^2 + T_B T_S/Q_L Q_T + T_S^2] + s(T_B/Q_L + T_S/Q_T) + 1} \quad (13)$$

where $s = \sigma + j\omega$ is the complex frequency variable, the diaphragm displacement function

$$X(s) = \frac{s^2 T_B^2 + s T_B/Q_L + 1}{D(s)} \quad (14)$$

where $D(s)$ is the denominator of Eq. (13), the displacement constant

$$k_x = 1 \quad (15)$$

and the voice-coil impedance function

$$Z_{VC}(s) = R_E + R_{ES}\frac{s(T_S/Q_{MS})(s^2T_B^2 + sT_B/Q_L + 1)}{D'(s)} \quad (16)$$

where $D'(s)$ is the denominator of Eq. (13) but with Q_T wherever it appears replaced by Q_{MS}.

3. ENCLOSURE LOSSES

In any vented-box loudspeaker system, three kinds of enclosure losses are present: absorption losses, leakage losses, and vent losses. These losses correpond to the resistances R_{AB}, R_{AL}, and R_{AP} in Fig. 1. The magnitude of each of these losses may be established by defining a value of Q for the enclosure-vent resonant circuit at f_B, considering each loss one at a time. Thus for the leakage losses,

$$Q_L = \omega_B C_{AB} R_{AL} \quad (5)$$

for the absorption losses,

$$Q_A = 1/(\omega_B C_{AB} R_{AB}) \quad (17)$$

and for the vent losses

$$Q_P = 1/(\omega_B C_{AB} R_{AP}). \quad (18)$$

The total Q of the enclosure–vent circuit at f_B is then defined as Q_B, where

$$1/Q_B = 1/Q_L + 1/Q_A + 1/Q_P. \quad (19)$$

It is this Q_B that is measured in a practical system using the method of Thiele described in [10, sec. 14] and in Section 7 (Part II) of this paper.

This paper deals only with systems in which enclosure losses are kept to a practical minimum. Systems making use of deliberately enlarged enclosure losses (e.g., large leaks, resistively damped vents, heavily damped or filled enclosures) will be treated in a later paper.

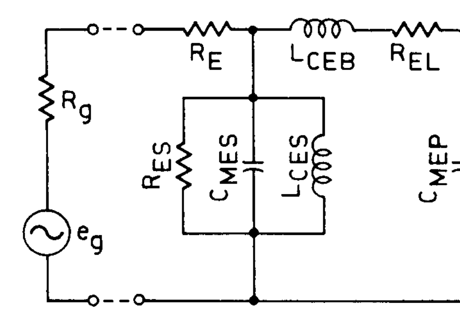

Fig. 3. Simplified electrical equivalent circuit of vented-box loudspeaker system.

Assessment of the contribution of enclosure losses to system performance requires meaningful answers to two questions. First, what is the effect of each kind of loss on system performance? Second, what are the typical magnitudes of the three kinds of losses in practical enclosures?

The answer to the first question has been obtained by constructing the circuit analog of a vented-box system and observing the change in response as a "lossless" enclosure is provided successively with individual leakage, absorption, and vent losses corresponding to a given value of Q. The results for the fourth-order Butterworth (B4) alignment given by Thiele in [10, Table I] are shown in Fig. 4 for Q values of 5. As indicated by Thiele [10, eq. (90)], the maximum response loss occurs at f_B and to a very close approximation depends only on Q_B and not on the actual nature of the loss or losses present. Above f_B absorption losses have the greatest effect and vent losses the least effect on response, while below f_B the relative effects are reversed. The effect of leakage losses is intermediate both above and below f_B. The relative effects are the same for other alignments given in [10], except that, as stated by Thiele, the response loss for a given value of Q_B is greater for alignments having a lower compliance ratio and smaller for alignments having a higher compliance ratio.

The second question has troubled a great many authors because measured losses tend to be higher than the values predicted from theory. Both Beranek [3, p. 257] and Thiele [10, footnote to sec. 14] suspected that absorption losses were to blame for their low measured values of Q_B, and Nomura's paper [11] is based on the assumption that these losses are dominant. Van Leeuwen found that neither lining nor bracing of the enclosure affected his loss measurements [4] and concluded that absorption losses were not significant. He suspected that his extra losses arose in the vent and could be explained only by assuming an increased value for the coefficient of viscosity of air—about 30 times larger than the normally accepted value.

It is possible to determine the magnitude of each kind of loss in practical systems by an extension of Thiele's measurement method as described in Appendix 3. From measurements of this type on a number of commercial and experimental systems, the following was found.

1) Losses in unobstructed vents are usually about the same as or a little greater than the values calculated from viscous theory [10, eq. (7)]. Typical values of Q_P for unobstructed vents are in the range of 50–100. If the vent is obstructed by grill cloth or lining materials, the value of Q_P can fall considerably, but with reasonable care in design need not fall below 20.

2) Absorption losses in unlined enclosures are quite small, giving Q_A values of 100 or more. Typical lining materials placed on the enclosures walls where air particle velocity is low do not extract very much energy [13, p. 383] but can reduce Q_A to a range of 30–80. Very thick linings or damping partitions reduce Q_A even further.

3) Leakage losses are usually the most significant, giving Q_L values of between 5 and 20.

The last result is surprising, because the enclosures tested were well built and appeared to be quite leak-free. In fact, some of the more serious leaks were traced to the drivers. These leaks were caused by imperfect gasket seals and/or by leakage of air through a porous dust cap and past the voice coil. However, the few systems having drivers with solid dust caps and perfect gaskets still had dominant measured leakage losses.

Confidence in the measurement method, based on its ability to detect with reasonable accuracy the deliberate introduction of small additional enclosure losses, leads to the conclusion that the measured leakage in apparently leak-free systems is not an error of measurement but an indication that the actual losses in the system enclosure are not constant with frequency as assumed in the method of measurement (Appendix 3).

The analog circuit simulator has proved to be an invaluable aid in reaching and supporting this conclusion and also in establishing the practical meaning and usefulness of the total-loss measurement. First, it has shown that vent losses which increase with frequency and absorption losses which decrease with frequency do indeed appear in the measurement results as apparent leakage. Second, it has shown that where such frequency-varying losses are present, the system response is predicted with extremely high accuracy from the measured values of Q_A, Q_L, and Q_P as defined.

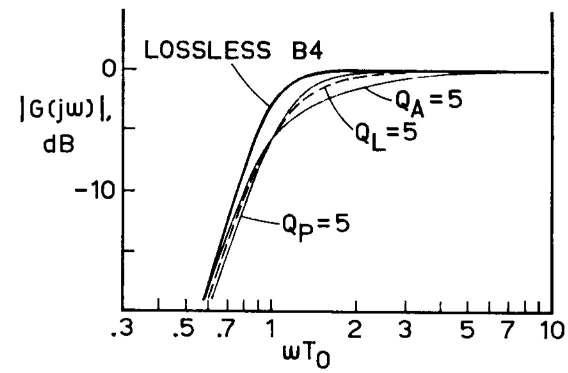

Fig. 4. Effects of enclosure-circuit losses on response of a lossless B4-aligned vented-box loudspeaker system (from simulator).

Finally, and not surprisingly in view of Fig. 4, it has shown that approximately equal values of Q_A and Q_P in the range of values normally measured in practical enclosures have a combined effect on system response which is effectively indistinguishable from the same total value of Q_L.

The above findings lead to the conclusion that even where *actual* leakage is not dominant, the enclosure losses present in a normal vented-box system may be adequately approximated, for purposes of evaluation or design, by a single frequency-invariant leakage resistance. The value of this equivalent leakage resistance is such that the corresponding value of Q_L is equal to the total Q_B that would be measured in the real system by Thiele's method. This approximation is reflected in Figs. 2 and 3 and in the system describing functions Eqs. (13), (14), and (16).

4. RESPONSE

Response Function

The response function of the vented-box system is given by Eq. (13). This is a fourth-order (24-dB per

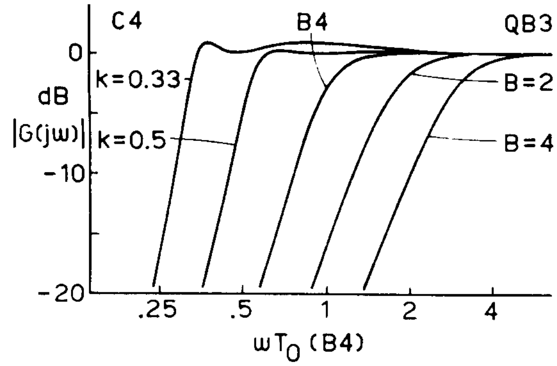

Fig. 5. Normalized response curves for B4 and selected C4 and QB3 alignments of vented-box loudspeaker system.

octave cutoff) high-pass filter function which may be expressed in the general form

$$G(s) = \frac{s^4T_0^4}{s^4T_0^4 + a_1s^3T_0^3 + a_2s^2T_0^2 + a_3sT_0 + 1} \tag{20}$$

where T_0 is the nominal filter time constant and a_1, a_2, a_3 are coefficients which determine the behavior of the filter response.[1]

The behavior of Eq. (13) may be assessed by studying Eq. (20) and then using the relationships which make the corresponding terms of the two expressions identical. Using Eq. (11), these are

$$T_0 = (T_BT_S)^{1/2} = T_S/h^{1/2} \tag{21}$$

$$a_1 = \frac{Q_L + hQ_T}{h^{1/2}Q_LQ_T} \tag{22}$$

$$a_2 = \frac{h + (\alpha + 1 + h^2)Q_LQ_T}{hQ_LQ_T} \tag{23}$$

$$a_3 = \frac{hQ_L + Q_T}{h^{1/2}Q_LQ_T}. \tag{24}$$

Frequency Response

Alignment

The frequency response $|G(j\omega)|$ of Eq. (20) is examined in Appendix 1. Coefficient data are given for a variety of useful response characteristics which may be used to align the vented-box system.

Three very useful types of alignments are given by Thiele in [10]. These are the fourth-order Butterworth maximally flat alignment (B4), the fourth-order Chebyshev equal-ripple alignment (C4), and the alignment which Thiele has dubbed "quasi-third-order Butterworth" (QB3). Alternative alignments include the degenerated Chebyshev responses of Nomura [11] and the sub-Chebyshev responses of Thiele [14], although the latter provide less effective use of enclosure volume in relation to the efficiency and low-frequency cutoff obtained, i.e., a lower value of the efficiency constant described in Section 5.

[1] This normalization of the filter function follows the example of Thiele [10]. The relationships between this form of normalization and others, e.g., that used by Weinberg [18], including relative pole locations are given by Benson in [32, pp. 422-438 and Appendix 7].

Both the C4 and QB3 alignments provide a wide range of realizable response characteristics with gradually changing properties. Also, both as a limiting case coincide with the unique B4 alignment, so a completely continuous span of alignments is mathematically possible. A few of these alignments are illustrated in Fig. 5. The frequency scale of Fig. 5 is normalized to the nominal time constant of the B4 alignment; the other curves are plotted to the same scale but displaced horizontally for clarity. In this paper, the C4 alignments are specified by the value of k used by Thiele and defined in Appendix 1. The QB3 alignments are specified by the value of B defined in Appendix 1.

Inspection of Eqs. (21–24) reveals that the four mathematical variables needed to specify a given alignment, T_0, a_1, a_2, and a_3, are related to five independent system variables (or parameters), T_S, h, α, Q_L, and Q_T. This means that specification of a particular alignment does not correspond to a unique set of system parameters but may be obtained in a variety of ways. For any given alignment, one parameter may be assigned arbitrarily (within limits of realizability) and the rest may then be calculated.

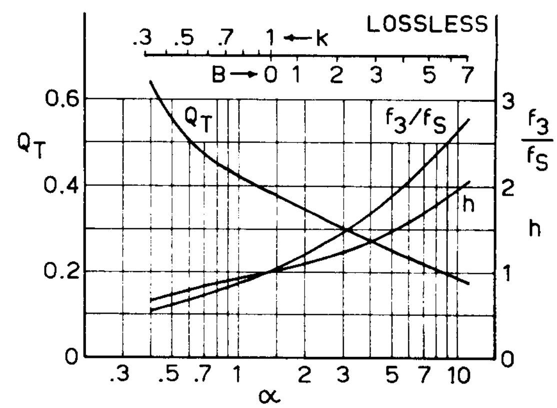

Fig. 6. Alignment chart for lossless vented-box systems.

A basic understanding of the behavior of the vented-box system is quickly obtained if the enclosure losses are ignored, i.e., Q_L is taken to be infinite. In this case, Eqs. (22–24) are simplified and all alignments become unique in terms of the system parameters. This is the process followed by Thiele in [10].

Fig. 6 is an alignment chart for systems with lossless enclosures based on the C4, B4, and QB3 alignments. The compliance ratio α is chosen as the primary independent variable and plotted as the abscissa of the figure. The corresponding values of k and B which specify the C4 and QB3 alignments are also given on the figure. Because each alignment is unique, every value of α corresponds to a specific alignment and requires specific values of the other system parameters to obtain the correct response. Thus the figure gives the values of Q_T and the tuning ratio $h = f_B/f_S$ required for each value of α, as well as the normalized cutoff frequency f_3/f_S at which the response is 3 dB down from its high-frequency asymptotic value.

Misalignment

The effect of an incorrectly adjusted parameter on the

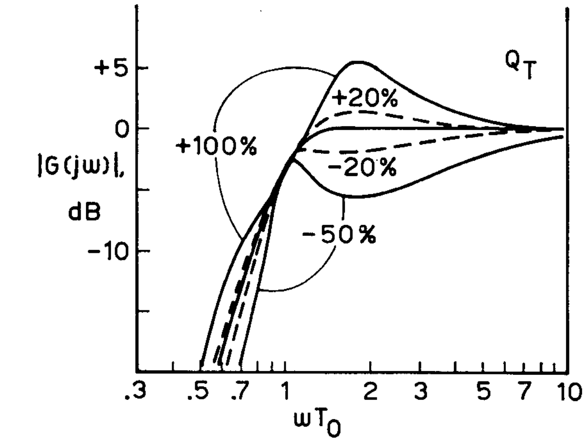

Fig. 7. Variations in frequency response of lossless B4-aligned vented-box system for misalignment of Q_T (from simulator).

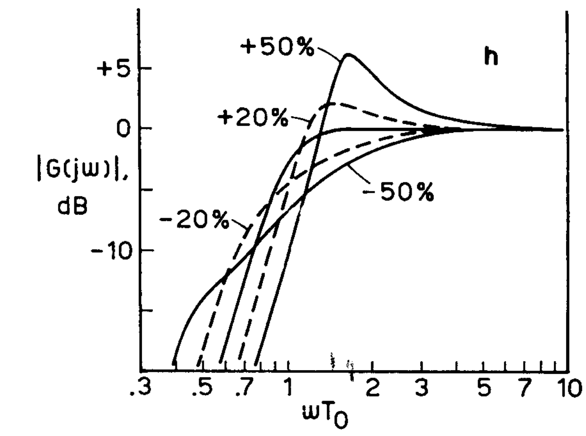

Fig. 8. Variations in frequency response of lossless B4-aligned vented-box system for misalignment of h (from simulator).

frequency response of a vented-box system is easily observed using the analog circuit simulator. Fig. 7 shows the variation produced in the response of a lossless system aligned for a B4 response by changes in the value of Q_T of $\pm 20\%$, -50%, and $+100\%$. This agrees exactly with [10, eqs. (42) and (43)] which indicate that the response at the frequencies f_L and f_H of the voice-coil impedance peaks is directly proportional to Q_T, while the response at f_B is independent of Q_T. Fig. 8 shows the variations produced in the same alignment by mistuning (changing the value of h) of $\pm 20\%$ and $\pm 50\%$.

Similar effects occur with other alignments. It is not difficult to see why the vented enclosure is sometimes scorned as a "boom box" when it is realized that the values of Q_T required are much lower than the majority of woofers provide [15, Table 13] and that a historical emphasis on unity tuning ratio regardless of compliance ratio often results in erroneously high tuning.

Alignment with Enclosure Losses

Using the approximation arrived at in Section 3, the parameter relationships required to provide a specified response in the presence of enclosure losses may be calculated as described in Appendix 1. Compared to lossless alignments, a particular response characteristic generally requires a larger value of Q_T and a smaller value of α.

Alignment charts for the C4, B4, and QB3 responses are presented in Figs. 9–13 for systems having enclosure losses corresponding to a Q_L of 20, 10, 7, 5, and 3, respectively. These values are representative of real enclosures, for which the most commonly measured values of Q_B are in the range of 5–10.

Transient Response

Keibs [8], [16] offered alignment solutions for what he considered to be the optimum transient response of a fourth-order filter. The same alignment parameters were later advocated by Novak [17]. The step responses of various fourth-order high-pass filter alignments are illustrated in Fig. 14. The alignments range from Chebyshev to sub-Chebyshev types and include the alignment recommended by Keibs.

The transient response of any minimum-phase network is of course directly related to the frequency response. For the vented-box system, the alignments which have more gradual rolloff also have less violent transient ringing. If transient response is considered important, then it would appear that the QB3 alignments are to be preferred over the B4 and C4 alignments. The SC4 alignments (Appendix 1) provide a further improvement in

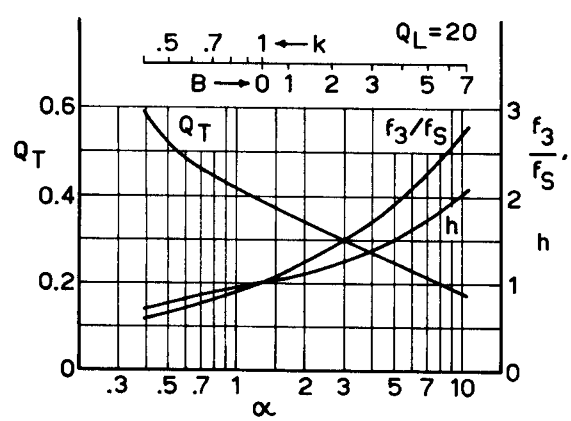

Fig. 9. Alignment chart for vented-box systems with $Q_B = Q_L = 20$.

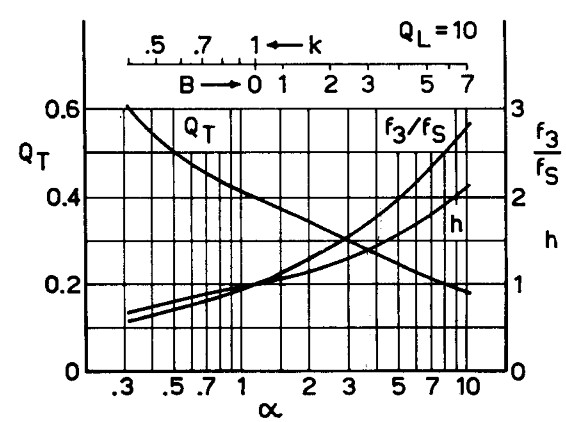

Fig. 10. Alignment chart for vented-box systems with $Q_B = Q_L = 10$.

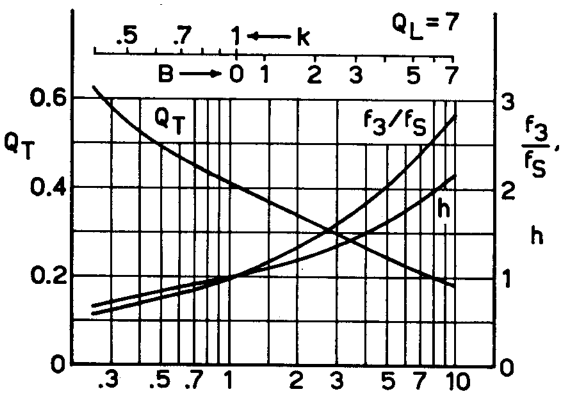

Fig. 11. Alignment chart for vented-box systems with $Q_B = Q_L = 7$.

transient response but have a less attractive frequency response.

Phase and Delay Response

Weinberg [18] shows how the conditions of maximal flatness or equal-ripple behavior may be imposed on any property of a response function, including phase response and group delay. The condition of maximally flat passband group delay is provided by the Bessel filter. The polynomial coefficients of the fourth-order Bessel filter are calculated in Appendix 1 from the pole locations given in [19].

General Response Realization

Any physically realizable minimum-phase fourth-order response characteristic which can be described in terms of the coefficients of Eq. (20) can be realized in a vented-box loudspeaker system. Using the method of Appendix 1, the coefficients may be processed into system alignment parameters which will produce the specified response.

5. EFFICIENCY

Reference Efficiency

The piston-range reference efficiency of a vented-box loudspeaker system is the reference efficiency of the system driver when the total air-load mass seen by the driver diaphragm is the same as that imposed by

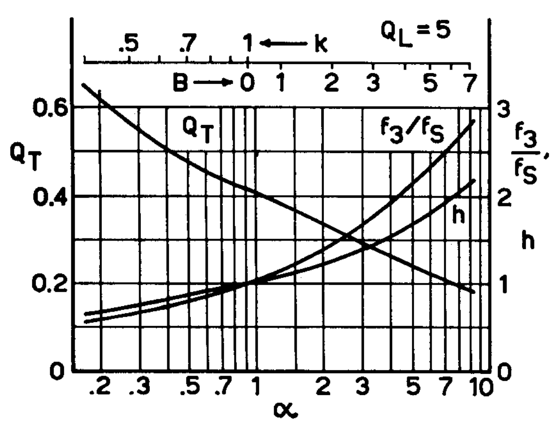

Fig. 12. Alignment chart for vented-box systems with $Q_B = Q_L = 5$.

the enclosure. Thus if the driver parameters are measured under or adjusted to correspond to this condition, the system reference efficiency η_0 is [12, eq. (32)]

$$\eta_0 = \frac{4\pi^2}{c^3} \cdot \frac{f_S{}^3 V_{AS}}{Q_{ES}}. \tag{25}$$

For SI units, the value of $4\pi^2/c^3$ is 9.64×10^{-7}.

Efficiency Factors

Eq. (25) may be written

$$\eta_0 = k_\eta f_3{}^3 V_B \tag{26}$$

where f_3 is the cutoff (half-power or -3 dB) frequency of the system, V_B is the net internal volume of the system enclosure, and $k\eta$ is an efficiency constant given by

$$k_\eta = \frac{4\pi^2}{c^3} \cdot \frac{V_{AS}}{V_B} \cdot \frac{f_S{}^3}{f_3{}^3} \cdot \frac{1}{Q_{ES}}. \tag{27}$$

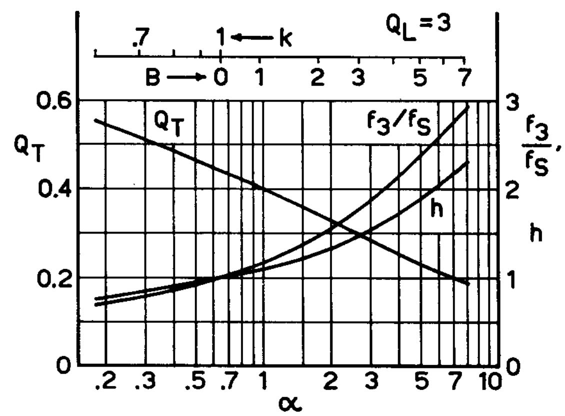

Fig. 13. Alignment chart for vented-box systems with $Q_B = Q_L = 3$.

The efficiency constant k_η may be separated into two factors, $k_{\eta(Q)}$ related to driver losses and $k_{\eta(G)}$ related to the response characteristic and enclosure losses. Thus,

$$k_\eta = k_{\eta(Q)} k_{\eta(G)} \tag{28}$$

where

$$k_{\eta(Q)} = Q_T/Q_{ES} \tag{29}$$

$$k_{\eta(G)} = \frac{4\pi^2}{c^3} \cdot \frac{V_{AS}}{V_B} \cdot \frac{f_S{}^3}{f_3{}^3} \cdot \frac{1}{Q_T}. \tag{30}$$

Driver Loss Factor

The value of Q_T for systems used with modern high-damping-factor amplifiers ($R_g = 0$) is equal to Q_{TS}, where [12, eq. (47)]

$$Q_{TS} = \frac{Q_{ES} Q_{MS}}{Q_{ES} + Q_{MS}}. \tag{31}$$

Eq. (29) then reduces to

$$k_{\eta(Q)} = Q_{TS}/Q_{ES} = 1 - Q_{TS}/Q_{MS}. \tag{32}$$

This expression has a maximum value of unity which is approached only when mechanical driver losses are negligible (Q_{MS} infinite) and all required damping is

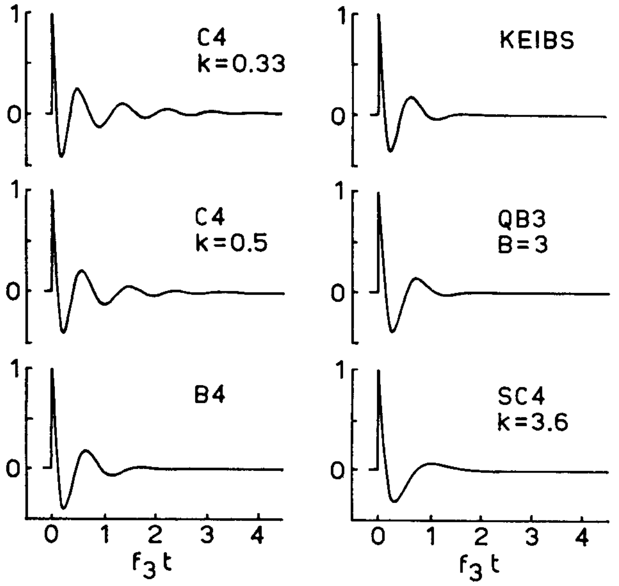

Fig. 14. Normalized step response of vented-box loudspeaker system (from simulator).

provided by electromagnetic coupling ($Q_{ES} = Q_{TS}$).

The value of $k_{\eta(Q)}$ for typical vented-box system drivers is in the range of 0.8–0.95.

System Response Factor

Normally, vented enclosures contain only a small amount of damping material used as a lining. Under these conditions [3, p. 129],

$$C_{AB} = V_B/\rho_0 c^2 \tag{33}$$

and, using Eqs. (9) and (10), Eq. (30) can be written in terms of the system parameters as

$$k_{\eta(G)} = \frac{4\pi^2}{c^3} \cdot \frac{\alpha}{Q_T (f_3/f_S)^3}. \tag{34}$$

The relationships between α, Q_T, and f_3/f_S for the C4–B4–QB3 alignments have already been calculated and plotted in Figs. 6 and 9–13. Thus the value of $k_{\eta(Q)}$ for any of these alignments can also be calculated. Fig. 15 is a plot of the value of $k_{\eta(G)}$ as a function of α for several values of Q_L. For reference, the location of the B4 alignment is indicated on each curve by a short vertical bar.

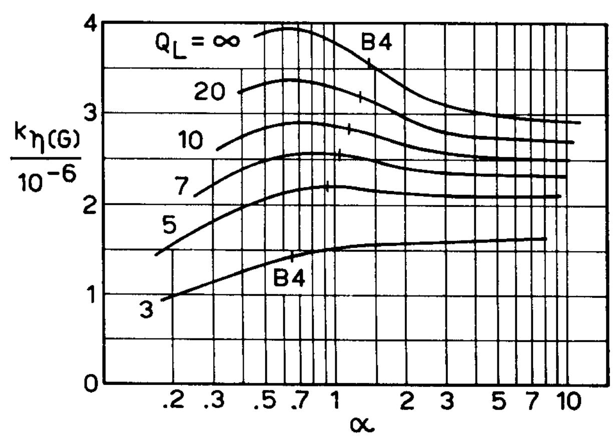

Fig. 15. Response factor $k_{\eta(G)}$ of efficiency constant for vented-box loudspeaker system as a function of α (system compliance ratio) for several values of enclosure Q.

It is clear that enclosure losses significantly reduce the value of $k_{\eta(G)}$ for a correctly aligned system. The maximum possible value of $k_{\eta(G)}$ is 3.9×10^{-6} and occurs when the enclosure losses are negligible and the system compliance ratio is adjusted to about 0.6. This is a $k = 0.5$ C4 alignment which has a ripple of about 0.2 dB.

Maximum Reference Efficiency, Cutoff Frequency, and Enclosure Volume

Taking the maximum theoretical values of $k_{\eta(Q)}$ and $k_{\eta(G)}$, the maximum reference efficiency $\eta_{0(\max)}$ that could be obtained from a lossless vented-box system for specified values of f_3 and V_B is, from Eqs. (26) and (28),

$$\eta_{0(\max)} = 3.9 \times 10^{-6} f_3{}^3 V_B \tag{35}$$

with f_3 in Hz and V_B in m^3. This relationship is illustrated in Fig. 16, with V_B (given here in cubic decimeters: 1 dm^3 = 1 liter = 10^{-3} m^3) plotted against f_3 for various values of $\eta_{0(\max)}$ expressed in percent.

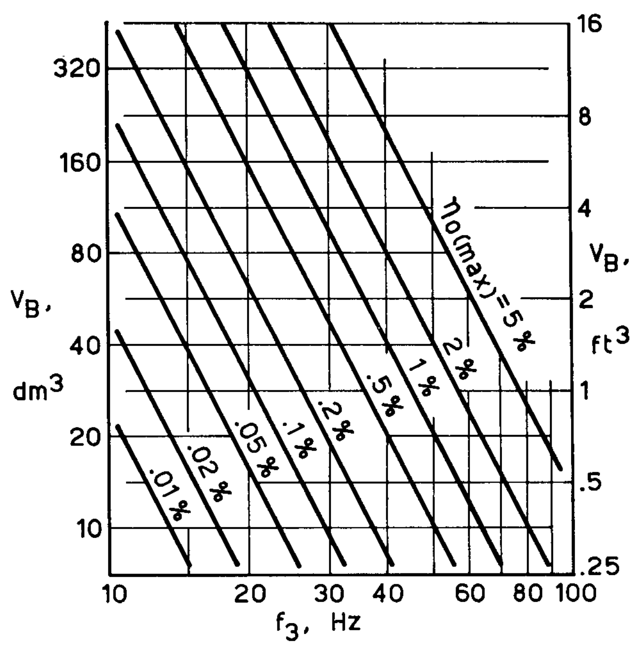

Fig. 16. Relationship between cutoff frequency, enclosure volume, and maximum reference efficiency for vented-box loudspeaker system.

Fig. 16 represents the physical efficiency–cutoff frequency–volume limitation of vented-box system design. A practical system having given values of f_3 and V_B must always have an actual reference efficiency lower than the corresponding value of $\eta_{0(\max)}$ given by Fig. 16. Similarly, a system of specified efficiency and volume must have a cutoff frequency higher than that indicated by Fig. 16, and so on.

Actual vented-box systems have an efficiency lower than the maximum given by Eq. (35) because of driver mechanical losses, enclosure losses, and the use of alignments other than that which gives maximum efficiency for a given value of Q_L. Typical practical efficiencies are 40–50% (2–3 dB) lower than the theoretical maximum given by Eq. (35) or Fig. 16. For most systems, the driver parameters can be measured and the reference efficiency calculated directly from Eq. (25).

The physical limitation imposed by Eq. (35) or Fig. 16 may be overcome in a sense by the use of amplifier assistance, i.e., networks which raise the gain of the amplifier in the cutoff region of the system [10], [20]. While the overall response of the complete system is thus extended, there is no change in the driver-enclosure efficiency in the cutoff region. The amplifier must deliver more power, and the driver must dissipate this power.

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Editor's Note: Dr. Small's biography appeared in the December issue.

Vented-Box Loudspeaker Systems Part II: Large-Signal Analysis

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The power capacity of a vented-box loudspeaker system is shown to be directly related to the system frequency response and to the volume of air that can be displaced by the system driver. The vent area must be made large enough to prevent noise generation or excessive losses; the required area is shown to be quantitatively related to enclosure tuning and to driver displacement volume. Mutual coupling between driver and vent is found to be of negligible importance in most cases.

The basic performance characteristics of a vented-box system may be determined from knowledge of a number of fundamental system parameters. These parameters can be evaluated from relatively simple measurements. The vented-box system is shown to possess two important performance advantages compared with the closed-box system.

Editor's Note: Part I of Vented-Box Loudspeaker Systems appeared in the June issue.

6. DISPLACEMENT-LIMITED POWER RATINGS

Diaphragm Displacement

The vented-box system displacement function given by Eq. (14) is a low-pass filter function which has a notch at f_B contributed by the numerator and an ultimate cutoff slope of 12 dB per octave at high frequencies. The behavior of this function is examined at the end of Appendix 1.

The normalized diaphragm displacement magnitude $|X(j\omega)|$ is plotted in Fig. 17 for a few common alignments. For convenience, the frequency scale is normalized to f_B. Note that the effect of moving from the C4 alignments toward the QB3 alignments (i.e., increasing α) is to reduce the diaphragm displacement near and above f_B relative to the displacement at zero frequency, and that the principal effect of enclosure losses is to increase the displacement near f_B, i.e., reduce the sharpness of the notch.

Acoustic Power Rating

Assuming linear large-signal diaphragm displacement, the steady-state displacement-limited acoustic power rating P_{AR} of a loudspeaker system, from [12, eq. (42)], is

$$P_{AR} = \frac{4\pi^3\rho_0}{c}\cdot\frac{f_S{}^4 V_D{}^2}{k_x{}^2\,|X(j\omega)|_{\max}{}^2} \qquad (36)$$

where $|X(j\omega)|_{\max}$ is the maximum magnitude attained by the displacement function and V_D is the peak displacement volume of the driver diaphragm, given by

$$V_D = S_D\,x_{\max} \qquad \mathbf{(37)}$$

x_{max} being the peak linear displacement of the driver diaphragm, usually set by the amount of voice-coil overhang.

For the vented-box system, Eq. (15) gives $k_x = 1$. The displacement-limited acoustic power rating of the vented-box system then becomes

$$P_{AR(VB)} = \frac{4\pi^3\rho_0}{c} \cdot \frac{f_S{}^4 V_D{}^2}{|X(j\omega)|_{max}{}^2}. \quad (38)$$

For SI units, the value of $4\pi^3\rho_0/c$ is 0.424.

Power-Rating Constant

Eq. (38) may be written in the form

$$P_{AR(VB)} = k_P f_3{}^4 V_D{}^2 \quad (39)$$

where k_P is a power-rating constant given by

$$k_P = \frac{4\pi^3\rho_0}{c} \cdot \frac{1}{(f_3/f_S)^4 |X(j\omega)|_{max}{}^2}. \quad (40)$$

The value of f_3/f_S is already established for any alignment in the C4–B4–QB3 range. But from Fig. 17, $|X(j\omega)|$ has *two* maxima. The first occurs outside the system passband; this has a value of unity and is located at zero frequency for the QB3, B4, and moderate C4 alignments but slightly exceeds unity and is located below f_B for the extreme C4 alignments. The second maximum occurs within the system passband, above f_B, and is always smaller than the first.

There are thus two possible values for k_P, one if the system driving signal is allowed to have large-amplitude components at frequencies well below cutoff, and another, which is substantially larger, if the signal is restricted so that all significant spectral components are within the system passband.

Fig. 18 is a plot of the values of k_P for each of the above driving conditions as a function of the alignment parameters k and B for systems with lossless enclosures. The crosses in Fig. 18 indicate the values of k_P for a few selected alignments with $Q_L = 5$. The effect of this relatively severe amount of enclosure loss on k_P is negligible for the QB3 alignments but gradually increases as the extreme C4 alignments are approached. For these alignments, k_P is slightly reduced for the passband-drive case but slightly increased for the wideband-drive case.

Program Acoustic Power Rating

In most program applications, a portion of the driving signal spectrum lies below the system passband. The lower value of k_P given by Fig. 18 is then in general conservative, while the higher value is comparatively optimistic. A truly realistic value of k_P for program material can be evaluated only if the actual spectral power distribution of the particular driving signal is known. Thiele for example has obtained comparative power handling data for a number of system alignments (including amplifier-assisted alignments) based on a particular random-noise driving signal [20].

In most cases, provided that the program spectrum is principally within the system passband, a satisfactory program rating is obtained by setting k_P equal to 3.0, regardless of the alignment used. This is indicated by the broken line in Fig. 18. This compromise value for k_P is arrived at by considering, for the entire range of alignments, the passband and wideband values of k_P, the ratio of maximum displacements for passband- and wideband-drive conditions, and the degree to which the driving signal spectrum may extend below system cutoff before the displacement exceeds the passband maximum (see Fig. 17).

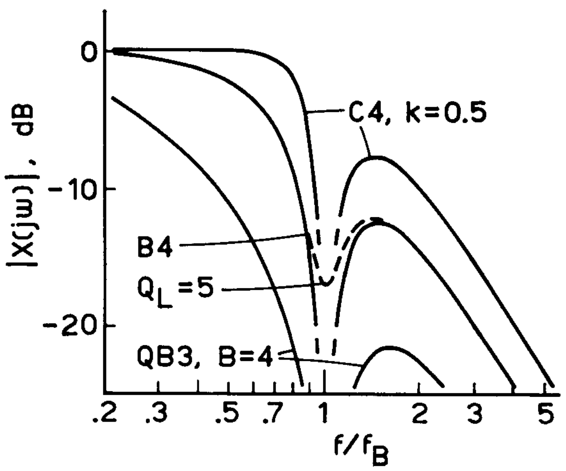

Fig. 17. Normalized diaphragm displacement of vented-box system driver as a function of normalized frequency for several typical alignments (from simulator).

With this value of k_P, Eq. (39) becomes

$$P_{AR(VB)} = 3.0 f_3{}^4 V_D{}^2. \quad (41)$$

This relationship is generally applicable to all vented-box alignments for which the system passband includes the major components of the program signal spectrum. Whenever the signal and alignment properties are accurately known, a more exact relationship may be obtained with the help of Fig. 18 or by using Eq. (38) directly.

Power Output, Cutoff Frequency, and Displacement Volume

Eq. (41) is illustrated in Fig. 19. P_{AR} is expressed in both watts (left scale) and equivalent sound pressure

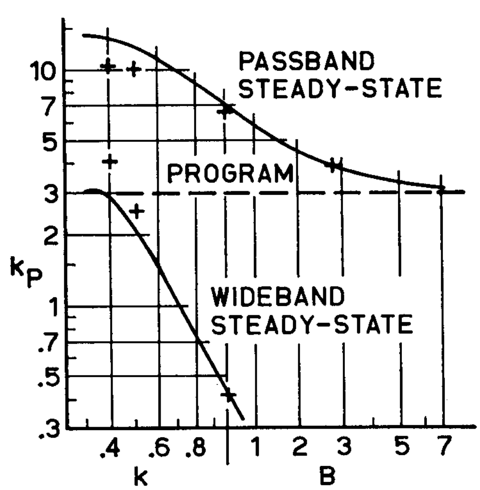

Fig. 18. Power rating constant k_P for vented-box loudspeaker system as a function of response shape. Solid lines are for lossless systems; crosses represent systems with $Q_L = 5$.

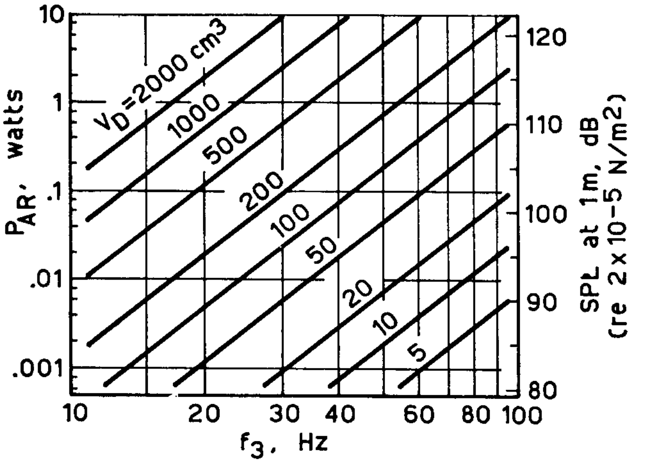

Fig. 19. Relationship between cutoff frequency, driver displacement volume, and rated acoustic power for a vented-box loudspeaker system operated on program material.

level (SPL) at 1 meter [3, p. 14] for 2π-steradian free-field radiation conditions (right scale). This is plotted as a function of f_3 for various values of V_D (note 1 $cm^3 = 10^{-6} m^3$). The SPL at 1 meter given on the right-hand scale is a rough indication of the SPL produced in the reverberant field of an average listening room for a radiated acoustic power given by the left-hand scale [3, p. 318]. For particular listening environments such as large halls, the reference just cited gives methods for computing the acoustic power required to obtain a specified SPL.

Fig. 19 represents the approximate physical large-signal limitation of vented-box system design. It may be used to determine the maximum performance tradeoffs (P_{AR} versus f_3) for a given voice-coil/suspension design or to find the minimum value of V_D which is required to meet a given specification of f_3 and P_{AR}.

Power ratings calculated from Eq. (41) or Fig. 19 apply only for "typical" program material which does not drive the system hard at frequencies below cutoff. For other circumstances the applicable rating may be higher or lower. Even where the condition of passband drive is met with regard to the intended program material, the vented-box system is clearly vulnerable to extraneous signals such as turntable rumble and subsonic control tones. These normally inaudible signals may produce audible harmonics or cause noticeable modulation distortion [21]. In cases where such signals are particularly troublesome and cannot otherwise be eliminated, the use of a closed-box design or one of the higher order amplifier-assisted vented-box alignments described by Thiele [10], [20] may provide relief.

Electrical Power Rating

The displacement-limited electrical power rating P_{ER} of the vented-box system is obtained by dividing the acoustic power rating Eq. (38) by the system reference efficiency Eq. (25). Thus,

$$P_{ER(VB)} = \frac{P_{AR(VB)}}{\eta_0} = \pi \rho_0 c^2 \frac{f_S Q_{ES}}{V_{AS}} \cdot \frac{V_D^2}{|X(j\omega)|_{\max}^2}. \quad (42)$$

This rating is subject to the same adjustments for program material as used above. Its dependence on the performance factors already discussed is easily observed from the form obtained by dividing Eq. (39) by Eq. (26):

$$P_{ER} = \frac{k_P}{k_\eta} f_3 \frac{V_D^2}{V_B}. \quad (43)$$

In practice, the values of P_{AR} and η_0 are much more important; these would normally be specified or calculated first. P_{ER} is then obtained directly from these numbers as indicated by Eq. (42). P_{ER} describes only the amount of nominal power which may be absorbed from an amplifier if thermal design of the voice-coil permits. It gives no indication of acoustic performance unless reference efficiency is known.

Enclosure and driver losses reduce η_0 without much effect on P_{AR} and thus lead to a higher value of P_{ER}. Driver displacement nonlinearity for large signals also has the effect of reducing efficiency at high levels, i.e., increasing the electrical input required to actually reach the driver displacement limit. In both cases, the extra input power is only dissipated as heat.

7. PARAMETER MEASUREMENT

The direct dependence of system performance characteristics on system parameters provides a simple means of assessing or predicting loudspeaker system performance from a knowledge of these parameters. The important small-signal parameters can be found with satisfactory accuracy from measurement of the voice-coil impedance of the system and its driver.

The voice-coil impedance function of the vented-box system is given by Eq. (16). A plot of the steady-state magnitude $|Z_{VC}(j\omega)|$ of this function against frequency has the shape illustrated in Fig. 20; the measured impedance curve of a practical vented-box system has this same characteristic shape.

The impedance magnitude plot of Fig. 20 has a minimum at a frequency near f_B (labeled f_M) where the impedance magnitude is somewhat greater than R_E. The additional resistance is contributed primarily by enclosure losses and is designated R_{BM} on the plot axis. There are two maxima in the impedance plot, located at frequencies below and above f_M. These are labeled f_L and f_H. At these frequencies, the magnitudes of the impedance maxima depend on both driver losses and enclosure circuit losses and are seldom equal.

Where only normal enclosure losses are present, the basic system parameters and the total enclosures loss Q_B may be found with satisfactory accuracy using the method developed by Thiele in [10]. The indicated value of Q_B may then be used to check the measurement approximations. Thiele's method is based on an initial assumption of negligible enclosure losses and may be summarized as follows. The relationships are derived in Appendix 2.

1) Measure the three frequencies f_L, f_M, and f_H where the impedance magnitude is maximum or minimum. The accurate identification of these frequencies may be aided by measuring the impedance phase; if this passes through zero at the appropriate maximum or minimum, the frequency of zero phase (which may be located with high precision) may be taken as the center of the maximum or minimum. However, if zero phase is not closely coincident with maximum or minimum magnitude, as may occur for moderate to high enclosure losses, the frequency of actual maximum or minimum impedance mag-

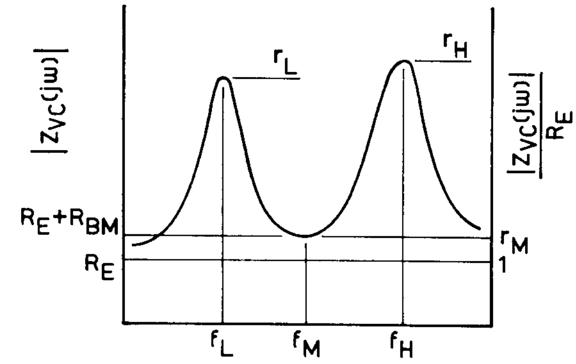

Fig. 20. Voice-coil impedance magnitude of vented-box loudspeaker system as a function of frequency.

nitude must be located as carefully as possible. Experience with many systems and experiments with the analog circuit simulator have shown that where the frequencies of zero phase and maximum or minimum magnitude do not coincide, the latter always provide more accurate values of the system parameters. Bypass any crossover networks for this measurement, and keep the measuring signal small enough so that both voltage and current signals are undistorted sinusoids. For the following calculations, assume that $f_B = f_M$.[2]

2) Calculate f_{SB}, the resonance frequency of the driver for the air-load mass presented by the enclosure, from the relationship

$$f_{SB} = \frac{f_L f_H}{f_B}. \tag{44}$$

3) Calculate the compliance ratio α from the relationship

$$\alpha = \frac{(f_H + f_B)(f_H - f_B)(f_B + f_L)(f_B - f_L)}{f_H^2 f_L^2}. \tag{45}$$

If the enclosure contains little or no lining material, the driver compliance equivalent volume V_{AS} may be calculated in terms of the enclosure net volume V_B. The relationship is, from Eqs. (9), (10), and (33),

$$V_{AS} = \alpha V_B. \tag{46}$$

4) Calculate the tuning ratio h from

$$h = f_B / f_{SB}. \tag{47}$$

5) Remove the driver from the enclosure, measure the driver parameters f_S, Q_{MS}, and Q_{ES} by the method of [12, Appendix],[3] and correct the driver Q values if necessary to correspond to the driver resonance frequency in the enclosure. This is done by multiplying the measured values of Q_{MS} and Q_{ES} by the ratio f_S/f_{SB}, where f_S is the resonance frequency for which Q_{MS} and Q_{ES} have been measured and f_{SB} is the resonance frequency in the enclosure found from Eq. (44). Usually if the driver parameters are measured on a test baffle of suitable size, the two resonance frequencies are almost identical and the correction is not required.

6) Calculate Q_{TS} from

$$Q_{TS} = \frac{Q_{ES} Q_{MS}}{Q_{ES} + Q_{MS}}. \tag{31}$$

7) Measure the minimum system impedance magnitude $R_E + R_{BM}$ at f_M and calculate

$$r_M = \frac{R_E + R_{BM}}{R_E}. \tag{48}$$

Then, using the corrected values of Q_{ES} and Q_{MS} obtained above, determine the total enclosure loss Q_B from the relationship

$$Q_B = \frac{h}{\alpha} \left[\frac{1}{Q_{ES}(r_M - 1)} - \frac{1}{Q_{MS}} \right]. \tag{49}$$

The term $1/Q_{MS}$ can usually be neglected.

8) The accuracy of the approximation $f_B \approx f_M$ on which the above method is based may be checked by calculating the approximate error introduced by the enclosure losses. Assuming that leakage losses are dominant in effect and that f_M is the measured frequency of zero phase, the error correction factor is

$$\frac{f_B}{f_M} = \sqrt{\frac{\alpha Q_B^2 - h^2}{\alpha Q_B^2 - 1}}. \tag{50}$$

This factor is usually quite close to unity. If it is significantly different from unity, it may be used to correct the value of f_B used in the above calculations to obtain better accuracy in the calculated parameter values.

The estimation or measurement of driver large-signal parameters is discussed in [22, Sec. 6].

With values determined for all important system parameters, system performance may be determined from the relationships given in earlier sections. The system frequency response may be calculated manually or using a digital computer but is most easily obtained by introducing the system parameters to an analog circuit simulator. The design of a simple simulator suitable for this purpose will be published in the future.

8. VENT REQUIREMENTS

The vent of a vented-box system must provide the necessary small-signal enclosure resonance frequency f_B; it must also provide the maximum required large-signal volume velocity without excessive losses or generation of spurious noises.

The second requirement can be satisfied by adjusting the vent area to a value which prevents the vent air velocity from exceeding a specified limit. An experimentally determined limit which avoids excessive noise generation is about 5% of the velocity of sound, provided that the inside of the vent is smooth and that the edges are rounded off with a reasonable radius. This velocity

[2] In [32, Appendix 4] Benson shows that if a large voice-coil inductance (or crossover inductance) is present, the measured value of f_M is lower than the true value of f_B, while f_L and f_H are negligibly affected. A much better approximation to f_B is obtained by carefully blocking the vent aperture and measuring the resonance frequency f_C of the resulting closed-box system [22]. Then, from [32, eq. (A4-6)], $f_B = (f_L^2 + f_H^2 - f_C^2)^{1/2}$. Because this relationship is true, f_C can be used directly in place of f_B in Eq. (45) to determine the system compliance ratio.

[3] Again, if the driver voice-coil inductance is large, Benson [32, Appendix 2] shows that the accuracy of determination of the Q values is improved if f_S in [12, eq. (17)] is replaced by the expression $\sqrt{f_1 f_2}$.

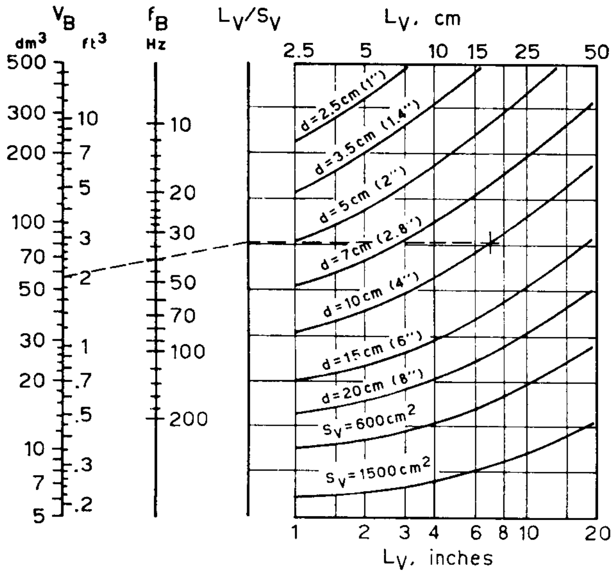

Fig. 21. Nomogram and chart for design of ducted vents.

limitation generally ensures acceptable losses as well, provided that the vent is not unduly obstructed.

The alignment, response, and power rating data of this paper combine to yield a relationship between vent area and maximum vent velocity for any given system. For program power ratings this relationship reduces to a simple approximate formula for vent area which limits the peak vent velocity, at maximum rated power input and at the frequency of maximum vent velocity, to 4½ % of the velocity of sound. This formula, which is accurate within ±10% for the entire C4–B4–QB3 range of alignments, is

$$S_V \geqq 0.8\, f_B\, V_D \tag{51}$$

or

$$d_V \geqq (f_B V_D)^{1/2} \tag{52}$$

where S_V is the area of the vent in m² or d_V is the diameter of a circular vent in meters; V_D must be expressed in m³ and f_B in Hz. Because the noise generated depends on factors other than velocity (e.g., edge roughness), and because the annoyance caused by vent noise is subjective, this formula should be regarded as a general guide only, not as a rigid rule.

Once the area of the vent is determined, the length must be adjusted to satisfy the first requirement, i.e., correct enclosure tuning. There are many popular formulas and nomograms for doing this. Using Thiele's formulas [10, eqs. (60)–(65)], the nomogram and chart of Fig. 21 were constructed to simplify the calculation process for ducted vents.

To use Fig. 21, lay a straight-edge through the enclosure volume on the V_B line and the desired resonance frequency on the f_B line and find the intersection with the L_V/S_V line. This is illustrated on the figure with lightly dashed lines for $V_B = 57$ dm³ (2 ft³) and $f_B = 40$ Hz. Next, move horizontally to the right from this intersection point until a curve is reached on the chart which corresponds to the required minimum size determined from Eqs. (51) or (52). The intersection of the horizontal projection with this curve indicates on the horizontal scale the required duct length L_V for a vent of the prescribed size. For the example illustrated, if the minimum duct diameter is 100 mm (4 inches), the required length is about 175 mm (7 inches). End corrections for one open end and one flanged end are included in the construction of the chart. For intermediate vent areas the chart may be interpolated graphically.

For some proposed systems a satisfactory vent design cannot be found. This is particularly the case for small enclosures when a low value of f_B is desired. Also, tubular vents for which the length is much greater than the diameter tend to act as half-wave resonant pipes, and any noise generated at the edge is selectively amplified. In these cases it is better to use a drone cone or passive radiator in place of the vent [2], [23]. Systems of this type will be discussed in a later paper.

9. DIAPHRAGM–VENT MUTUAL COUPLING

Mutual Coupling Magnitude

The acoustical analogous circuit of a lossless vented-box system, modified to include mutual coupling [2], [6], is presented in Fig. 22. The mutual coupling components are inside the dashed lines. (The mutual coupling resistance [2] is equal to the radiation load resistance and is therefore neglected [4], [12].)

The acoustic mutual coupling mass M_{AM} has a maximum magnitude when the diaphragm–vent spacing is a minimum. A practical minimum spacing between the centers of diaphragm and vent is about $1.5a$, where a is the diaphragm radius. Using this value, and assuming radiation conditions of a 2π-steradian free field, the maximum value of M_{AM} is about $0.13/a$ [2]. This value is reduced for a 4π-steradian free-field load [6].

For a 12-inch driver with an effective diaphragm radius of 0.12 m, the mechanical equivalent M_{MM} of the acoustic mass M_{AM} has a maximum value of 2.2g. The mechanical diaphragm mass M_{MD} for 12-inch drivers varies from about 20g for older types used in large enclosures to more than 100g for newer types designed for use in compact enclosures. Thus the mutual coupling mass may have a magnitude of from 2 to 8% of the total moving mass of the driver when all of the diaphragm air-load mass is accounted for [3, pp. 216-217].

The effect of these values of mutual coupling mass was investigated using the analog circuit simulator. A "lossless" system aligned for a B4 response was compared to the same circuit with the driver and vent masses reduced by the amount of the mutual coupling

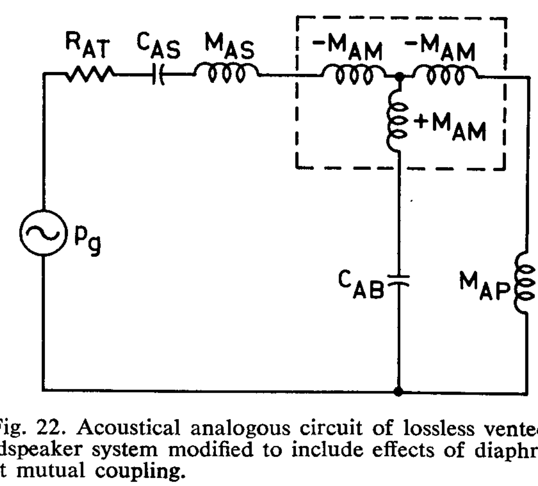

Fig. 22. Acoustical analogous circuit of lossless vented-box loudspeaker system modified to include effects of diaphragm–vent mutual coupling.

mass and the same amount of mass then introduced into the enclosure branch in agreement with Fig. 22.

Effect on Response

The effect of 2% mutual coupling mass on the frequency response could not be observed. The effect of 4% mutual coupling mass could be observed but was hardly worth taking into account. With 8% mutual coupling mass, the cutoff frequency was lowered by about 5% and the corner of the response curve became sharper as described by Locanthi. Similar effects were observed for other alignments.

It would appear that in most cases the effect of mutual coupling on system response is negligible. Only when a driver with a light diaphragm is mounted very close to the vent is the effect on response significant. It then amounts to a slight alignment shift with a very small decrease in cutoff frequency.

Effect on Measurement

Mutual coupling alters the location of the frequencies f_L and f_H of Fig. 20 but does not affect the location of f_M [2]. The shift in f_L and f_H toward each other upsets the calculation of the compliance ratio from Eq. (45), giving a value lower than the true value.

This suggests that if it is desired to measure the true compliance ratio of a system for which the magnitude of mutual coupling is very high, the vent should be blocked and the compliance ratio measured by the closed-box method described in [22]. However, if the parameters of a system are being measured only to evaluate the response of the system, the presence of mutual coupling may be ignored. Experiments on the analog circuit simulator show that the response of a system having the false calculated value of α and no mutual coupling is essentially identical to that of the actual system with its mutual coupling.

10. DISCUSSION

Features of Vented-Box Loudspeaker Systems

The vented-box loudspeaker system acts as a fourth-order high-pass filter. This basic fact determines the available range of amplitude, phase, and transient response characteristics. By suitable choice of parameters, the response may be varied from that of an extreme C4 alignment with passband ripple and very abrupt cutoff to that of an extreme QB3 alignment for which the response is effectively third order. The cost of the gentler cutoff slope and improved transient response of the QB3 alignment is a reduced value of the system efficiency factor $k_{\eta(G)}$, although this reduction is relatively small for real systems with typical enclosure losses. A further sacrifice in the value of this efficiency factor permits the use of SC4 alignments for which the transient response may approach that of a second-order system.

Perhaps the most important feature of the vented-box loudspeaker system is the very modest diaphragm excursion required at frequencies near the enclosure resonance frequency f_B. This feature is responsible for the relatively high displacement-limited power capacity of the system; it also helps to maintain low values of nonlinear distortion and modulation distortion [21].

The "misalignment" curves of Figs. 7 and 8 indicate the necessity for careful alignment of the vented-box system. The plurality of variables makes it very difficult to obtain optimum adjustment by trial-and-error methods, although simulators or computers may be used to speed up the process.

Comparison of Vented-Box and Closed-Box Systems

Most direct-radiator loudspeaker systems use or are based on either the closed-box or vented-box principle. It is therefore of interest to compare these two fundamental systems, and to observe the advantages and disadvantages of each.

One obvious difference is that the vented-box system is more complex, i.e., has more variables requiring adjustment, than the closed-box system. This difference means that satisfactory designs are relatively easier to obtain with the closed-box system and probably accounts for much of the popularity of this system.

The performance relationships derived in this paper for the vented-box system and in [22] for the closed-box system make possible a number of interesting quantitative comparisons which follow.

Response

The response of the vented-box system can typically be adjusted from fourth-order Chebyshev to quasi-third-order maximally flat; that of the closed-box system can be adjusted from second-order Chebyshev to an overdamped second-order condition approaching first-order behavior. This means the closed-box system is nominally capable of better transient response, but Thiele [10, Sec. 13] suggests the differences among correctly adjusted systems of both types are likely to be inaudible.

Efficiency

A comparison of Fig. 16 or Eq. (35) with [22, Fig. 7 or eq. (28)] reveals that the vented-box system has a maximum theoretical value of k_η which is 2.9 dB greater than that of the closed-box system. Both systems suffer to a similar degree from the combined effects of driver and enclosure losses, and both must sacrifice efficiency to make use of alignments which have better transient response than the maximum-efficiency alignment (see Fig. 15 and [22, Fig. 8]).

Typical values of k_η for practical designs still favor the vented-box system by about 3 dB. The larger efficiency constant may be used to obtain higher efficiency for the same size and cutoff frequency, a smaller enclosure size for the same efficiency and cutoff frequency, a lower cutoff frequency for the same size and efficiency, or any proportional combination of these [22, Sec. 4].

Power Capacity

The reduced diaphragm excursion of the vented-box system near the enclosure resonance frequency gives the vented-box system a higher power rating constant k_P than a comparable closed-box system. Comparing Eq. (41) with [22, eq. (35)], the advantage in favor of the vented-box system for average program material is a factor of 3.5, or 5½ dB; for particular applications it may be larger.

However, except for the extreme C4 alignments, **this** advantage is limited to the passband; at frequencies well below cutoff, the vented-box system has a higher relative displacement sensitivity and is therefore more vulnerable to turntable rumble and other subsonic signals.

Driver Requirements

For a given specification of enclosure size and system cutoff frequency, the driver of a vented-box system requires a lighter diaphragm and greater electromagnetic coupling in the magnet-voice-coil assembly compared to the same size driver used in a closed-box system (cf. example of Section 12, Part III, with that of [22, Sec. 10]). These differences are physically consistent with the higher efficiency of the vented-box system. However, for equivalent acoustic power rating, the peak displacement volume V_D and therefore the peak diaphragm displacement $x_{\max}$ is substantially smaller for the vented-box driver. Because $x_{\max}$ determines required voice-coil overhang, total amount of magnetic material required for the vented-box driver is not necessarily greater.

The closed-box system driver must have high compliance relative to the enclosure if maximum efficiency is to be achieved. While high driver compliance may be beneficial to the vented-box design in terms of transient response, it is not necessary. In fact, a maximum efficiency constant is obtained for the vented-box system with a relatively low value of compliance ratio, and maximum displacement-limited power capacity is obtained with very low values.

Enclosure Size

It is stated above that the larger value of k_η for the vented-box system may be used to obtain a size advantage, i.e., the enclosure may be smaller than that of a closed-box system having the same efficiency and cutoff frequency. Then, despite the smaller enclosure size, if the drivers have equal peak displacement volume, the larger value of k_P for the vented-box system must give a higher acoustic power rating.

This is theoretically correct, but it is practically possible only so long as V_B remains very much larger than the maximum volume displacement required. The maximum air-volume displacement from the enclosure of a vented-box system is larger than V_D because of the contribution of the vent; if this *total* volume displacement exceeds a small percentage of V_B, the compression of air within the enclosure becomes nonlinear to such a degree that the system must produce distortion regardless of the driver linearity [3, p. 274].

In most practical loudspeaker system designs, V_D is indeed very much smaller than V_B, and power capacity is not limited by enclosure size. However, if extreme miniaturization is attempted or if a driver is specifically designed to obtain a very large value of V_D, this limitation may become relevant.

It is important to realize that two direct-radiator loudspeaker systems operated at the same frequency and *acoustic* power level have the same total output volume velocity and displacement regardless of the type of system [12, eq. (2)]. Thus for both closed-box and vented-box systems, adequate enclosure volume is essential to the production of high acoustic output power with low distortion at low frequencies. Some size reduction is possible for closed-box systems if motional feedback is used to control distortion [24], but this technique can be difficult to apply successfully [25].

Typical System Performance

A sampling of commercial vented-box loudspeaker systems was tested in late 1969 by measuring the system parameters as described in Section 7 and programming these into the analog simulator to obtain the system response. For a few systems, the response obtained in this way was checked by indirect measurement [26].

Most of the samples tested fitted into the same two categories previously described for closed-box systems [22, Sec. 8]: systems with a volume of 40 dm^3 (1.5 ft^3) or more, a cutoff frequency of 50 Hz or lower, and relatively flat response; and smaller systems with a cutoff frequency above 50 Hz and several decibels of peaking in the response above cutoff. There was, however, a greater tendency for these two categories to overlap.

While most of the systems were probably designed by traditional trial-and-error methods, the general objectives of system manufacturers appear remarkably consistent. The larger systems fulfill the traditional requirements for high-fidelity reproduction, while the smaller systems suit the apparent requirements of the mass marketplace.

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Vented-Box Loudspeaker Systems Part III: Synthesis

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The analytical relationships developed in Parts I and II which relate the performance characteristics of the vented-box loudspeaker system to the basic parameters of its components make possible the straightforward design of loudspeaker systems meeting specific performance goals. A set of desired system performance specifications may be checked for realizability and then used to determine the required physical properties of all the system components. The most suitable enclosure design for a particular driver may also be readily determined.

Editor's Note: Part I of Vented-Box Loudspeaker Systems appeared in the June issue and Part II in July/August.

11. SYSTEM SYNTHESIS

System-Component Relationships

The relationships between response and system parameter adjustment are given in Part I by Figs. 6 and 9–13 for the "flat" C4–B4–QB3 alignments. Enclosure losses cannot be known exactly in advance but can be predicted from experience. For example, for numerous commercial systems and laboratory enclosures in the range of 25–100 dm^3 (1–4 ft^3) measured in the course of this research, the most commonly measured values of Q_B are between 5 and 10 with a general tendency for Q_B to fall with increasing enclosure volume.

For enclosures of moderate size, the assumption of an equivalent Q_L value of 7 is a very satisfactory starting point for design purposes. In this case Fig. 11 is used to represent the basic relationships between driver parameters, system parameters, and system response. If a higher or lower value of Q_B is expected with some confidence, one of the other figures is used.

The appropriate alignment and response relationships (Fig. 11 or otherwise) and the efficiency, power capacity, and vent design relationships established in Parts I and II permit the design of vented-box systems in complete detail. Procedures are described and illustrated below for two important cases, design of an enclosure to suit a particular driver and design of a complete system starting from required performance specifications.

Design with a Given Driver

The design of an enclosure to suit a given driver

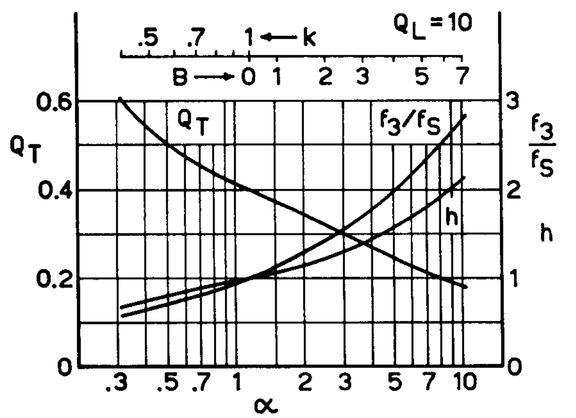

Fig. 10. Alignment chart for vented-box systems with $Q_B = Q_L = 10$.

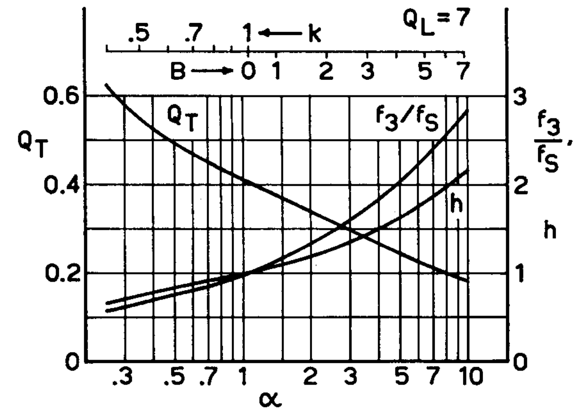

Fig. 11. Alignment chart for vented-box systems with $Q_B = Q_L = 7$.

starts with a knowledge of the driver small-signal parameters f_S, Q_{TS}, and V_{AS}; f_S and Q_{TS} must be adjusted if necessary to correspond to enclosure mounting conditions. If these parameters are not already known, they may be measured by the methods given in [10] or [12] using a standard baffle to provide air-mass loading as for an enclosure (see also Section 7 in Part II of the present paper, including Footnote 3).

The value of Q_{TS} is of primary importance. If the loudspeaker system is to be used with a modern amplifier having very low output (Thevenin) resistance, then Q_T for the system will be equal to Q_{TS} for the driver. From Figs. 6 and 9–13 it is clear that Q_T must be no larger than about 0.6 for successful application in a vented enclosure.

If Q_{TS} has a reasonable value, then the optimum value of α for a system using the driver is found from, say, Fig. 11 by locating the measured value of Q_{TS} on the Q_T curve in the figure and observing the corresponding value of α on the abscissa. This value of α then determines the optimum value of V_B using Eq. (46). It also determines the required value of h (and therefore f_B) and the corresponding value of f_3 for the system as indicated on the same figure. If the resulting system design is not acceptable (f_3 too high, V_B too large, etc.), then it is probable that the driver is not suitable for use in a vented-box system.

The design process may alternatively be begun by selecting an enclosure size V_B which suits aesthetic or architectural requirements. This determines α and hence the required enclosure tuning f_B, the required value of Q_T, and the resulting cutoff frequency f_3. If the value of f_3 is not satisfactory, then the driver and the enclosure size chosen are not compatible. If f_3 is satisfactory but the required Q_T is very different from Q_{TS}, it may be possible to use the driver as discussed below.

There are limited ways of salvaging a driver having unsatisfactory parameter values. If the value of Q_{TS} is too high to fit an alignment which is otherwise desirable in terms of enclosure size and bandwidth, an acoustically resistive material such as bonded acetate fiber may be stretched over the rear of the driver frame to reduce the effective value of Q_{MS}, thus lowering Q_{TS} [17], [27]. The correct amount of resistive material is determined experimentally by remeasurement of Q_{TS} as material is added. Q_T may also be reduced by using a negative value of amplifier output resistance R_g [10, Sec. 12], [28] to produce a low value of Q_E, where [12, eq. (21)]

$$Q_E = Q_{ES}\frac{R_g + R_E}{R_E} \tag{53}$$

because in this case [12, eq. (22)]

$$Q_T = Q_E Q_{MS}/(Q_E + Q_{MS}). \tag{54}$$

Both methods reduce Q_T without changing Q_{ES}; thus the value of $k_{\eta(Q)}$ from Eq. (29), and therefore η_0 for the system, will be lower than could be achieved by altering the magnet design to reduce Q_{ES} directly.

Sometimes the value of Q_{TS} is found to be undesirably low. This may be remedied by placing a resistor in series with the voice coil to increase R_E and therefore Q_{ES} or by using a positive value of R_g to increase Q_E.

If the driver proves satisfactory and an acceptable system design is found, the system reference efficiency is calculated from the basic driver parameters using Eq. (25). The approximate displacement-limited acoustic power rating of the system is computed from Eq. (41) if V_D is known. V_D usually can be evaluated as described in [22, Sec. 6]. The approximate displacement-limited input power rating is then found by dividing the acoustic power rating by the reference efficiency as indicated by Eq. (42). The vent design is carried out in accordance with Section 8 of Part II.

Example of Design with a Given Driver

The following small-signal parameters were measured for an 8-inch wide-range driver manufactured in the United States:

$$f_S = 33 \text{ Hz}$$
$$Q_{MS} = 2.0$$
$$Q_{ES} = 0.45$$
$$V_{AS} = 57 \text{ dm}^3 \text{ (2 ft}^3\text{)}.$$

The large-signal characteristics specified by the manufacturer are as follows.

1) "Total linear excursion of one-half inch." From this, $x_{max} = 6$ mm, and, assuming a typical effective diaphragm radius of 0.08 m,

$$V_D = 120 \text{ cm}^3.$$

2) "Power capacity 25 watts program material." From this it is assumed that for program material the thermal capacity of the driver is adequate for operation with amplifiers of up to 25-watt continuous rating.

By calculation from Eqs. (31) and (25).

$$Q_{TS} = 0.37$$
$$\eta_0 = 0.44\% .$$

Assuming that the amplifier to be used with the system has negligible Thevenin output resistance, Q_T for the system will be 0.37. Taking $Q_B = 7$ initially, Fig. 11 indicates that the enclosure volume will be relatively small; a more likely value of Q_B is thus about 10. Using Fig. 10 then, a QB3 response with $B = 1.0$ can be obtained for which the system parameters are

$$\alpha = 1.55$$
$$h = 1.07$$
$$f_3/f_S = 1.16.$$

Thus the required enclosure volume is

$$V_B = V_{AS}/\alpha = 37 \text{ dm}^3 \text{ } (1.3 \text{ ft}^3).$$

The enclosure must be tuned to

$$f_B = hf_S = 35 \text{ Hz}$$

and the system cutoff frequency is

$$f_3 = 38 \text{ Hz}.$$

From Eq. (41) the displacement-limited program acoustic power rating of the system is

$$P_{AR} = 3.0\, f_3{}^4\, V_D{}^2 = 90 \text{ mW}.$$

The corresponding displacement-limited program input power rating is

$$P_{ER} = P_{AR}/\eta_0 = 20 \text{ W}.$$

Because this is less than the manufacturer's input power rating, it should be quite safe to operate the system with an amplifier having a continuous power rating of 20 watts.

From Eq. (52) the minimum diameter of a tubular vent is $(V_D f_B)^{1/2}$ or 65 mm (2.6 inches). From Fig. 21, the required vent length is 175 mm (7 inches) for a tubing of this diameter.

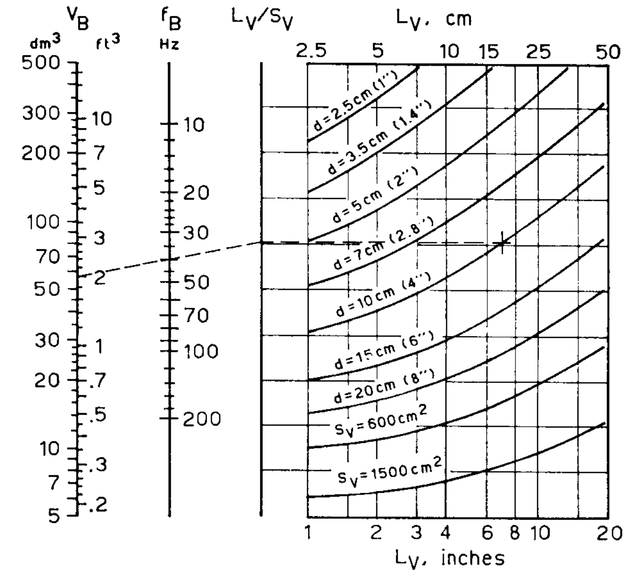

Fig. 21. Nomogram and chart for design of ducted vents.

Design from Specfications

The important performance specifications of a loudspeaker system include frequency response, efficiency, power capacity, and enclosure size. The complexity of the vented-box system makes control of all these specifications quite difficult when traditional trial-and-error design techniques are used. In contrast, the analytical relationships developed in this paper make possible the direct synthesis of a vented-box system to meet any physically realizable set of small-signal and large-signal specifications and even provide a check on realizability before design is begun.[4]

Specification of system frequency response basically amounts to specification of an alignment type and a cutoff frequency f_3. While the emphasis in this paper is on the "flat" C4–B4–QB3 alignments, any other desired alignment may be specified, e.g., the degenerated Chebyshev type 2 (DT2) alignment used by Nomura which provides passband peaking [11]. Appendix 1 shows how the required system alignment parameters may be calculated from the polynomial coefficients of any desired alignment based on the assumed or expected value of Q_B. For any alignment in the C4–B4–QB3 range, the necessary alignment data are provided in Figs. 9–13. The frequency response specification thus fixes the values of the parameters α, Q_T, f_S, and f_B.

For a specified frequency response, the designer may specify also the enclosure size or the reference efficiency; but he may not specify both unless the values satisfy the realizability requirements of Section 4. If the enclosure volume V_B is specified, the required driver compliance is then

$$V_{AS} = \alpha V_B. \qquad (46)$$

The required value of the driver parameter Q_{ES} is found from the required value of Q_T by allowing for reasonable values of R_g (typically zero) and Q_{MS} (typically 5, but varies greatly depending on the amount of mechanical damping deliberately added to the suspension to suppress higher frequency resonances). The system efficiency is then calculated from Eq. (25).

The power capacity of the system may be specified in terms of either P_{ER} or P_{AR}, but not both unless the values agree with the attainable system efficiency. It is possible to specify both independently only if neither V_B nor η_0 are separately specified; then the required value of η_0 is given by the ratio of P_{AR} to P_{ER}, and the required enclosure volume which will provide this efficiency for the specified frequency response is found from Eqs. (26) and (28) using values of $k_{\eta(Q)}$ and $k_{\eta(G)}$ obtained from Eq. (32) and Fig. 15 and based on the estimated or expected values of Q_{MS} and Q_B.

Assuming that V_B and P_{AR} are specified and that η_0 has been determined from Eq. (25), P_{ER} is given by

$$P_{ER} = P_{AR}/\eta_0. \qquad (42)$$

The required value of V_D for the driver is found from

[4] See [32, Sec. 5 and 6] for an extensive discussion of the principles of system small-signal response synthesis.

Eq. (41) using the given values of f_3 and P_{AR}. Check that $V_D << V_B$. The thermally limited maximum input power rating of the driver $P_{E(\max)}$ must be not less than the value of P_{ER} divided by the peak-to-average power ratio of the program material to be reproduced.

The vent is designed so that the area S_V satisfies Eq. (51) and the effective length-to-area ratio gives the required f_B in combination with the enclosure volume V_B as determined from Fig. 21.

The driver is completely specified by the parameters calculated above and may be designed by the method given in Section 12.

Example of System Design from Specifications

A loudspeaker system to be used with an amplifier having very low output resistance must meet the following specifications:

$$f_3 = 40 \text{ Hz}$$
$$\text{Response} = \text{B4}$$
$$V_B = 57 \text{ dm}^3 \ (2 \text{ ft}^3)$$
$$P_{AR} = 0.25 \text{ W program peaks; expected peak-to-average power ratio 5 dB.}$$

It is assumed that the enclosure losses will correspond to $Q_B = Q_L = 7$ and that the driver mechanical losses will correspond to $Q_{MS} = 5$.

Using Fig. 11, the B4 response is located at a compliance ratio of

$$\alpha = 1.06$$

for which the required system parameters are

$$h = 1.00$$
$$f_3/f_S = 1.00$$
$$Q_T = 0.40.$$

Therefore the required driver parameters are

$$V_{AS} = 60 \text{ dm}^3 \ (2.1 \text{ ft}^3)$$
$$f_S = 40 \text{ Hz}$$
$$Q_{TS} = 0.40$$

and the required enclosure tuning is

$$f_B = 40 \text{ Hz}.$$

Taking $Q_{MS} = 5$ and using Eq. (31),

$$Q_{ES} = 0.44.$$

From Eq. (25) the reference efficiency of the system is then

$$\eta_0 = 0.84\%$$

and from Eq. (42) the displacement-limited electrical power rating is

$$P_{ER} = 30 \text{ W}.$$

This requires that the system amplifier have a continuous power rating of at least 30 watts. For the 5-dB expected peak-to-average power ratio of the program material, the thermal rating $P_{E(\max)}$ of the driver must be at least 9.5 watts [22, Sec. 5].

From Eq. (41), the displacement volume of the driver must be

$$V_D = 180 \text{ cm}^3.$$

This is only about 0.3% of V_B. Then, from Eq. (52), a tubular vent should be at least 85 mm (3.4 inches) in diameter. From Fig. 21, the length should be 115 mm (4.5 inches) for a tubing of this diameter.

12. DRIVER DESIGN

Driver Specification

The process of system design leads to specification of the required driver in terms of the basic design parameters f_S, Q_{ES}, V_{AS}, V_D, and $P_{E(\max)}$. To complete the physical specification of the driver, the arbitrary physical parameters S_D and R_E must be selected and the resulting mechanical parameters calculated. This process is described in [22, Sec. 10] and is illustrated by the example below.

Example of Driver Design

The basic design parameters of the driver required for the system in the example of the previous section are

$$f_S = 40 \text{ Hz}$$
$$Q_{ES} = 0.44$$
$$V_{AS} = 60 \text{ dm}^3$$
$$V_D = 180 \text{ cm}^3$$
$$P_{E(\max)} = 9.5 \text{ W}.$$

These specifications could be met by drivers of 8–15-inch advertized diameter [15].

Choosing a 12-inch driver, the effective diaphragm radius a will be approximately 0.12 m, giving

$$S_D = 4.5 \times 10^{-2} \text{ m}^2$$

and

$$S_D^2 = 2.0 \times 10^{-3} \text{ m}^4.$$

The required mechanical compliance and mass of the driver are then [22, eqs. (61) and (62)]

$$C_{MS} = V_{AS}/(\rho_0 c^2 S_D^2) = 2.14 \times 10^{-4} \text{ m/N}$$
$$M_{MS} = 1/[(2\pi f_S)^2 C_{MS}] = 74 \text{ g}.$$

M_{MS} is the total moving mass including air loads. Assuming that the driver diaphragm occupies one third of the area of the front baffle of the enclosure and using [3, pp. 216-217] to evaluate the air loads, the mass of the voice coil and diaphragm alone is

$$M_{MD} = M_{MS} - (3.15a^3 + 0.65\pi\rho_0 a^3) = 64 \text{ g}.$$

The electromechanical damping resistance must be [22, eq. (64)]

$$B^2l^2/R_E = 2\pi f_S M_{MS}/Q_{ES} = 42 \text{ N}\cdot\text{s/m}.$$

For the popular 8Ω rating impedance, R_E is usually about 6.5 Ω. The required Bl product for such a driver is then

$$Bl = 16.5 \text{ T}\cdot\text{m}.$$

For the required displacement volume of 180 cm³, the peak linear displacement of the driver must be

$$x_{\max} = V_D/S_D = 4.0 \text{ mm}.$$

This is approximately the amount of voice-coil overhang required at each end of the magnetic gap. The total "throw" of the driver is then 8.0 mm (0.32 inch). This requirement presents no great difficulty so far as the design of the suspension is concerned.

The choice of a smaller driver diameter results in a lighter diaphragm and a less costly magnetic structure, but a greater peak displacement is then required, e.g., 9 mm (18-mm total throw) for an 8-inch driver.

The voice coil must be able to dissipate 9.5 watts nominal input power without damage.

13. DESIGN VERIFICATION

The suitability of a prototype driver designed in accordance with the above method may be checked by measuring the driver parameters as described in [12].

One of the driver parameters which is difficult to control in production is the mechanical compliance C_{MS}. Any shift in this compliance changes the measured values of both f_S and Q_{ES} as well as V_{AS}. Fortunately, system response is not critically sensitive to the value of C_{MS} so long as M_{MS} and B^2l^2/R_E have the correct values. Thus if the measured value of V_{AS} is not too far off its specified value, the driver will be satisfactory provided the quantities $f_S{}^2V_{AS}$ and f_S/Q_{ES}, which together indicate the effective moving mass and magnetic coupling, correspond to the same combinations of the specified parameters.

The effect of variations in C_{MS} on the response of a vented-box system is shown in Fig. 23 for a B4 alignment. The ±50% variation illustrated is larger than that commonly encountered. The relative effects are smaller for higher compliance ratios (i.e., QB3 alignments) and larger for lower compliance ratios (C4 alignments).[5]

The completed system may be checked by measuring its parameters as described in Section 7 and comparing these to the initial specifications. The actual system performance may also be verified by measurement in an anechoic environment or by an indirect method [26].

14. SPECIFICATIONS AND RATINGS

Drivers

The moving-coil or electrodynamic driver has long been the workhorse of the loudspeaker industry. However, system designers have not been fully aware of the importance or usefulness of a knowledge of the important fundamental parameters of these drivers. They have instead used trial-and-error design techniques and relied on acoustical measurements of a completed system to determine the performance characteristics of the system.

The most important message of this paper and those that have preceded it is that trial-and-error design techniques are not only wasteful but unnecessary. Design may be carried out by direct synthesis provided the system designer either knows the parameters of a given driver or can obtain a desired driver by specifying its parameters.

It is essential for a driver manufacturer to specify all the important parameters of a driver so that system designers can completely evaluate the small-signal and large-signal performance obtainable from that driver. In addition to the specific physical properties of diaphragm

[5] A very recent paper by Keele [33] contains exact calculations of the sensitivity factors of vented-box alignments to all important driver and system parameters. The sensitivity to driver compliance is shown to be extremely low compared to that for most other parameters over a wide range of alignments.

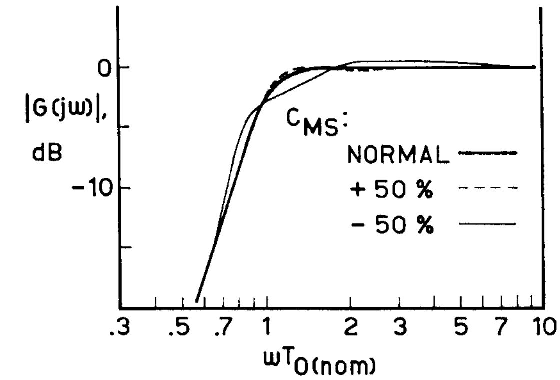

Fig. 23. Variation in frequency response of a B4-aligned vented-box system for changes in driver compliance C_{MS} of ±50% (from simulator).

size and voice-coil resistance (or rating impedance), the designer needs to know the values of the parameters f_S, Q_{ES}, Q_{MS}, V_{AS}, V_D, and $P_{E(max)}$. Conversely, where the designer needs a driver having particular values of these parameters, the driver manufacturer must be able to work from such specifications to produce the driver.

Because the basic design parameters above are directly related to the fundamental mechanical parameters such as M_{MD}, C_{MS}, B, and l, which the driver manufacturer has long used, there need be no difficulty in supplying these parameters. There is every likelihood that feedback from system designers will be helpful to driver manufacturers in improving their products, particularly in finding the best tradeoffs among response, efficiency, and power capacity requirements which can be obtained for a given cost.

Systems

Because the frequency response, reference efficiency, and displacement-limited power capacity of a vented-box loudspeaker system are all directly related to a relatively small number of easily measured system and driver parameters, there is every incentive for system manufacturers to provide complete data on these fundamental performance characteristics with the basic system specifications.

The theoretical relationships developed here refer to a standard radiation load of a 2π-steradian free field. This is only an approximation to average listening-room conditions [29], but ratings and specifications based on these relationships are of unquestionable value in comparing the expected performance of different systems in a particular application.

There is little doubt that buyers and users of loudspeaker systems would appreciate an increase in the amount of quantitative and directly comparable data supplied with such systems, especially in the categories of reference efficiency and acoustic power capacity.

15. CONCLUSION

The vented-box loudspeaker system has been popular for decades but has recently been shunned in favor of the more easily designed closed-box system.

The quantitative relationships presented in this paper make the design of vented-box systems a relatively simple task, despite the complexity of these systems.

They also indicate that the vented-box system has substantial advantages over the closed-box system in terms of the attainable values of the efficiency and power-rating constants, although these advantages are gained at the expense of transient response and immunity to subsonic signals.

As the design of vented-box systems becomes better understood, interest in these systems may be expected to increase again. This does not mean that the popularity of well-designed closed-box systems will diminish. The choice of one or the other will depend on the requirements of a particular application.

The ease with which the low-frequency performance of a loudspeaker system may be specified in terms of simply measured system parameters should encourage more complete specification by manufacturers of the important frequency response, reference efficiency, and power capacity characteristics of their products.

16. ACKNOWLEDGMENT

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Vented-Box Loudspeaker Systems Part IV: Appendices

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The appendices present a method of calculating the system parameters required to obtain a desired alignment defined by transfer-function polynomial coefficients in the presence of enclosure losses together with diaphragm displacement data for that alignment, a derivation of the parameter-impedance relationships that permit parameter evaluation from voice-coil impedance measurements, and a method of evaluating the amounts of absorption, leakage, and vent losses present in a vented-box loudspeaker system.

Editor's Note: Part I of Vented-Box Loudspeaker Systems appeared in the June issue, Part II in July/August, and Part III in September.

APPENDIX 1 FOURTH-ORDER FILTER FUNCTIONS AND VENTED-BOX SYSTEM ALIGNMENT

General Expressions

The general form of a prototype low-pass fourth-order filter function $G_L(s)$ normalized to unity in the passband is

$$G_L(s) = \frac{1}{1 + a_1 sT_0 + a_2 s^2 T_0^2 + a_3 s^3 T_0^3 + s^4 T_0^4} \quad (55)$$

where T_0 is the nominal filter time constant and the coefficients a_1, a_2, and a_3 determine the actual filter characteristic.

Tables of filter functions normally give only the details of a low-pass prototype function; the high-pass and bandpass equivalents are obtained by suitable transformation. For the high-pass filter function $G_H(s)$, the transformation (retaining the same nominal time constant) is

$$G_H(sT_0) = G_L(1/sT_0). \quad (56)$$

This leads to the general high-pass form of Eq. (20):

$$G_H(s) = \frac{s^4 T_0^4}{s^4 T_0^4 + a_1 s^3 T_0^3 + a_2 s^2 T_0^2 + a_3 s T_0 + 1}. \quad (57)$$

Study of the magnitude-versus-frequency behavior of filter functions is facilitated by the use of the magnitude-squared form

$$|G_H(j\omega)|^2 = \frac{\omega^8 T_0^8}{\omega^8 T_0^8 + A_1 \omega^6 T_0^6 + A_2 \omega^4 T_0^4 + A_3 \omega^2 T_0^2 + 1} \quad (58)$$

where

$$A_1 = a_1^2 - 2a_2$$
$$A_2 = a_2^2 + 2 - 2a_1 a_3$$
$$A_3 = a_3^2 - 2a_2. \quad (59)$$

Using Eq. (58) it can be shown that the magnitude response of G_H is down 3 dB, i.e., $|G_H|^2 = ½$, at a frequency f_3 given by

$$f_3/f_0 = d^{½} \quad (60)$$

where

$$f_0 = 1/(2\pi T_0) \quad (61)$$

and d is the largest positive real root of the equation

$$d^4 - A_1 d^3 - A_2 d^2 - A_3 d - 1 = 0. \quad (62)$$

Coefficients of Some Useful Responses

Butterworth Maximally Flat Amplitude Response (B4)

This well-known response is characterized by [10], [18]

$$a_1 = (4+2\sqrt{2})^{1/2} = 2.6131$$
$$a_2 = 2+\sqrt{2} = 3.1412$$
$$a_3 = a_1 = 2.6131$$
$$A_1 = A_2 = A_3 = 0$$
$$f_3/f_0 = 1.0000$$

Bessel Maximally Flat Delay Response (BL4)

The normalized roots are given in [19]. They yield

$$a_1 = 3.20108 \quad A_1 = 1.4638$$
$$a_2 = 4.39155 \quad A_2 = 1.2857$$
$$a_3 = 3.12394 \quad A_3 = 0.9759.$$
$$f_3/f_0 = 1.5143$$

Chebyshev Equal-Ripple (C4) and "Sub-Chebyshev" (SC4) Responses

These responses are both described in [14]; the $C4$ responses are further described in [32]. The pole locations may be derived from those of the Butterworth response by multiplying the real part of the Butterworth pole by a factor k which is less than unity for the $C4$ responses and greater than unity for the $SC4$ responses. The filter-function coefficients are then given by

$$a_3 = \frac{k(4+2\sqrt{2})^{1/2}}{D^{1/4}}$$
$$a_2 = \frac{1+k^2(1+\sqrt{2})}{D^{1/2}}$$
$$a_1 = \frac{a_3}{D^{1/2}}\left[1 - \frac{1-k^2}{2\sqrt{2}}\right] \qquad (63)$$

where

$$D = \frac{k^4+6k^2+1}{8}.$$

For the $C4$ responses, the passband ripple is given by

$$\text{dB ripple} = 10\log_{10}[1+K^4/(64+28K+80K^2+16K^3)] \qquad (64)$$

where

$$K = 1/k^2 - 1.$$

Quasi-Third-Order Butterworth Responses (QB3)

This class of response is described in [10] and [32]. In this paper, the response is varied as a function of the parameter B given by

$$B = A_3^{1/2}. \qquad (65)$$

The other coefficients are given by

$$A_1 = A_2 = 0$$
$$a_2 > 2+\sqrt{2}$$
$$a_1 = (2a_2)^{1/2}$$
$$a_3 = (a_2^2+2)/(2a_1). \qquad (66)$$

Because the direct relationships between B and the a coefficients are very involved, the range of responses is computed by taking successive values of a_2 and then computing a_1, a_3, A_3, and B.

Other Possible Responses

Other fourth-order responses which can be obtained with the vented-box system include transitional Butterworth–Thompson [18], transitional Butterworth–Chebyshev [30], Thiele interorder [31], and degenerated Chebyshev [11].

The degenerated Chebyshev responses of the second kind ($DT2$) described by Nomura [11] look particularly appealing for cases where a smooth bass lift (similar to an underdamped second-order response, but with a steeper cutoff slope) is desired. Nomura's design parameters are readily convertible into those of this paper.

Computation of Basic Alignment Data

The basic alignment data are obtained by using the coefficient–parameter relationships given by Eqs. (21)–(24). The steps are as follows.

1) For a given response and value of Q_L calculate

$$c_1 = a_1 Q_L$$
$$c_2 = a_3 Q_L. \qquad (67)$$

2) Find the positive real root r of

$$r^4 - c_1 r^3 + c_2 r - 1 = 0. \qquad (68)$$

3) Then, using Eqs. 60–62 to obtain f_3/f_0, the alignment parameters are

$$h = r^2$$
$$f_3/f_S = h^{1/2}(f_3/f_0)$$
$$\alpha = a_2 h - h^2 - 1 - (1/Q_L^2)(a_3 h^{1/2} Q_L - 1)$$
$$Q_T = hQ_L/(a_3 h^{1/2} Q_L - 1). \qquad (69)$$

For infinite Q_L the above expressions reduce to Thiele's formulas:

$$h = a_3/a_1$$
$$f_3/f_S = h^{1/2}(f_3/f_0)$$
$$\alpha = a_2 h - h^2 - 1$$
$$Q_T = 1/(a_1 a_3)^{1/2}. \qquad (70)$$

Computation of Displacement Maxima

Eq. (14) may be written in the generalized form

$$X(s) = \frac{b_1 s^2 T_0^2 + b_2 s T_0 + 1}{s^4 T_0^4 + a_1 s^3 T_0^3 + a_2 s^2 T_0^2 + a_3 s T_0 + 1} \qquad (71)$$

where T_0, a_1, a_2, and a_3 are given by Eqs. (21)–(24) or by the alignment specification and

$$b_1 = 1/h$$
$$b_2 = 1/(h^{1/2} Q_L). \qquad (72)$$

The magnitude-squared form of this expression is

$$|X(j\omega)|^2 = \frac{B_1\omega^4 T_0^4 + B_2\omega^2 T_0^2 + 1}{\omega^8 T_0^8 + A_1\omega^6 T_0^6 + A_2\omega^4 T_0^4 + A_3\omega^2 T_0^2 + 1} \qquad (73)$$

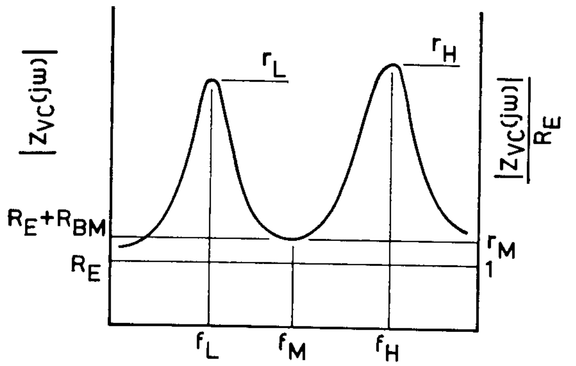

Fig. 20. Voice-coil impedance magnitude of vented-box loudspeaker system as a function of frequency.

where the A_i coefficients are given by Eq. (59) and

$$B_1 = b_1^2$$
$$B_2 = b_2^2 - 2b_1. \qquad (74)$$

The value of $|X(j\omega)|_{\max}^2$ for any alignment is found by differentiating Eq. (73), setting the result equal to zero, solving for the value of $\omega^2 T_0^2$, and then replacing this solution in Eq. (73) and evaluating the expression. There are always at least three frequencies of zero slope for Eq. (73): zero, near f_B, and above f_B. For the extreme $C4$ alignments, there is a fourth frequency, below f_B. The first of these frequencies gives unity displacement; the second is not of interest because it gives a displacement minimum. The third frequency gives the displacement needed to evaluate the displacement-limited power capacity for bandwidth-limited drive conditions. The procedure is as follows.

1) For a given alignment and value of Q_L, calculate

$$C_4 = (1/2B_1)(A_1B_1 + 3B_2)$$
$$C_3 = (1/B_1)(A_1B_2 + 2)$$
$$C_2 = (1/2B_1)(3A_1 + A_2B_2 - A_3B_1)$$
$$C_1 = (1/B_1)(A_2 - B_1)$$
$$C_0 = (1/2B_1)(A_3 - B_2). \qquad (75)$$

2) Find the largest positive real root G of

$$G^5 + C_4G^4 + C_3G^3 + C_2G^2 + C_1G + C_0 = 0. \qquad (76)$$

(The normalized frequency of maximum passband displacement is then $f_{X\max}/f_0 = G^{1/2}$).

3) Calculate

$$|X(j\omega)|_{\max}^2 = \frac{B_1G^2 + B_2G + 1}{G^4 + A_1G^3 + A_2G^2 + A_3G + 1}. \qquad (77)$$

The same procedure is used to determine the frequency of maximum displacement below f_B for the extreme $C4$ alignments by finding the smallest nonzero positive real root in 2). The corresponding maximum value of the displacement function magnitude is then determined as in 3).

APPENDIX 2 PARAMETER-IMPEDANCE RELATIONSHIPS

Determination of f_{SB} and α

For infinite Q_L, the steady-state form of Eq. (16) becomes

$$Z_{VC}(j\omega) = R_E + R_{ES}\frac{j(\omega T_S/Q_{MS})(1-\omega^2T_B^2)}{\omega^4T_B^2T_S^2 + 1 + \omega^2[(\alpha+1)T_B^2 + T_S^2] + j(\omega T_S/Q_{MS})(1-\omega^2T_B^2)}. \qquad (78)$$

This expression has minimum magnitude and zero phase when the numerator of the second term is zero, i.e., when $\omega = 1/T_B$. Thus for this case, the frequency f_M of Fig. 20 is equal to f_B. The expression also has zero phase, with maximum magnitude, when the real part of the denominator of the second term is zero, i.e., for

$$\omega^2 = \frac{T_S^2 + (\alpha+1)T_B^2 \pm \sqrt{T_S^4 + (\alpha+1)^2T_B^4 + (2\alpha-2)T_B^2T_S^2}}{2T_B^2T_S^2}. \qquad (79)$$

Let the solution using the plus sign be ω_H^2 and the solution using the minus sign be ω_L^2. Then

$$\omega_H^2 + \omega_L^2 = \omega_B^2 + (\alpha+1)\omega_S^2 \qquad (80)$$

and

$$(\omega_H^2 - \omega_L^2)^2 = \omega_B^4 + (\alpha+1)^2\omega_S^4 + (2\alpha-2)\omega_B^2\omega_S^2. \qquad (81)$$

Combining Eqs. (80) and (81), it can be shown that

$$(\omega_H^2 - \omega_L^2)^2 = (\omega_H^2 + \omega_L^2)^2 - 4\omega_B^2\omega_S^2 \qquad (82)$$

which simplifies to

$$\omega_H^2\omega_L^2 = \omega_S^2\omega_B^2$$

or [10, eq. (105)]

$$f_S = \frac{f_Hf_L}{f_B} \qquad (83)$$

where $f_S = f_{SB}$ is the resonance frequency of the driver for the particular air-load mass presented by the enclosure.

With f_S known, α can be found by rearranging Eq. (80) into

$$\alpha = \frac{f_H^2 + f_L^2 - f_B^2}{f_S^2} - 1. \qquad (84)$$

Alternatively, substituting Eq. (83) into Eq. (80), it is easily shown that [10, eq. (106)]

$$\alpha = \frac{(f_H^2 - f_B^2)(f_B^2 - f_L^2)}{f_H^2f_L^2}. \qquad (85)$$

This expression factors into

$$\alpha = \frac{(f_H + f_B)(f_H - f_B)(f_B + f_L)(f_B - f_L)}{f_H^2f_L^2}. \qquad (45)$$

Approximate Determination of Q_B

From Fig. 3, Z_{VC} will be resistive when the portion of the circuit to the right of R_{ES} is resistive. The steady-state impedance of this portion of the circuit is

$$Z(j\omega) = R_{EL}\frac{(\alpha T_BQ_L)[-\omega^2T_B/Q_L + j\omega(1-\omega^2T_B^2)]}{\omega^4T_B^2T_S^2 + 1 - \omega^2[(\alpha+1)T_B^2 + T_S^2] + j\omega(T_B/Q_L)(1-\omega^2T_S^2)} \qquad (86)$$

At a frequency of zero phase, the magnitude of $Z(j\omega)$ may be evaluated by taking the ratio of either the real or the imaginary parts of the numerator and denominator, because these ratios must be equal. That is, for zero phase,

$$|Z(j\omega)| = R_{EL}(aT_BQ_L)\frac{-\omega^2 T_B/Q_L}{\omega^4 T_B^2 T_S^2 + 1 - \omega^2[(a+1)T_B^2 + T_S^2]}$$

$$= R_{EL}(aT_BQ_L)\frac{1-\omega^2 T_B^2}{(T_B/Q_L)(1-\omega^2 T_S^2)}. \qquad (87)$$

Setting the real and imaginary ratios equal in the normal way leads to a very complex set of solutions for the exact frequencies of zero phase. However, it can be seen that the first ratio varies relatively slowly with frequency near ω_B (as indeed does $|Z_{VC}(j\omega)|$) and hence can be expected to have about the same magnitude at the frequency of zero phase ω_M very near to ω_B as it has at ω_B. This gives

$$|Z(j\omega_M)| \approx |Z(j\omega_B)| = R_{EL}. \qquad (88)$$

The resistive voice-coil impedance measured at f_M, defined as $R_E + R_{BM}$ in Fig. 20, is thus made up of R_E plus the parallel combination of R_{ES} and R_{EL}. Evaluating this resistance and using Eqs. (5), (7), (8), (10), and (11), it can be shown that

$$Q_L = \frac{h}{a}\left[\frac{1}{Q_{ES}(r_M-1)} - \frac{1}{Q_{MS}}\right] \qquad (49)$$

where r_M is $(R_E + R_{BM})/R_E$ as defined in Eq. (48) and Fig. 20. In many cases the $1/Q_{MS}$ term can safely be neglected.

Now, if the two ratios in Eq. (87) are equal at ω_M, the second must give the same value as the first. This requires that

$$\omega_M^2 = \frac{1-aQ_L^2}{T_S^2 - aT_B^2 Q_L^2} \qquad (89)$$

which may be rearranged to give Eq. (50). The approximation made earlier in Eq. (88) seems justified by Eq. (50) for Q_B values as low as 5, because the difference between f_M and f_B is then at most a few percent. For lower values of Q_B (which are unusual), substantial inaccuracy must be expected. Inaccuracy can also be contributed by a significant voice-coil inductance (see [32]).

APPENDIX 3
MEASUREMENT OF ENCLOSURE LOSSES

Measurement Principle

In this method of measurement the system driver is used as a coupling transducer between the enclosure impedances and the electrical measuring equipment. The driver losses are subtracted from the total measured losses to obtain the enclosure losses. Greatest accuracy is therefore obtained where the driver mechanical losses are small and stable.

The method assumes that R_E remains constant with frequency (i.e., voice-coil inductance losses are negligible), that the individual enclosure circuit losses correspond to Q values of about 5 or more (so that $Q^2 >> 1$), and that any variation with frequency of the actual losses present can still be represented effectively by a combination of the three fixed resistances R_{AB}, R_{AL}, and R_{AP} of Fig. 1.

System Loss Data

From the system impedance curve, Fig. 20, find the three frequencies f_L, f_M, and f_H, and the ratio of the corresponding maximum or minimum impedance to R_E, designated r_L, r_M, and r_H.

Using the methods of Section 7 (Part II) or [32], determine the system compliance ratio a. Measure independently the driver resonance frequency f_S and the corresponding value of Q_{ES} as described in [12] or [32]. The driver mounting conditions for the latter measurements do not matter, because the product f_SQ_{ES} which will be used is independent of the air-load mass present.

Driver Loss Data

Let the symbol ρ be used to define the ratio

$$\rho = (R_{ES} + R_E)/R_E. \qquad (90)$$

Because R_{ES} is in fact a function of frequency for real drivers, so too is ρ. Typically the variation is of the order of 2 to 4 dB per octave increase with increasing frequency.

At the resonance frequency of the driver, ρ is the ratio of the maximum voice-coil impedance to R_E which is defined as r_0 in [12]. The value of ρ for frequencies down to f_L may be measured by weighting (mass loading) the driver diaphragm and measuring the maximum voice-coil impedance at resonance for a number of progressively lower frequencies as more and more mass is added. A convenient nondestructive method of weighting is to stick modeling clay or plasticene to the diaphragm near the voice coil.

Unfortunately, there is no comparable simple way to reduce mass or add stiffness which will raise the driver resonance frequency without affecting losses. For simplicity, it is necessary to extrapolate the low-frequency data upward to f_H. This is risky if f_H is more than an octave above f_S but gives quite reasonable results for many drivers.

Under laboratory conditions, it is possible to fabricate a low-mass driver which is "normally" operated with a fixed value of added mass. This mass is selected so that the unloaded driver resonance occurs at a frequency equal to or greater than the value of f_H for the loaded driver in a particular enclosure. In this case the value of ρ can be accurately determined for the entire required frequency range by adding and removing mass.

Measure and plot (extrapolating if necessary) the value of ρ over the frequency range f_L to f_H. Find the values at f_L, f_M, and f_H and designate these ρ_L, ρ_M, and ρ_H.

These measurements should be carried out at the same time and under the same conditions as those for the system loss data above. The signal level should be the same and should be within small-signal limits at all times.

Enclosure Loss Calculation

Define:

$$H = f_H/f_M$$
$$L = f_M/f_L$$
$$F = f_M/(af_SQ_{ES}). \qquad (91)$$

Calculate:

$$k_L = \frac{1}{r_L - 1} - \frac{1}{\rho_L - 1}$$

$$k_M = \frac{1}{r_M - 1} - \frac{1}{\rho_M - 1}$$

$$k_H = \frac{1}{r_H - 1} - \frac{1}{\rho_H - 1} \tag{92}$$

$$C_L = Fk_L(L^2 - 1)\left(1 - \frac{1}{L^2}\right)$$

$$C_M = (Fk_M)^{-1}$$

$$C_H = Fk_H(H^2 - 1)\left(1 - \frac{1}{H^2}\right) \tag{93}$$

$$\Delta = \left(H^2L^2 - \frac{1}{H^2L^2}\right) - \left(H^2 - \frac{1}{L^2}\right) - \left(L^2 - \frac{1}{H^2}\right)$$

$$N_L = C_M\left(H^2L^2 - \frac{1}{H^2L^2}\right) - C_H\left(L^2 - \frac{1}{L^2}\right) - C_L\left(H^2 - \frac{1}{H^2}\right)$$

$$N_A = -C_M\left(L^2 - \frac{1}{H^2}\right) + C_H(L^2 - 1) + C_L\left(1 - \frac{1}{H^2}\right)$$

$$N_P = -C_M\left(H^2 - \frac{1}{L^2}\right) + C_H\left(1 - \frac{1}{L^2}\right) + C_L(H^2 - 1). \tag{94}$$

Then the values of Q_L, Q_A, and Q_P which apply at the frequency f_M are found from

$$Q_L = \Delta/N_L$$
$$Q_A = \Delta/N_A$$
$$Q_P = \Delta/N_P. \tag{95}$$

Using the same data, the total enclosure loss Q_B at the frequency f_M is

$$Q_B(f_M) = 1/C_M = Fk_M. \tag{96}$$

The approximate formula for $Q_B = Q_L$ given in Eq. (49) differs from Eq. (96) only in that R_{ES} is assumed constant, i.e., that $\rho_M = r_0$. However, because ρ_M is seldom very different from r_0, and particularly because $r_M - 1$ is usually much less than $\rho_M - 1$, Eq. (49) provides an adequately accurate measurement of total losses for normal evaluation purposes.

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Loudspeakers in Vented Boxes: Part II*

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VIII. LOUDSPEAKER EFFICIENCY

In Eq. (12) an expression was derived for the efficiency of a loudspeaker in a box, which consists of three parts. We have considered, in the meantime, the third part which varies with frequency. We now consider the first two parts. Thus the basic efficiency

$$\eta_{ob} = (\rho_o/4\pi c)(B^2l^2S_d^2/R_eM^2_{ms}). \quad (66)$$

If this experience is compared with Beranek's Eq. (7.19) it will be seen to give one quarter of his value, after the differences in notation are allowed for.

1) Multiplication by 100 to give percentage.

2) The definition of "nominal input power" in Eq. (10) of this paper as the power delivered by the amplifier into the nominal speaker impedance R_e[3]. Beranek's treatment is based on the idea of maximum power transfer when the load impedance is equal to the generator impedance, as in his Eq. (7.14). If this condition, $R_g = R_e$, is substituted in his Eq. (7.19), one of the conditions for agreement with Eq. (66) is satisfied. However, in dealing with the output power from an amplifier, the writer prefers to consider the power delivered into the load without regard to the output impedance R_g, for the relationship of R_g to the optimum load impedance depends in the first place on the nature of the output device, transistor, pentode, or triode. Furthermore, R_g can be manipulated by feedback techniques (see Section XII) to almost any desired value without affecting the condition for optimum output power. Hence the treatment in this paper.

3) The lumping in this paper of all mechanical mass into M_{ms}.

The additional multiplication factor of one quarter arises from the following.

4) Beranek's figure being for the radiation from *both* sides of the diaphragm, giving twice the output from one side.

5) The assumption in this paper that the radiation resistance in a box is that of a piston at the end of a long tube [3, p. 216]. This radiation resistance is one half of that of a piston in an infinite baffle.

Thus the results are consistent. We will continue here to use η_{ob}, unless stated otherwise. But it is important to define efficiency in terms of actual use and to remember that the value of η_{ob}, being the basic efficiency in a box, is one half the efficiency on an infinite baffle and one quarter of the efficiency, if radiation from both the front and back of a speaker in an infinite baffle is considered.

To simplify the understanding of Eq. (66), we make a further substitution. It can be shown that

$$l^2/R_e = V_{cu}/2\sigma \quad (67)$$

where σ is the resistivity of the conductor and V_{cu} is the volume of the conductor assumed to be completely within the air gap. In so far as the conductor overlaps the air gap a correction factor would be applied. Then

[3] The nominal impedance of a loudspeaker is usually taken as the minimum impedance at mid-frequencies, at f_n in Fig. 5. This is a little greater than R_e; but for simplicity, and it is hoped without too much confusion, the nominal impedance is taken here as R_e.

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Eq. (66) becomes

$$\eta_{ob} = (\rho_o/8\pi c\sigma)(B^2 S_d^2 V_{cu}/M^2_{ms}) \quad (68)$$

that is, once the voice coil conductor material, and therefore σ, is chosen, the loudspeaker efficiency depends on the four parameters in the second bracket. Without digressing too far into the problem of loudspeaker design, it is noted that this shows the two basic questions in loudspeaker design for good efficiency at low frequencies.

1) How to make the product $B^2 V_{cu}$ a maximum for a given magnet, since the larger V_{cu} is made, the wider and/or deeper is the air gap, and hence the lower is B.

2) How to make S_d^2/M_{ms}^2 a maximum, since the larger the area the greater the mass for a given cone thickness. If thickness is reduced, break-up problems increase due to nonlinearity of the piston drive. In conventional designs the mass of the voice coil is small (less than 20%) compared with the mass of the cone, so there is little interaction between V_{cu} and M_{ms}.

The writer prefers to express efficiency as an electroacoustic conversion loss

$$\mathrm{dB}_{ea} = 10 \log_{10} \eta. \quad (69)$$

For example, 1% efficiency is equivalent to 20-dB electroacoustic conversion loss. This facilitates comparisons between different designs and estimations of the acoustic level (in phons) which a speaker will provide with a given amplifier and listening room (see Appendix).

IX. RELATIONSHIP OF EFFICIENCY η, Q, AND BOX VOLUME

First we take Eq. (57) and break Q_t into two component parts, one due to the acoustic resistances and the other due to electrical damping, so that

$$1/Q_t = 1/Q_a + (1/Q_e)[R_e/(R_g+R_e)]. \quad (70)$$

Then from Eqs. (8) and (57), the acoustic Q of the loudspeaker

$$Q_a = \omega_s M_{as}/R_{as} \quad (71)$$

and the electrical Q of the loudspeaker

$$Q_e = \omega_s M_{as} R_e S_d^2/B^2 l^2 \quad (72)$$

i.e.,

$$Q_e = 2\sigma\omega_s M_{ms}/B^2 V_{cu}. \quad (73)$$

Again if we consider the approximate relationship established in Table I that

$$C_{as} f_s^2/C_{ab} f_3^2 \cong \sqrt{2} \quad (74)$$

thus, converting the acoustic compliance of the box into the equivalent volume of air, the box volume

$$V_b \cong (\rho_o c^2/\omega_3^2\sqrt{2})(S_d^2/M_{ms}) \quad (75)$$

remembering that this approximate relationship holds only in the absence of amplifier assistance.

Now considering together Eqs. (68), (73), and (75), the following points emerge.

1) The same considerations that ensure high efficiency also ensure a low Q_e, except that Q_e is independent of the projected piston area S_d and depends only on the *first* power of the cone mass M_{ms} instead of the second power.

2) The box volume depends, apart from the choice of cutoff frequency f_3, only on S_d^2 and M_{ms}. Reduction of box volume by reduction of S_d involves an increased cone excursion, which is inversely proportional to S_d and ω_b^2, for a given acoustic power. If the box volume is reduced by increasing M_{ms}, η is decreased even more (see Eq. (68)), necessitating increased amplifier power. It would seem that the well-known R–J enclosure works this way. The opening in front of the cone is restricted, and this increases the air mass loading M_{a1} of Fig. 1 in the same manner as a vent. Thus M_{ms} is increased and the box volume V_b, i.e., C_{ab}, for a given low-frequency cutoff is reduced, but at the price of reduced efficiency throughout the piston range.

3) The best way of increasing η and lowering Q_e is to increase the flux density B. But if one starts with a reasonably high value of B in the first place, the cost of obtaining an extra decibel of efficiency increases rapidly. So again to obtain a given amount of acoustic power at a given price, a compromise must be struck between the sizes of magnet, box, and amplifier. However, this discussion does show the reason for the large magnet, long throw, heavy cone designs used overseas in small "bookshelf boxes."

Note that Q_a in Eq. (71) depends only on acoustic reactance and resistance, that is, Q_a is independent of B.

Substituting Eqs. (58) and (73) in (68), we obtain the interesting relationship

$$\eta_{ob} = \omega_s^3 V_{as}/4\pi c^3 Q_e \quad (76)$$

where V_{as} is the volume of air equivalent to the acoustic compliance of the loudspeaker, or

$$\eta_{ob} = 8.0\times 10^{-12} f_s^3 V_{as}/Q_e \quad (77)$$

where V_{as} is in cubic inches. *Thus the basic efficiency of the speaker can be calculated from the three parameters which are used for the design of the box.* A physical explanation of the variation of η and Q_e is given at the end of Section XII.

X. EXCURSION OF LOUDSPEAKER CONE

In the derivation of Eq. (12) it was found that

$$U_c/(U_c - U_p) = 1 - 1/\omega^2 M_{ar} C_{ab} = 1 - (\omega_b/\omega)^2. \quad (78)$$

Thus the acoustic output power radiated by the cone alone is

$$W_{aoc} = W_{ei}\eta_{ob}[1-(\omega_b/\omega)^2]^2|E(j\omega)|^2. \quad (79)$$

Now starting from the relationship

$$W_{aoc} = (R_{ma}\dot{x}^2)10^{-7} \quad (80)$$

which is [4, Eq. 6.13], where R_{ma} is the mechanical radiation resistance and $\dot{x}$ is the rms velocity of the piston in cm/s, it is possible to derive an expression for peak cone movement,

$$x_{pk} = 1.31\times 10^5\sqrt{W_{aoc}}/f^2 S_d \quad (81)$$

or

$$x_{pk} = 5.17\times 10^6\sqrt{W_{aoc}}/\omega^2 S_d \quad (82)$$

where x_{pk} is in inches (note that this x which stands for excursion is unrelated to the shape parameter x of Eq.

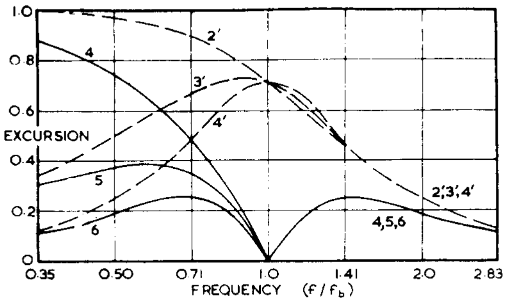

Fig. 10. Normalized cone excursion versus normalized frequency for various orders of Butterworth response with loudspeaker in vented box (solid curves) and in infinite Baffle (dashed curves). Curves are numbered for order of response. Normalized excursion is $|(f_b/f)^2 - (f_b/f)^4| \cdot |E(j\omega)|$, part of Eq. (84).

(21) *et seq.*), S_d is in square inches, and W_{aoc} is in watts. Again allowance is made for the fact that the loudspeaker is mounted in a box so that the radiation resistance is half the value for an infinite baffle. Thus Eqs. (81) and (82) will give values for displacements which are $\sqrt{2}$ times those given in [4, Fig. 6.9]. Thus

$$x_{pk} = 5.17\times 10^{6}(\eta_{ob}W_{ei})^{1/2}[1-(\omega_b/\omega)^2]|E(j\omega)|/\omega^2 S_d. \qquad (83)$$

If we write this expression as

$$x_{pk} = [1.31\times 10^{5}(\eta_{ob}W_{ei})^{1/2}/f_b^2 S_d][(\omega_b/\omega)^2-(\omega_b/\omega)^4] |E(j\omega)| \qquad (84)$$

it is apparent that there are two parts, one fixed for a given speaker and box (note frequency f_b in this expression) and one that varies with frequency. This latter expression is plotted in Fig. 10 for various Butterworth responses, in which box, speaker, and cutoff frequencies are identical. The solid curve 4 gives the excursion of the classical fourth-order Butterworth alignment no. 5 of Table I. Solid curve 5 refers to the fifth-order Butterworth alignment no. 10, which includes a simple auxiliary filter. Solid curve 6 refers to the sixth-order Butterworth alignment which is identical for nos. 15, 20, and 26, since both frequency response and box resonant frequency are the same in each. For comparison, the dotted curves give the excursions for the same speaker in an infinite baffle (totally enclosed box) with the same power. Dotted curve 2 applies to a speaker with a second-order Butterworth response ($Q_t = 0.707$). Dotted curve 3 applies to a third-order Butterworth response ($Q_t = 1$, with a simple auxiliary filter). Dotted curve 4 applies to a fourth-order Butterworth response ($Q_t = 1.307$, with a second-order auxiliary filter). The frequency response is the same as solid curve 4, but it is obtained by different means. The curves show the following.

1) The excursion below resonance is reduced greatly in both vented box and infinite baffle when an auxiliary highpass filter is used. The first-order auxiliary filter gives a good improvement especially in view of its simplicity. The second-order auxiliary filter not only allows a greater reduction of cone excursion, it also allows the use of three separate box alignments for the same response and allows box volume to be traded for amplifier power in the case of the vented box. The Butterworth curves with second-order auxiliary filters are symmetrical about the center frequency. There seems little need therefore to use more elaborate filtering.

2) Even more important, the excursion of the cone is reduced greatly when the loudspeaker is placed in a vented box. The curve predicts zero excursion at the box frequency. This arises from the assumption that the Q of the box circuit is infinite. While this cannot be achieved completely in practice, the excursion at the box frequency will be low so long as the ratio of Q of the box to Q of the speaker is high, as demonstrated in Section II.

Of course, if resistance is deliberately introduced into the box circuit, as by making the vent from a number of small holes or by stretching fabric across the vent, the Q will be greatly reduced and some of the advantage of the vented box will be lost, as shown in the next section. Fig. 10 refers only to Butterworth responses. In Fig. 11, a plot is made of the function $|(\omega_b/\omega)^2-(\omega_b/\omega)^4|$ against frequency. If, for example, in a Chebyshev response the frequency response is known, the excursion at different frequencies can be found by reading off the function at a given frequency on Fig. 11 and multiplying it with the frequency response. The rapid rise of the function between normalized frequencies of 1 and 0.71 shows why responses should be preferred in which f_b is not too much greater than f_3. Thus with respect to cone excursion, an alignment in the group 20–25 would be preferred to its counterpart in the group 15–19 which has a lower value of f_3/f_b.

It would seem that in published ratings of loudspeakers, the maximum excursion x_{max} would be more useful than the conventional rating of maximum input power. The latter might save the loudspeaker from a melted voice coil, but when mechanical damage or undistorted acoustic output are of interest, x_{max}, along with the kind of baffle and the alignment, determine the performance.

XI. BOXES WITH RESISTIVE LOADING OF VENT

Good results have been reported with resistively loaded vents [1]. These were therefore investigated using both series and parallel loading of the vent as shown in Fig. 12. In both cases, the resistance was assumed to be constant with respect to frequency and the response function was found to be of third order.

This, by the way, explains a discrepancy between the statements in [3, p. 244] and in [2, p. 11] that the drop in response below cutoff is 18 dB per octave, even though [2, Eq. 15)], which is equivalent to Eq. (20) of this

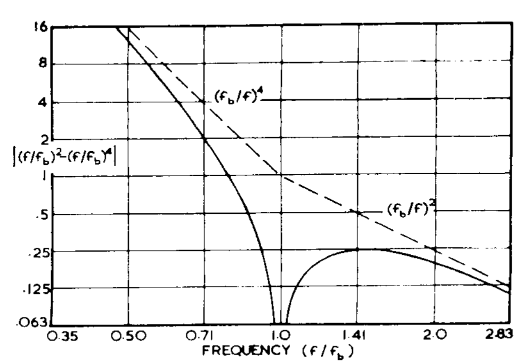

Fig. 11. Function $|(f_b/f)^2 - (f_b/f)^4|$ versus normalized frequency f/f_b. The function, part of Eq. (84), is used to compute excursion when frequency response $|E(j\omega)|$ is known.

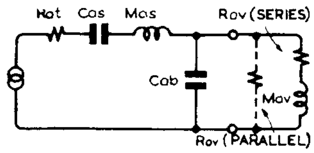

Fig. 12. Equivalent acoustic circuit of loudspeaker and box showing added acoustic damping in series or parallel with vent.

paper, obviously has an asymptotic slope of 24 dB per octave. In the practical case, where resistance loading of the vent however small will be encountered, the asymptotic slope will eventually be 18 dB per octave; but so long as the original simplifying assumptions hold, the response in the region that concerns us will be effectively 24 dB per octave.

The expressions are, for the case of series resistance loading,

$$E(p) = 1/\{1+(1/p)(1/Q_bT_b+Q_bT_b/T^2_s)+(1/p^2)(1/T_s^2+1/T_b^2+C_{as}/C_{ab}T_s^2)+Q_b/p^3T_s^2T_b\} \quad (85)$$

when

$$1/Q_t = T_s/Q_bT_b+Q_bT_b/T_s \quad (86)$$

and Q_b is defined as the ratio of acoustic mass resistance to series acoustic resistance of the vent at the *box* resonant frequency.

For the case of parallel resistance loading,

$$E(p) = 1/\{1+(1/p)(Q_bT_b/T_s^2+1/Q_bT_b+C_{as}T_b/C_{ab}T_s^2Q_b)+(1/p^2)(1/T_s^2+1/T_b^2+C_{as}/C_{ab}T_s^2)+Q_b/p^3T_s^2T_b\} \quad (87)$$

when

$$1/Q_t = Q_bT_b/T_s+T_s/Q_bT_b+C_{as}T_b/C_{ab}T_sQ_b \quad (88)$$

and Q_b in this case is the ratio of parallel acoustic resistance across the vent (series resistance being assumed negligible) to acoustic mass reactance. Note the inversion of the expression for parallel Q_b compared with that for series Q_b. Since these equations are of third order and there is one extra variable Q_b, there are two extra degrees of freedom in the design. However, one is removed if an all-pole function is desired, hence Eqs. (86) and (88). Before an alignment is commenced, one other parameter must be fixed arbitrarily. The ratio C_{as}/C_{ab} seems the easiest to handle for this purpose. Thus in a third-order Butterworth alignment, if C_{as}/C_{ab} is made 1.414, for comparison with the fourth-order Butterworth alignment no. 5 of Table I, the results are as given in Table II.

Table II. Parameters for third-order Butterworth alignment with resistive vented loading.

Method of Loading	f_3/f_s	f_3/f_b	C_{as}/C_{ab}	Q_t	Q_b
Series Resistance	1.317	1.285	1.414	0.379	2.22
Parallel Resistance	1.420	1.120	1.414	0.352	2.25
No Resistance (Alignment No. 5, for comparison)	1.000	1.000	1.414	0.383	∞

It will be seen that although the box had the same volume, the cutoff frequencies for the resistively loaded alignments are 1.32 and 1.42 times higher than no. 5 of Table I. Compared with previous alignments (no. 1–9 of Table I) those of Table II are most inefficient in utilization of box volume, there is no compensating freedom to use a larger value of Q_t, in fact it needs to be a little smaller, finally and more important, the excursion of the speaker near cutoff frequency is greatly increased. For these reasons, the use of acoustic damping seems to be unjustified. It is realized that the cases treated here use resistances which are constant with frequency. Some acoustic resistances, as described for example in [3, Eqs. (5.54) and (5.56)], vary with frequency and might have a somewhat different effect. However, the use of added damping with the attendant dissipation of input power seems to be wrong in principle, unless a suitable alternative cannot be found. It is believed that the method outlined already provides the suitable alternative.

Effect of Losses in Box and Vent

Having established that intentional loading of the vent is undesirable, it is of interest to know the effect on the ideal response, obtained by assuming zero loss, of small unavoidable losses in the box and vent. We will only consider performance at the box resonant frequency, since at this frequency 1) the box circuit contributes most, in the ideal case all, of the acoustic output, and 2) the losses in the box circuit are greatest.

In the ideal case, the transfer impedance connecting the input force $E_gBl/S_d(R_g+R_e)$ with the vent volume velocity U_p in Fig. 2, at the box resonant frequency ω_b, is $j\omega_bM_{av}$. If now we express all the losses in the vent *and* the box as Q_b, the "Q of the box and vent circuit," the transfer impedance, and thus the frequency response at ω_b is reduced by a factor which we will call the maximum box loss $(A_b)_{max}$. Then, to a close approximation,

$$(A_b)_{max} = 1/[1+(1/Q_tQ_b)(C_{ab}/C_{as})(\omega_b/\omega_s)]. \quad (89)$$

If we apply the approximations of parts 1) and 2) of Section VI for the "unassisted" alignments no. 1–9 of Table I, Eq. (89) is simplified to

$$(A_b)_{max} = 1/[1+(1.85/Q_b)(f_b^2/f_3^2)] \quad (90)$$

that is, for a given value of Q_b, the box loss increases with higher values of f_b/f_3 and thus, larger box sizes.

To illustrate the effect of box loss, Eq. (89) is applied to various alignments. Taking first the classical alignment, no. 5 of Table I, the maximum box loss is 0.5 dB when Q_b is 30 and 1.5 dB when Q_b is 10. Taking other, extreme, alignments when Q_b is 30, the losses for alignments no. 1, 9, 19, and 25 are 0.3 dB, 0.7 dB, 0.5 dB, and 0.7 dB, respectively. Thus it can be seen that a Q_b of 30 will have little effect on any alignment. With a Q_b of 10, the losses are 0.9 dB, 1.9 dB, 1.5 dB, and 2.2 dB, respectively, i.e., when the box Q is reduced three times, the maximum box loss is increased approximately three times in each case. A method of measuring Q_b is given at the end of Section XIV and illustrated in the Appendix.

Table III. Change of output impedance R_g with type of feedback.

	Negative	Positive
Voltage Feedback	R_g Decreases	R_g Increases
Current Feedback	R_g Increases	R_g Decreases

XII. AMPLIFIER CIRCUITS

Negative Output Impedance

It is essential to the method that the overall Q_t of the loudspeaker plus amplifier be properly controlled within ±10% for ±1 dB accuracy of response. As explained in Section V, if Q_t is twice the optimum value, a 6-dB peak results. Similarly if Q_t is too small, there will be a dip in the response. Thus it is important that the speaker Q_e be known, either from information supplied by the manufacturer or by measurement, and that the amplifier output impedance be then adjusted to give the required overall value of Q_t. It is assumed in the following that the available speaker Q_e is larger than the required Q_t. This is the more usual case, especially with lower priced loudspeakers. But if it is smaller, a suitable adjustment can easily be made, for example, by changing the positive current feedback to negative current feedback.

The subject of amplifier output impedance control properly requires another paper, which it is hoped will be presented later. For the present only some general results will be given.

If feedback is applied to an amplifier, not only does its gain change, but its effective output impedance R_g changes also; not its optimum load impedance which remains unchanged by feedback but the impedance which is seen when looking back into the amplifier output terminals. The effect of applying different kinds of feedback is shown in Table III.

The terms voltage feedback and current feedback refer of course to feedback of a voltage which is proportional to output voltage and output current, respectively. In the latter case, this is usually achieved by placing a small resistor in series with the load, and taking the voltage drop across it for feedback. It will be seen that not only does negative voltage feedback reduce the output impedance R_g, positive current feedback reduces R_g also, and to the greater extent that R_g can be made zero or negative.

Negative output impedance is characteristic of oscillators; one therefore tends to be wary of it as tending to instability. But this can only happen when the positive output impedance presented by the load is less than the negative impedance presented by the amplifier. Now the impedance of a loudspeaker in a box, typified by Fig. 5, can never be less than its dc resistance R_e of Fig. 4. The only exception is at very high frequencies, where the shunt capacitance of the connecting leads takes effect. But unless the leads are very long and the nominal impedance of the speaker is high, this will not usually take effect within the bandwidth of the amplifier. And in any case, we will want to eliminate the negative impedance characteristic at the higher audio frequencies for reasons that will be discussed later. Thus a negative impedance amplifier can be made completely stable apart from gross misadjustment, such as connecting a loudspeaker of much lower impedance than the design figure or short-circuiting the output leads.

The method of applying mixed feedback is shown in Fig. 13. It will be seen that if the sense of the voltage developed across the potential divider R_3 and R_4 is negative, then the voltage developed across the current feedback resistor R_2, usually made less than 1/10 the nominal impedance of the speaker to minimize power loss, will be positive. The circuit shows why this method is sometimes described as bridge feedback. Usually the circuit is arranged to be unbalanced at all frequencies so that the net feedback is always negative, but it need not necessarily be so. For example, if no net negative feedback is desired, so that there is no overall gain reduction with nominal load, the bridge will be balanced at nominal load.

Physically, the circuit can be thought of as having a certain amount of feedback with nominal load, in which the negative voltage feedback is partially neutralized by the voltage from the positive current feedback resistor. If the impedance Z_1 is open-circuited, the current feedback from R_2 disappears leaving a greater amount of negative feedback. Thus the output voltage may be less on open circuit than on nominal load. This is the effect we describe as negative output impedance. Its extent, or whether it is seen at all, will depend on the original gain and output impedance of the amplifier and the value of the feedback resistor R_2. Thus if we have, as in Fig. 4, a loudspeaker resistance R_e, and make the effective output impedance of the amplifier R_g equal to, say, $-0.6R_e$, the total effective impedance of R_g+R_e becomes $+0.4R_e$. And if the Q_e of the loudspeaker is 1.0, this will make the overall Q_t a value of 0.4 by applying a maximum of 1.0/0.4 times, i.e., 8.0 dB, extra gain reduction by negative feedback when the impedance of the speaker becomes high, as at f_h and f_l of Fig. 5. (Need it be emphasized that this form of damping does not dissipate amplifier output power, except in the small current feedback resistor. It reduces power by feedback at the source.)

This fact necessitates a degree of additional care in the design of negative impedance amplifier. For when the load is open-circuited, the negative feedback rises to the maximum; in this case a gain reduction of 8 dB above the nominal value, and the stability margin will be reduced. The size of the negative impedance will in practice be limited either by this consideration or by the need for a feedback resistor so large that it dissipates an appreciable part of the output power.

An alternative method of control damping uses a feedback winding closely coupled to the voice coil. In this way, feedback can be taken effectively from the junction of R_e and L_e in Fig. 4. Simple negative feedback

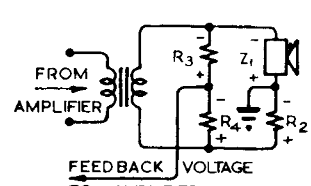

Fig. 13. Method of applying mixed feedback (positive current and negative voltage).

then reduces an effective output impedance which is the sum of R_g+R_e. Thus Q_t is reduced in the same way as before. Since the impedance of the feedback circuit is usually high compared with the voice coil impedance, the feedback winding can be made of very fine wire. In fact, if it is wound bifilar with the main winding with wire 16 B&S gauges smaller, it will fit into the air spaces between the larger wires. It thus takes up no more space in the air gap and adds less than 3% to the mass of the copper in the voice coil. Unfortunately, such a winding is difficult to achieve in production and is thus rarely, if ever, used.

If negative impedance is applied, it reduces the output voltage whenever the load impedance is high, i.e., not only in the region of f_l and f_h in Fig. 5, but also at frequencies above f_n where the impedance rise is due to the inductance L_e of Fig. 4. At high frequencies, this contributes nothing to the acoustic damping of the speaker, but simply reduces the high-frequency response, in the case quoted above, a maximum of 8 dB. This is usually undesirable, so the negative impedance should be eliminated at the higher audio frequencies. One method among several possible is shown in Fig. 14**a**. Here an inductance L_2 is added to the feedback resistor R_2 with a time constant L_2/R_2 matching that of the speaker, usually in the range of 30–60 μs. This can be easily done by winding a solenoid of copper wire which combines resistance R_2 and inductance L_2. However, since this achieves its result by feeding back an increasing positive voltage to neutralize an increasing negative voltage, quite small unbalance between the two can cause instability at high frequencies.

On the other hand, consider the circuit of Fig. 14**b** where the lower resistor of the negative feedback potential divider R_4 becomes two resistors R_5 and R_6 in series. Suppose that a suitable set of resistors R_2, R_3, and R_4 has been found to give the correct gain and output impedance for low frequencies with the dotted connection open-circuited. It is then possible to find a tapping point on R_4 (i.e., the junction of R_5 and R_6) such that the same gain is obtained on nominal load whether the dotted connection is open circuit or short circuit. This is done by connecting the nominal load and making R_5 and R_6 a potentiometer whose wiper is grounded through a switch. The wiper is adjusted until the gain is the same with the switch open or closed. In the open-circuit condition, the output impedance will be the value originally chosen, but on short circuit, most of the positive current feedback will be eliminated. If then a capacitor is substituted for the switch as shown in Fig. 14**b**, the output impedance will change from a negative value at low frequencies to a small value, either positive or negative depending on the particular circuit. The frequency of changeover, which should be, say, two octaves above f_h, depends on the capacitance C and the resistances R_5 and R_6. At the same time, the gain of the amplifier on nominal load stays constant over the whole audio range.

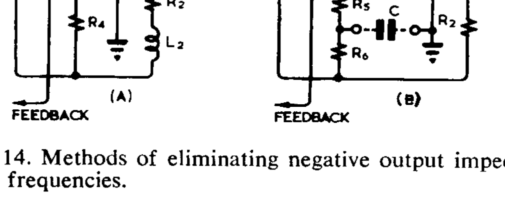

Fig. 14. Methods of eliminating negative output impedance at high frequencies.

Auxiliary Filters

The auxiliary filtering needed for sixth-order alignments is best provided by circuits using RC networks in a feedback loop ahead of the main amplifier. In general it is unwise to use the main amplifier feedback loop to provide both negative impedance and high-pass filtering. It is hoped to deal with this in a later paper, but for the moment the reader's attention is directed to the extensive literature, of which [9] and [10] are examples, concerning low-frequency filters without inductors, which use resistors, capacitors, and tubes in comparatively inexpensive combinations.

Maximum Power at Maximum Impedance

The electrical impedance seen at the terminals of a loudspeaker varies greatly with frequency, but output stages deliver maximum power into a comparatively narrow range of impedances. To consider the maximum acoustic power that can be delivered by an amplifier through a loudspeaker, we return to the equivalent electrical circuit of Fig. 4, together with the impedance curve of Fig. 5. For this purpose, we ignore for the moment the inductance L_e with its electrical shunt loss R_{sh} and assume that the curve of Fig. 5 reaches a final value of R_e above f_n.

The acoustic output depends on the voltage across R_{es}, which includes the electrical equivalent of the radiation resistance R_{ar1}. Since R_{ar1} varies with frequency squared, the voltage across R_{es} needs to vary inversely with frequency to maintain constant acoustic power. At the higher frequencies the motional impedance is much lower than R_e and is controlled by the reactance of C_{mes}, which is equal to B^2l^2/M_{ms}. Thus the condition for flat response is achieved, often described as mass control.[4]

If B is varied while R_e remains constant, the motional impedance at any given high frequency within the piston range will increase with B^2. The electrical equivalent of radiation resistance, though small, will increase and with it the ratio, again small, of acoustic power radiated to electrical power input. Thus efficiency varies with B^2. At the same time the increase of motional impedance while the resistance R_e remains constant causes Q_e, the electrical Q, to decrease inversely with increasing B^2.

But as the frequency decreases, the motional impedance rises, reaching at f_h and again at f_l a maximum value of R_{es} which is usually several times the resistance R_e. Thus at these peaks the motional impedance, which at high frequencies was negligible compared with R_e, is now the major part of the total impedance. Suppose for simplicity that it comprises all of the speaker impedance. This time when B is varied and the motional impedance

[4] This should not be confused with the technique of mass control practiced by politicians and advertising people. In that context, the reactance is usually assumed to result from the equivalent of a compliance, and hence to decrease with signal frequency.

varies as B^2, then for a given acoustic power output the voltage across R_{es}, which is virtually the input voltage, will need to increase with increasing B. Summarizing, for a fixed acoustic power output, an increase of B will decrease the input voltage required at high frequencies, and increase the input voltage required at the impedance peaks. Also Q_e will decrease.

With a load impedance much larger than nominal, the criterion of performance of the amplifier becomes, not output power, but the undistorted output voltage on open circuit. This will always be larger than the undistorted output voltage at nominal load; how much larger will depend on the design of the amplifier.

Now if the Q_t required for a flat frequency response is identical with the Q_e of the loudspeaker, then if we ignore Q_a, the generator impedance R_g must be zero. Thus for a constant acoustic power output the same voltage will be required at the loudspeaker terminals at all frequencies, and all impedances, so that at the frequency f_h somewhat more maximum acoustic power is available than at higher frequencies.

If the Q_t required is less than Q_e, R_g will need to be negative, and for constant acoustic power and amplifier output voltage, at the junction of R_g and R_e in Fig. 4, will fall at f_h. But if the Q_t required is greater than Q_e, R_g will need to be positive, and the amplifier output voltage for constant acoustic power will rise at f_h. If the ratio of increase of voltage required is greater than the ratio of amplifier undistorted output voltages on open circuit to on-load, it is possible for less maximum acoustic power to be available in the region of f_h than at other frequencies in the useful band. But since low values of Q_e are normally associated with high efficiency, this is only likely to occur with high-efficiency, usually high-quality speakers. It should not cause trouble until Q_e is less than half Q_t, and even then the maximum acoustic power in most program material is less at frequencies below 100 Hz than around 400 Hz.

Thus there is a paradox that a highly efficient speaker may deliver less power around f_h than at higher frequencies, while a less efficient speaker delivers more. This will depend on the ratio of Q_e to Q_t and of amplifier undistorted output voltage off-load to on-load.

Related to this topic is the flattening of the impedance characteristic which is usually considered to be a good feature of vented boxes. Reference to Fig. 5, and comparison with Fig. 16, shows that, with the simplifying assumption that the resistive losses in the box and vent are negligible, the height of the impedance peak $R_e + R_{es}$ peaks at f_h and f_l and raise the minimum impedance at f_b. But this is incidental, and the relative heights are of little importance. Thus the idea of tuning the box so that the impedance peaks at f_h and f_l are equal, misses the real point. In the impedance curve of a loudspeaker in a box, the most useful information is not the values of the impedances, so long as box and vent damping is not too severe, but the values of the frequencies f_h, f_b, and f_l. Knowledge of these three frequencies alone enables a box alignment to be checked by Eqs. (105) and (106).

It should be clear that flatness of the impedance characteristic is no indication of flatness of acoustic response. Take as an analogy a coupled pair of tuned circuits. When the output voltage, or more exactly the transfer impedance, is maximally flat, the input impedance has two peaks. If one parameter is known, say the ratio of primary to secondary Q, the transfer impedance can be deduced from the input impedance, just as we do for loudspeakers in Eqs. (105) and (106). But a flat input impedance characteristic does not indicate a flat transfer impedance. In a loudspeaker, the impedance characteristic has greater peaks, whose height depends purely on the acoustic damping, though this contributes little to the overall system damping, and thus the overall frequency response.

XIII. EFFECTIVE REVERBERATION TIME

An objection sometimes made to the use of vented boxes is that the slope of attenuation beyond cutoff, 24 dB per octave, is much steeper than the 12 dB per octave of a speaker on an infinite baffle, and therefore the transient response is worse. In a low-pass filter, the ringing associated with steep attenuation slope is virtually removed by the use of Thompson or critically damped responses. But in high-pass filters such as are considered here, there is always some overshoot with filters of order two or more. To estimate its effect on a listener we use the concept of "effective reverberation time."

Imagine that we have a source of sound in a room which has built up a steady field. The source is then stopped. The sound in the room does not stop immediately, but dies away gradually. The time taken for the sound to decay is called the reverberation time, defined as the time taken for the sound pressure in the room to fall 60 dB from its original value. In small rooms the reverberation time will probably lie between 400 ms for a highly damped room to 1 s or more for a live one.

When the sound passes through two reverberant rooms in cascade, the law of the resulting overall reverberation time is not well establishd, but calculations on cascaded high-pass filters suggest that rms addition gives at least a guide. In any case it would appear that an added reverberation time of 200–300 ms should not appreciably color the reproduction.

When a transient is applied to a filter and it rings, the effect is perceived by the ear, or brain, as an extension of the transient event in time. Hence the expression "hang-over." To express the effect of the ringing then, an idea is borrowed from architectural acoustics, and the effective reverberation time of a filter is defined as the time taken, after a step function is applied, for the amplitude of the envelope of ringing to fall 60 dB below the amplitude of the original step function.

For the higher order filter functions, with two or more second-order factors, only the most lightly damped factor need be considered. For, by the time the ringing due to the most lightly damped factors is 60 dB down, the ringing due to the more heavily damped factors is negligible. This eases computation greatly.

Actually, at low frequencies the reverberation time defined above will be rather longer than the time the sound is perceived by the listener. To see why, we consult the much abused Fletcher–Munson curves [4, Fig. 12.11].

Suppose, for example, that the original sound is at 100-phon level. This is probably the maximum a system could reproduce, or a listener tolerate. Now at 50 Hz the threshold of hearing is 51 dB above reference level, that is, 49 dB below our arbitrary listening level. At 25 Hz the threshold of hearing is 67 dB above reference

Table IV. Reverberation times for various alignments.

Type of response	B_2	C_2	C_2	B_4	C_4	B_6	C_6	C_6	C_6
Q_t (for second order alignments)	0.707	1.000	1.414	—	—	—	—	—	—
k (for sixth order alighments)	—	—	—	—	—	1.000	0.600	0.414	0.268
Alignment numbers	—	—	—	5	8	15, 20, 26	17, 22	19, 24	25, 27
Time (in periods of cutoff frequency)	1.63	2.24	3.17	2.87	7.09	4.77	6.79	9.67	14.86
Time for 50 c/s cutoff (mS)	33	45	63	57	142	95	136	193	297

level, that is, only 33 dB below our arbitrary listening level. At 25 Hz, therefore, the effective reverberation time for the listener cannot be greater than the time in which the sound level falls 33 dB, i.e., about half the reverberation time as defined conventionally. Thus at low frequencies in general, the conventional definition based on a 60-dB fall in level yields a reverberation time rather longer than a listener will hear. (This is probably the reason for the observed increase in optimum reverberation time at low frequencies, see [4, Fig. 11.11].)

In a filter which cuts off sharply, the major ringing frequency will be close to the cutoff frequency. Also for a given shape of response curve the reverberation time can be expressed as a certain number of cycles of the cutoff frequency (see Table IV), i.e., the reverberation time increases with decreasing cutoff frequency. On the other hand, below, say, 50 Hz, its effect on the listener will decrease at approximately the same rate. Thus for all filters of a given response curve shape, the figure for 50 Hz should give a rough idea of the maximum reverberation time, as perceived by the listener.

Calculated reverberation times are given in Table IV. The first three alignments are of second order, corresponding to a loudspeaker on an infinite baffle. For these, the values of Q_t are shown. Note that the reverberation time, though low, doubles as Q_t increases from 0.707 to 1.414, that is, when the frequency response goes from maximally flat to a 4-dB peak. The times for 50-Hz cutoff are all below 200 ms, except for the last ($k = 0.268$), which is the very steepest.

It thus appears that a properly adjusted vented box, even with amplifier assistance (auxiliary filtering), need cause no perceptible coloration due to ringing. But it is important to emphasize that the adjustment must be correct. Table IV shows that the addition of a 4-dB peak to the response of a speaker on an infinite baffle can double the reverberation time. Being low in the first place it remains tolerable. But in the case of a vented box, particularly with an auxiliary filter, a doubled reverberation time would be more serious. Again, this emphasizes the importance of adequate damping (for correct value of Q_t) by the amplifier.

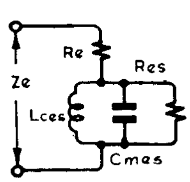

Fig. 15. Simplified equivalent electrical circuit of loudspeaker.

XIV. MEASUREMENT OF LOUDSPEAKER PARAMETERS

In earlier sections it was shown how the required response can be obtained from a loudspeaker and box if several parameters are known. The question remains, how are these parameters found?

Properly, this information should be available from the loudspeaker manufacturer. This is particularly important for equipment produced in quantity, where it is important to know not only the mean values but also the tolerances. However, in the absence of published figures, or to check them, the following procedure will provide the information.

Procedures for measuring Q are given in [2, p. 13], but the method used seems too laborious and inaccurate. The method outlined hereafter can be understood by considering Figs. 15 and 16. Figure 15 is derived from Fig. 4; only this time we omit the vented box and we ignore L_e and R_{sh} which take effect at much higher frequencies. Now

$$Q_a = \omega_s C_{mes} R_{es} \qquad (91)$$

$$Q_e = \omega_s C_{mes} R_e. \qquad (92)$$

These quantities, defined earlier in Eqs. (71) and (72) in terms of the acoustic equivalent circuit, are defined here in terms of the electrical equivalent circuit. We define r_0 as the ratio of the impedance at resonance, $R_{es} + R_e$, to the dc resistance of the voice coil R_e. Now we take another arbitrary impedance which is presented at two other frequencies f_1 and f_2 on the flanks of the curve, and we call its ratio to the dc resistance r_1. Then

$$f_1 f_2 = f_s^2. \qquad (93)$$

Physically, this means that the curve is symmetrical on a logarithmic frequency scale. In experimental work it provides a handy check. Now we can find

$$Q_a = [f_s/(f_2-f_1)][r_0^2-r_1^2)/(r_1^2-1)]^{1/2} \qquad (94)$$

and

$$Q_e = Q_a/(r_0-1). \qquad (95)$$

If additionally we choose r_1 such that

$$r_1 = \sqrt{r_0} \qquad (96)$$

then Eq. (94) is simplified to

$$Q_a = \sqrt{r_0}\, f_s/(f_2-f_1). \qquad (97)$$

The interesting feature of these expressions is that they involve no approximations, and thus hold for all values of Q. Furthermore around the value $\sqrt{r_0}$ the curve has its greatest slope. Thus the frequencies f_1 and f_2 can be

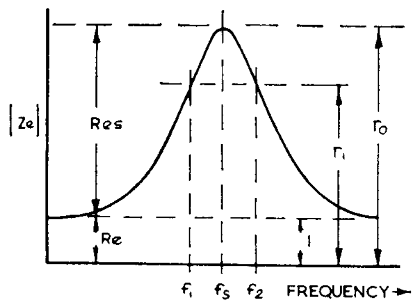

Fig. 16. Typical impedance curve of loudspeaker, modulus of Z_e in Fig. 15.

found most accurately. This is especially important since the calculation involves a comparatively small difference between large numbers f_2-f_1.

Usually Q_a takes account of the acoustic resistances in the loudspeaker. But if the voice coil has a short-circuited turn by accident or design, e.g., an aluminum former, this will appear in Q_a, even though its physical nature is similar to Q_e. (But eddy current losses in the pole piece or front plate appear in R_{sh}.)

Fig. 17 shows the test circuit. V is a voltmeter of impedance much higher than the loudspeaker. Throughout the readings, the generator is adjusted so that the reading of V is constant. The value is not of great importance, but a standard test figure is one volt. The accuracy of this voltmeter is not important so long as it is independent of frequency. A is an ac ammeter which reads the current into the speaker with the fixed voltage across its terminals. Again, since we are interested only in the shape of the impedance curve, the absolute accuracy of this instrument is not important so long as the meter reading is linear. However, to set the relative current due to R_e, first we measure R_e with dc on a Wheatstone bridge, and then a calibrating resistor R_c of similar value. Connecting R_c to the test terminals and applying the standard test voltage at say, f_s, a current value I_c is found on the ammeter A. Then the current I_e which corresponds to R_e is found by

$$I_e = I_c R_c / R_e. \qquad (98)$$

Now the loudspeaker is suspended in air as far from reflecting surfaces as is practical and connected to the test terminals instead of R_c. The generator is adjusted to the speaker resonant frequency f_s, indicated by minimum current I_o. Thus r_o is found:

$$r_o = I_e / I_o. \qquad (99)$$

Now the current $\sqrt{(I_e I_o)}$ is found corresponding to the ratio $\sqrt{r_o}$ and the frequencies either side of resonance, where this current value is read. These are f_1 and f_2 and they should be read to as close an accuracy as the test gear will allow. Eq. (93) provides a check on the

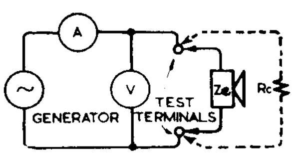

Fig. 17. Test circuit schematic for measurement of loudspeaker parameters.

method, and Eqs. (97) and (95) give Q_a and Q_e.

The next problem is to find the value of V_{as}, the volume of air equivalent to the loudspeaker compliance. For this, the loudspeaker is placed in a totally enclosed unlined box whose internal volume V_b is known, remembering that allowance must be made for bracing and the volume displaced by the speaker. It is important that this box be free of air leaks. If these occur we will read part of the curve of Fig. 5, around f_h. Thus care should be taken, not only in the construction of the box and in the mounting of the speaker, but also in the way the speaker leads are taken through the walls of the box. Solid terminals are preferred.

Another precaution may be necessary. In Figs. 15 and 16, from which we derived Eqs. (93), (94), (95), and (97), we assumed that the effect of the inductance L_e is negligible. In fact, L_e interacts with the parallel combination of L_{ces} and C_{mes} to produce a series resonance at f_n in Fig. 5, where the nominal impedance is measured. If this frequency, usually 400–600 Hz, is well above the speaker resonance f_s, so that there is little disturbance of the curve at f_2 of Fig. 16, the accuracy of the measurements will be unaffected. But if f_s is above 150 Hz, which can occur with small speakers and becomes even more likely when the speaker is placed in the box for the last test, the likelihood of inaccurate results increases.

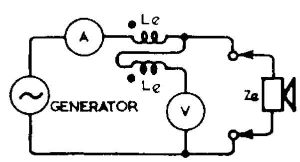

Fig. 18. Modification of Fig. 17 to cancel effect of loudspeaker inductance L_e.

This could be avoided by connecting in the circuit of Fig. 18 a bifilar inductance whose value L_e in each half is equal to the inductance of the voice coil. It is preferable, and not difficult, to wind this with an air core. In measuring L_e of the loudspeaker, it is important to measure it at a frequency well away from f_n, say, 10 kHz. Also it is important to measure it as an inductance in parallel with a resistance (D or tan δ scale, *not* the Q scale of a bridge), for the Q of the inductance at 10 kHz is usually of the order of one which can lead to serious error if the measurement is made as an inductance in series with a resistance. With a high-impedance voltmeter V, error due to series resistance of the inductor should be negligible.

If the new resonant frequency in the closed box f_{sc} is found, the ratio of volume is usually given as

$$V_{as}/V_b = (f_{sc}/f_{sa})^2 - 1 \qquad (100)$$

where f_{sa} is the resonant frequency of the speaker in air which we previously called f_s. However, this expression ignores the change in the acoustic mass M_{as} of 1.05 to 1.25 times which results from placing the speaker in the box. A more accurate method is to repeat the previous procedures for finding Q_e. Then if we call Q_{ea} and Q_{eo} the values of Q_e measured in air and in the closed box, respectively, then

$$V_{as}/V_b = [(f_{sc}Q_{ec})/(f_{sa}Q_{ea})] - 1. \qquad (101)$$

Also the ratio of the acoustic masses in air and in the closed box

$$M_{asa}/M_{asb} = f_{sc}Q_{ca}/f_{sa}Q_{ec} \qquad (102)$$

should lie between 0.8 and 0.95.

With V_b known, V_{as} can be calculated. The size of V_b is not critical, but should not be too large, otherwise the ratio f_{sc}/f_{sa} becomes close to unity, and the accuracy of the V_{as}/V_b calculation falls. This can be seen from Eq. (100). Finally the values of f_{sa} and Q_{ea} are adjusted to take account of the change in M_{as} when the speaker is placed in the box. Thus

$$f_{sb} = f_{sa}(M_{asa}/M_{asb})^{1/2} \qquad (103)$$

$$Q_{eb} = Q_{sa}/(M_{asa}/M_{asb})^{1/2}. \qquad (104)$$

Thus the efficiency η_{ob} can be calculated from Eq. (77). This gives the result, rather surprising at first sight, that the electroacoustic conversion efficiency of a loudspeaker in the piston range can be calculated from electrical measurements alone.

The following alternative method is useful, particularly when the loudspeaker has to be placed in a box whose size is already determined or as a final check on a previously calculated box, or again if it becomes too difficult to seal the loudspeaker in the test box.[5]

First the vent, if adjustable, is made to resonate with the box somewhere near the speaker resonant frequency, but this is not very important. Then the three frequencies f_l, f_b, and f_h of Fig. 5 are found as accurately as possible. Special care is needed in reading f_b as the curve has a flat bottom.

From these readings we find f_{sb}, the resonant frequency of the speaker when mounted in the box,

$$f_{sb} = f_h f_l / f_b \qquad (105)$$

and the compliance ratio C_{as}/C_{ab}, i.e.,

$$V_{as}/V_b = (f_h^2 - f_b^2)(f_b^2 - f_l^2)/f_h^2 f_l^2. \qquad (106)$$

With the speaker resonant frequency in air f_{sa} already known and f_{sb} known from Eq. (105), we find the mass ratio M_{asa}/M_{asb} from Eq. (103), and then Q_{eb} from Eq. (104). Q_a is adjusted to Q_{ab} in a similar manner. By reference to Table I and Fig. 7, a suitable alignment can be found, thus setting the final values of f_b and Q_t. Note that Q_t is due to the parallel combination of 1) Q_{ab} and 2) Q_{eb} modified by the amplifier.

To estimate the value of Q_b, the "Q of the box and vent circuit," we measure I_b, the current through the speaker at f_b, with the input voltage held constant as before. Then

$$Q_b = (\omega_b/\omega_s)(C_{ab}/C_{as})[(1/Q_e) + (1/Q_a)][(I_b - I_o)/(I_e - I_b)]. \qquad (107)$$

[5] Experience gained since the writing of this paper shows that accurate results are more easily obtained with this second method. Using a vented box is especially preferred if the speaker being measured has a low resonant frequency and if the testing box is fairly small. In such cases, small leaks in the "totally enclosed" box or around the loudspeaker pad ring can produce a virtual vent which produces the familiar twin peaks of loudspeaker impedance. But if the lower peak is below the limit of measurement, say, below 10 or 15 Hz, it could easily happen that the remaining upper peak would be taken as the single peak of a closed-box system with dire results.

Note that because the difference between I_e and I_b will be small, the readings must be taken carefully.

Comparing Eq. (107) with Eq. (89), it can be seen that

$$(A_b)_{max} = 1/\{1 + [Q_a Q_e / Q_t (Q_a + Q_e)][(I_e - I_b)/(I_b - I_o)]\}. \qquad (108)$$

This greatly simplifies the estimation of $(A_b)_{max}$.

A worked example of this method is given in the Appendix.[6]

XV. EXPERIMENTAL WORK

When the work was started from which this paper derived, it was necessary first to find the parameters for a number of loudspeakers. To date about fifty have been measured. In the case of one speaker, the effect of a number of modifications was observed; in the rest, usually one and occasionally two or three samples have been checked. The results obtained give confidence in the method. For example, from the readings and knowing other parameters, it is possible to calculate the flux density, and the values obtained give good correlation with readings on a flux meter. Changes of parameters during production can also be detected.

Some generalizations from the results have been mentioned earlier. For example, it was found that Q_a varies between about 3 and 10, which is high compared with the Q_t values of 0.2 to 0.6 required in Table I. Thus it was apparent that acoustic resistance usually has little effect on the damping of a speaker in a well-designed system. Values of Q_e varied from 0.2 to 0.5 in the case of high-quality speakers, through 0.5 to 1.0 in the better commercial grades of speakers, to 2 and even 3 in the case of some low-priced speakers.

Similarly efficiencies, for radiation from one side of an infinite baffle, ranged from −24 dB (0.4%) for low-priced speakers through −20 dB (1%) for medium-grade to −14 dB (4%) for high-quality speakers.

However, one must resist the tempting generalization that it is possible to rate the overall quality of a speaker by its Q_e or even its efficiency. For example, if efficiency is made higher and Q_e lower by reducing the cone mass M_{ms}, trouble with "break up" may result at middle frequencies. In fact while the best 8-in speaker tested had a Q_e of 0.33, there was one sample with good clean response at high frequencies with a high Q_e of 1.7 and another with Q_e below 1 which was less acceptable. It must be remembered that these readings, and the paper in general, are concerned only with low-frequency performance.

As a result of the design theory, a number of boxes have been made. In the absence of reliable measurements of sound pressure, all that can be said is that they gave a good improvement in clean low-frequency response, and that the cutoff frequencies are near the predicted values. Some particularly gratifying results have been obtained

[6] Experimental work, using the above method indicates that in practical boxes Q_b is often of the order of 10. This difference from the calculated values of 30 or more may be due to frictional losses in the timber. It is shown in Section XI that when Q_b is 10, the frequency response error is still only 1 to 2 dB. However, if there are sufficient air leaks, or if the cavity damping is excessive, as when the box is completely stuffed with underfelt, Q_b can fall below 5.

with 5-in speakers in modest boxes with response down to 80 Hz.

XVI. CONCLUSION

The work described herein was begun as an advanced development project in an attempt to obtain good low-frequency response from loudspeakers in small boxes. Unfortunately, no "revolutionary concept" was uncovered that offers something for nothing. On the other hand, it has provided a reasonably precise method of design that was previously lacking.

In general, a system with good flat response down to a predictable cutoff frequency can be designed, if the necessary parameters Q_e (and Q_a), V_{as}, and f_s are known for the loudspeaker. The box volume is closely proportional to the inverse square of cutoff frequency, which can be varied over a wide range. The output impedance R_g of the amplifier has a large effect in controlling the response, especially at f_h, the higher frequency of maximum impedance. Whether R_g needs to be positive, zero, or negative depends on the type of alignment and the Q parameters of the speaker. On the evidence available, acoustic resistance damping of the vent has no advantage, and is wasteful of box volume or bandwidth.

The advantages accruing from a predictable design include the possibility of optimum design of "rumble" filters. At frequencies below cutoff where negligible acoustic output is produced, these relieve the amplifier and loudspeaker of high signal amplitudes and thus minimize an annoying source of intermodulation distortion. Carried a step further, the use of auxiliary electrical filters makes it possible to trade box volume for low-frequency power capability of the amplifier.

Another way of reducing box volume is to increase the mass of the loudspeaker cone. But since this also reduces efficiency, it may be considered as a further example of trading amplifier size for box size, only this time the amplifier must deliver increased power over the whole audio spectrum. Again, the box volume may be reduced if a smaller diameter loudspeaker is used. The danger here is that the speaker excursion increases, but it is a good solution if the speaker is capable of a long linear excursion, or if the power output and/or low-frequency response is restricted.

The size of the magnet, or more precisely the flux density B, has a great influence on performance. Both efficiency, hence acoustic output, and Q_e vary with B^2; so it is clear that the saving of pennies on a smaller magnet can be poor economy.

The parameters needed for vented-box design can be measured with normal electrical measuring equipment together with a test box of known net internal volume. Nevertheless it is suggested to loudspeaker manufacturers that it is in their interest, as well as the user's, to publish typical values of Q_e, Q_a, V_{as}, and x_{max}, as well as f_s. These parameters are more useful to the system designer than, for example, flux density or total flux. Their publication would help ensure that the manufacturer's product is used to the best possible advantage.

The totally enclosed box has been mentioned only in passing, since it is well covered in [2]. But it should be noted that if a totally enclosed box is chosen with the same volume as that of alignment no. 5, the cutoff frequency is 1.55 times higher. With smaller boxes, the advantage decreases, though with practical sizes it is still appreciable. With larger totally enclosed boxes, the cutoff frequency can never fall below f_s, while the Chebyshev vented box alignments can extend the response considerably below f_s.

The greatest advantage of a vented box over an infinite baffle is the reduction of loudspeaker excursion, permitting higher power output or lower distortion. To this advantage, the present paper adds, it is hoped, a greater flexibility in design. The only apparent disadvantage of a vented box is in the transient response, but in fact the ringing is only perceptible with a misadjusted alignment. With proper adjustment, the effective reverberation time, though longer than that of a properly adjusted infinite baffle, is not long enough to appreciably color the sound in the listening room.

Finally, it is emphasized again that the acoustic response is due to the combination of speaker plus box plus amplifier as an integrated whole.

APPENDIX: WORKED EXAMPLE

This refers to a purely imaginary speaker, the readings being chosen to simplify the calculations. However, the readings would be typical of a medium-quality 8-in speaker.

Measurement of Speaker Parameters Q_a, Q_e, V_{as}, and f_s

With a Wheatstone bridge we find

dc resistance of speaker R_e = 4.00 ohms
dc resistance of calibrating resistor R_c = 5.00 ohms.

Now we place R_c in the test circuit of Fig. 17 and find that when V reads 1 volt,

$$I_c = 180 \text{ mA}.$$

Now

$$I_c R_c = 0.180 \times 5.00 = 0.900.$$

Since this is 10% below the observed reading of 1 volt, one or both of the meters is inaccurate, but this is unimportant so long as their readings are constant with frequency and the reading of ammeter A is linear.

Then from Eq. (98),

$$I_e = I_c R_c / R_e = (0.180 \times 5.00)/4.00 = 225 \text{ mA}.$$

We now suspend the loudspeaker in air as far from reflecting surfaces as possible and read the minimum current I_0 which is 25 mA at 55.0 Hz (f_{sa}, the speaker resonant frequency in air).

Then from Eq. (99),

$$r_0 = I_e/I_0 = 225/25 = 9$$

$$\sqrt{r_0} = \sqrt{9} = 3$$

$$\sqrt{(I_0 I_e)} = \sqrt{(225 \times 25)} = 75 \text{ mA}.$$

With the voltmeter V reading a constant 1 volt, the ammeter A reads 75 mA at 44.0 and 68.75 Hz.

First we use this reading to check $f_{sa} = \sqrt{(44.0 \times 68.75)}$ from Eq. (93) = 55.0 Hz as before. Then from Eq. (97),

$$Q_a = f_a \sqrt{r_0}/(f_2 - f_1) = (55 \times 3)/(68.75 - 44) = 6.67$$

and from Eq. (95),

$$Q_e = Q_a/(r_o-1) = 6.67/(9-1) = 0.833.$$

The speaker is now placed in a vented box whose net volume is 1000 in^3 and we read the frequencies defined in Fig. 5,

$$f_h = 100\,\text{Hz};\quad f_b = 60\,\text{Hz};\quad f_l = 30\,\text{Hz}.$$

Then from Eq. (105),

$$f_{sb} = f_h f_l/f_b = (100\times 30)/60 = 50\,\text{Hz}$$

and from Eq. (106),

$$V_{as}/V_b = (f_h^2-f_b^2)(f_b^2-f_l^2)/f_h^2 f_l^2.$$

Computation is easier if we rewrite Eq. (106) as

$$V_{as}/V_b = (f_h+f_b)(f_h-f_b)(f_b+f_l)(f_b-f_l)/f_h^2 f_l^2$$

i.e.,

$$\begin{aligned} V_{as}/V_b &= (100+60)(100-60)(60+30)(60-30)/100^2\times 30^2 \\ &= (160\times 40\times 90\times 30)/(100\times 30\times 100\times 30) \\ &= 1.92 \end{aligned}$$

i.e.,

$$V_{as} = 1.92\times 1000 = 1920\,\text{in}^3.$$

In the vented box, the speaker resonant frequency has dropped $f_{sb}/f_{sa} = 50/55 = 0.909$ times. Thus from Eq. (103),

$$M_{asa}/M_{asb} = (0.909)^2 = 0.826$$

and from Eq. (104),

$$Q_{ab} = 6.67/0.909 = 7.33$$

while

$$Q_{eb} = 0.833/0.909 = 0.917.$$

At f_b the current I_b was read as 220 mA. Then from Eq. (107), the Q of the box plus vent

$$\begin{aligned} Q_b &= (f_b/f_s)(C_{ab}/C_{as})[(Q_a+Q_e)/Q_aQ_e][(I_b-I_o)/(I_e-I_b)] \\ &= [60\times(7.333+0.917)\times(220-25)]/[50\times 1.92\times 7.33\times 0.917\times(225-220)] \\ &= (60\times 8.25\times 195)/(50\times 1.92\times 7.33\times 0.917\times 5) \\ &= 29.9. \end{aligned}$$

From Eq. (108) the maximum box loss in the quasi-Butterworth alignment described below, where $Q_t = 0.347$, is

$$\begin{aligned} (A_b)_{max} &= 1/\{1+[Q_aQ_e/Q_t(Q_a+Q_e)][(I_e-I_b)/(I_b-I_o)]\} \\ &= 1/\{1+(7.33\times 0.917\times 5)/(0.347\times 8.25\times 195)\} \\ &= 1/1.060 \end{aligned}$$

which is equivalent to 0.5 dB.

Efficiency η from Eq. (77)

$$\begin{aligned} \eta_{ob} &= 8.0\times 10^{-12} f_s^3 V_{as}/Q_e \\ &= (8.0\times 50^3\times 1920)/(10^{12}\times 0.917) \\ &= 2.09\times 10^{-3} \end{aligned}$$

which is equivalent to -26.6 dB in a box, or -23.6 dB on an infinite baffle (i.e., a true infinite baffle, not a totally enclosed medium-sized box which gives the same efficiency as a vented box), or -20.6 dB on a true infinite baffle, taking into account radiation from both front and back.

Thus if the speaker is mounted in a box and fed a 5-watt amplifier, the acoustic power output will

$$W_{ao} = \eta_{ob} W_{ei} = 5\times 2.09\times 10^{-3} = 0.0104\ \text{Wa}$$

If we assume a listening room of 16 × 12½ × 10 = 2000 ft^3, then from [4, p. 418, Fig. 11.12] an acoustic power of 0.003 watt provides +80-dB intensity level. Our output is 10.4/3 times, i.e., 5.4 dB greater than this; therefore the system is capable of a peak +85-dB intensity level.

Peak Excursion x_{pk}

We assume an alignment where the box is tuned to the same frequency as the loudspeaker, i.e., 50 Hz. This is typical of Butterworth alignments. Then the fixed part of the expression for x_{pk} in Eq. (84) is

$$(1.31\times 10^5\times\sqrt{W_{ao}})/f_b^2 S_d.$$

Now if the effective piston diameter is 7 in, i.e.,

$$S_d = \pi\times 3.5^2 = 38.5\,\text{in}^2$$

then the expression becomes

$$1.31\times 10^5\times\sqrt{0.104}/(50^2\times 38.5) = 0.139\ \text{in}.$$

Now the maximum value of the frequency-sensitive expression for a vented box in the useful band (above f_b) in Fig. 10 is approximately one quarter. Thus

$$x_{pk} = 0.139/4 = \pm 0.035\ \text{in}$$

compared with ±0.098 in in a totally enclosed box (infinite baffle).

Box Design

First suppose we wish to obtain the best results with the original 1000-in^3 box. Allowing 10% for the bracing and volume displaced by the speaker, the optimum inside dimensions would be $\sqrt[3]{1100}\times(0.8,\ 1.0,\ 1.25)$ in, i.e., 8.28 × 10.33 × 12.9 in, say 8¼ × 10¼ × 13 in. This would need to be checked in case the original assumption of 10% was incorrect. Assuming that the dimensions are

Table V. Computation of three Butterworth alignments for imaginary speaker.

Type of alignment		QB_3	B_4	B_5	B_6(i)
C_{as}/C_{ab}		1.92	1.414	1.000	2.732
V_b (cubic inches)		1000	1358	1920	704
Box	Height (in.)	13	14	16	11½
	Width (in.)	10¼	11½	13	9
	Depth "d" (in.)	8¼	9	10	7½
Cutoff frequency f_3 (c/s)		58.5	50	50	50
Box frequency f_b (c/s)		54.7	50	50	50
L_v/S_v (in.$^{-1}$)		1.56	1.37	0.97	2.65
S_v (in.^{2})		7.69	10.07	16.25	4.50
Vent height "l" (in.)		¾	⅞	1¼	½
Q_t		.347	.383	.447	.299
$(Q_e)_{total}$		.364	.404	.476	.312
R_g/R_e		−.600	−.560	−.481	−.660

then in a box similar to Fig. 9, the width of will be 10¼ in. The length of the tunnel will be ¾ in, together with two thicknesses of timber (say in each) plus a ½-in square stiffener on the top rear edge of the shelf, giving a total tunnel length of 9¾ in.

The simplest alignment for $C_{as}/C_{ab} = 1.92$ is a third-order quasi-Butterworth between alignments no. 4 and 5. From Fig. 7 (b),

$$f_3/f_s = 1.17, \text{ thus } f_3 = 50 \times 1.17 = 58.5 \text{ Hz}$$

$$f_3/f_b = 1.07, \text{ thus } f_b = 58.5/1.07 = 54.7 \text{ Hz.}$$

Thus

$$\omega_b^2 = 1.18 \times 10^5$$

and for the tunnel, from Eq. (61),

$$\begin{aligned}(L_v/S_v)_{required} &= 1.84 \times 10^8/\omega_b^2 V_b \\ &= 1.84 \times 10^8/1.18 \times 10^5 \times 10^3 \\ &= 1.56 \text{ in}^{-1}.\end{aligned}$$

Now if the tunnel height l = ¾ in, then area

$$S_v = 10\tfrac{1}{4} \times \tfrac{3}{4} = 7.69 \text{ in}^2$$

and

$$\begin{aligned}(L_v/S_v)_{end} &= 0.958/\sqrt{S_v} \\ &= 0.958/\sqrt{7.69} \\ &= 0.34 \text{ in}^{-1}\end{aligned}$$

$$\begin{aligned}(L_v/S_v)_{tunnel} &= 9.75/7.69 \\ &= 1.27 \text{ in}^{-1}.\end{aligned}$$

Thus

$$(L_v/S_v)_{available} = 1.61 \text{ in}^{-1}$$

which is about as close as can be obtained with the tolerances on the small dimension (¾ in) of l.

Amplifier Output Impedance R_g

Now by interpolation,

$$Q_t = 0.347$$

and since $Q_{ab} = 7.33$, $Q_{eb} = 0.917$, and from Eq. (70),

$$1/Q_t = 1/Q_a + 1/Q_e(1 + R_g/R_e).$$

Thus

$$1/0.347 = 1/7.33 + 1/0.917(1 + R_g/R_e).$$

Hence

$$R_g/R_e = -0.60.$$

Notes

1) $(L_v/S_v)_{end}$ is small compared with $(L_v/S_v)_{tunnel}$, and since the vent area is already small compared with the piston area, a simple hole in the front panel would be quite impractical as a vent. Its area would need to be about 1 in^2.

2) The dimension l (¾ in) is fairly critical.

3) Q_a has little effect on Q_t. The negative impedance required is fairly high but quite practical.

For comparison three Butterworth alignments have also been computed for this imaginary speaker so that the effect of amplifier filtering can be assessed (Table V). All three have cutoff frequencies of 50 Hz. But while B_4 has no filtering, B_5 has a simple CR filter which is -3 dB at 50 Hz ($CR = 3180$ μs), and B_6 has a peak 6 dB high at 53.5 Hz before it falls off at the rate of 12 dB per octave ($y = -1.732$, $f_{aux} = 50$ Hz).

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Loudspeakers on Damped Pipes*

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First Locanthi's horn analog is shown to be well suited for modeling transmission-line loudspeaker systems. The circuit can accommodate arbitrary flare shapes and allows damping to be included as any combination of series and parallel losses. Four empirical parameters are then developed to simulate the effects of lining or stuffing. Finally three optimized transmission-line geometries are presented, which can be described in terms of generalized alignments using familiar Thiele–Small parameters. Using one of these new alignments, a transmission line can match the frequency response and efficiency of a comparable closed box, but with reduced cone excursion.

0 INTRODUCTION

The Acoustic Labyrinth loudspeaker enclosure was patented by Olney in 1936 and analyzed in a paper published the same year [1]. Olney proposed to correct the defects of simple open-back loudspeaker cabinets by taking a different approach altogether: ". . . the major problem was taken to be the elimination of cavity resonance, and the course pursued was the direct but drastic one of abolishing the troublesome cavity." The loudspeaker was to be mounted at one end of a folded tube lined with a material whose absorption coefficient increased with frequency. In theory this would provide useful summation of front and rear radiation at low frequencies while attenuating higher order resonances.

Versions of the labyrinth were produced by Stromberg-Carlson and others as late as 1950. Interest in the design as a high-quality loudspeaker enclosure was revived by Bailey in 1965, who used fibrous stuffing instead of absorptive lining [2]. It remains a favorite of loudspeaker experimenters and audio enthusiasts. To its devotees, the damped transmission line delivers a neutral, uncolored performance that cannot be matched by vented or closed boxes.

The length and shape of the pipe, the density and kind of material used for damping, and the optimum loudspeaker characteristics have been debated for more than 30 years. However, objective information is rare and is confined mostly to the frequency response measurements of a few successful designs. The goals of this project were to develop a computer analog capable of modeling transmission-line systems, to validate the model by testing a variety of designs, and to develop basic performance relationships similar to the Thiele–Small analysis of vented boxes.

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1 SYMBOLS USED

f_3 = −3-dB corner frequency of low-frequency rolloff
f_P = nominal quarter-wave pipe resonance frequency
f_0 = actual pipe fundamental resonance frequency
f_S = loudspeaker resonance frequency
f_L = frequency of lower impedance peak
f_H = frequency of first upper impedance peak
Bl = loudspeaker force factor, N/A
Q_{MS} = Q of loudspeaker mechanical system
Q_{TS} = total Q of loudspeaker
R_{ES} = dc resistance of voice coil
R_{MS} = resistive component of loudspeaker mechanical system
L_{ES} = inductance of voice coil
L_{MS} = inductive component of loudspeaker mechanical system
C_{MS} = capacitive component of loudspeaker mechanical system
V_{AS} = volume of air having compliance equivalent to loudspeaker cone suspension
V_P = internal volume of pipe (including coupling chamber)
ρ = density of air, = 1.2 kg/m^3
c = velocity of sound, = 344 m/s.

These are familiar symbols, in most cases identical to those used for vented-box analysis. The symbol f_P is a reference frequency based on the physical length of the air path, such as "a nominal 100-Hz pipe." The pipe's actual fundamental resonance f_0 is affected by a number of additional factors, including end correction, pipe geometry, and stuffing material.

2 BASIC ANALOG CIRCUIT MODEL

A damped pipe can be analyzed as a horn with losses. At least three earlier papers describe one-dimensional horn analogs capable of modeling arbitrary flare shapes. In one study [3] the flare is approximated as a series of exponential sections. In another, conical sections are used [4]. A third approach, originated by Locanthi, predates the others and was originally built as an analog transmission-line model using real inductors and capacitors [5].

Locanthi's analog includes the familiar mobility model of the loudspeaker itself, followed by an *LC* ladder in which series inductors represent air compliance and shunt capacitors represent mass. Each *LC* section is equivalent to a cylindrical element of specified diameter and length. If the lengths of individual elements are very small in relation to wavelength, then the analog is surprisingly accurate.

Fig. 1 shows the circuit as modified to model transmission-line loudspeaker systems. The model uses 32 sections so that arbitrary flares can be entered element by element in a fairly short time, yet there are enough sections to handle a reasonable bandwidth. The usable upper frequency limit for a 32-element transmission line is roughly 800 divided by its length in meters.

Three modifications were made to Locanthi's circuit. First, shunt resistances were added to model damping losses. These are shown as variable resistors because damping may be frequency dependent. Series resistances could be included to represent leakage losses, but their effect is negligible.

Second, since the loudspeaker may not be mounted at the end of the pipe but at some intermediate location, an optional 16-element closed stub was inserted at the pipe throat.

Third, a transformer between the loudspeaker and the horn throat has been omitted. The transformer was there to explicitly match the throat area to a larger or smaller driver cone area. Its effect can be duplicated by scaling the circuit values for either the horn or the loudspeaker. If there is a coupling chamber between the cone and the pipe throat, it can be represented by an additional series inductor. In practice, the coupling chamber compliance is simply included in the value of L_1.

The easiest way to calculate loudspeaker circuit values is first to divide R_{ES} and L_{ES} by $(Bl)^2$. Then C_{MS} in farads is numerically equal to the moving mass in kilograms, and L_{MS} can be derived from f_S. With this information plus Q_{MS}, the value of R_{MS} can be found.

Calculating transmission-line values is not much more complicated. Let

s_d = driver cone area, m^2
S_0 = throat area, m^2
S_n = section area, m^2
K_s = S_n/S_0
x = section length, m.

Then,

$$L_n = \frac{K_s x}{\rho c^2 S_0}$$

$$C_n = \frac{\rho S_0 x}{K_s} .$$

If necessary, these values can then be impedance scaled to the ratio $(S_0/S_d)^2$. Detailed information about the derivation of the analog and the calculation of acoustic loads can be found in Locanthi [5].

Following Thiele–Small loudspeaker system analysis we assume that radiation loading is negligible in the (bass) frequency range of interest. In this range, however, acoustic loads for cone and pipe radiation can be accurately included as simple *RC* shunts if desired [6].

As with a vented-box analog, the net system output is equivalent to the complex sum (including phase reversal) of cone output and pipe output. In most practical transmission-line systems any effects of mutual coupling between the two are small enough to be ignored. The effects of pipe bends and folds are also ignored. In a typical transmission line, they can be expected to occur above the frequency range of useful pipe output.

The ability of this circuit to model a fairly compli-

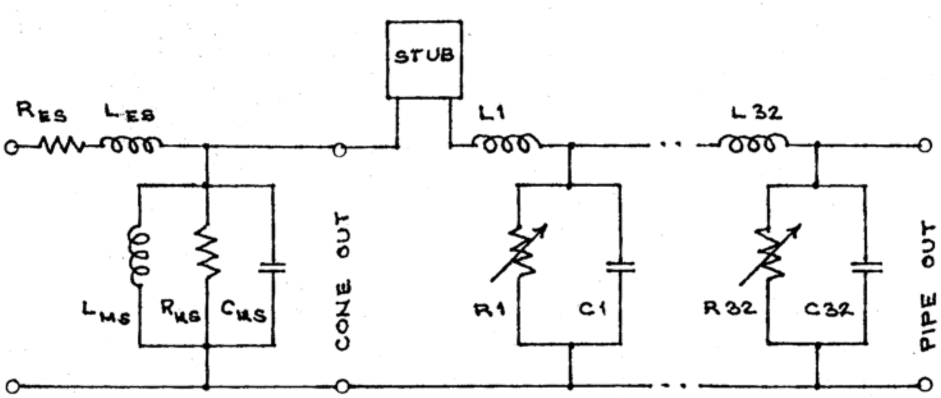


Fig. 1. Tranmission-line analog circuit.

cated, undamped transmission line is impressive. One of the test systems consists of a small loudspeaker mounted 0.14 m from the end of a 0.71-m tapered pipe. The measured cone output and pipe output are graphed in Fig. 2(a), and the corresponding analog circuit curves in Fig. 2(b).

Locanthi's horn analog has not received much attention, presumably because it is more complicated and less accurate than alternative computer models. However, a simple *RLC* ladder is easy to set up with any circuit modeling program and calculations are very fast. Moreover, using an electrical transmission line to model an acoustical transmission line seems particularly appropriate.

3 TEST PROCEDURE

The simplest transmission line is a straight pipe with a loudspeaker on one end. To check the accuracy of the analog and to study the behavior of damping materials, a number of these were built and tested. Four were cylindrical pipes varying in length from 0.6 to 1.8 m. The fifth was a reversible rectangular pipe with two slanted sides: a parabolic horn. Additional variants were built as the project went along.

To make response measurements, a given pipe was set horizontally on a trestle about 1 m above the floor. A calibrated Bruel & Kjaer 4134 microphone was connected to a TEF20 analyzer. Sweeps were run from 20 Hz to 1 kHz with a frequency resolution of 10 Hz, giving accurate readings down to about 25 Hz. Impedance curves were also run. Each test was saved to disk. The TEF system stores all measurements as sets of complex data points, preserving both magnitude and phase.

Frequency response measurements were made using near-field microphone placement [7] so that loudspeaker and pipe outputs could be measured separately with negligible flanking effects. By blocking the end of the pipe it was possible to measure leakage from the loudspeaker at the other end. Crosstalk in the 0.6-m pipe was about −25 dB. It was down more than 30 dB in the longer pipes.

Postprocessing allows the system response to be calculated as the complex sum of loudspeaker and pipe outputs. However, when a microphone is located very

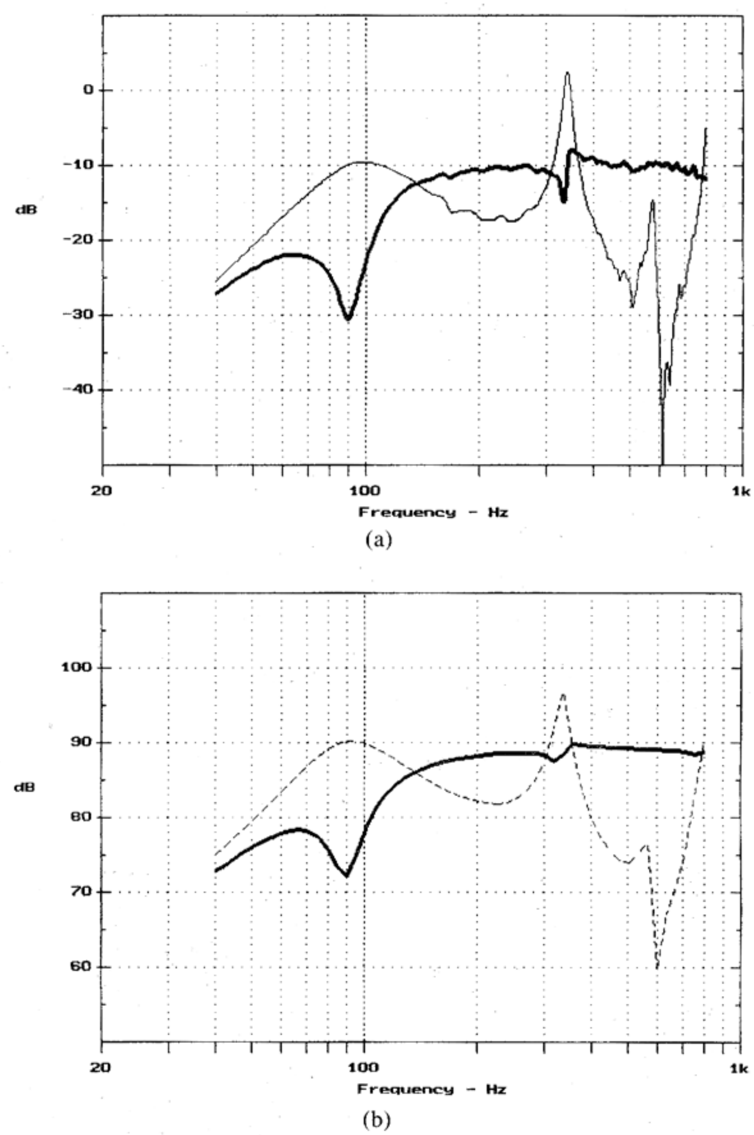


Fig. 2. (a) Measured response of undamped test system. (b) Response of analog circuit model. Cone output (bold) and pipe output.

near a small sound source, a movement of only 1 or 2 mm can shift the measured sound level by more than 1 dB. A number of preliminary runs were made to be sure that the test setup delivered repeatable results. To verify that the cone and pipe data could indeed be summed, several response measurements were made with the microphone equidistant from both loudspeaker and pipe, at one apex of an equilateral triangle.

It was later learned that this test setup closely parallels that of Letts' 1975 study [8]. To the extent that the tests overlap, results are in close agreement.

4 BEHAVIOR OF DAMPING MATERIALS

The available technical literature includes a great deal of information about the acoustical performance of absorptive materials. However, Bradbury's 1976 paper [9] is one of the few relating directly to transmission-line design. His study postulates that fibers are set in motion as sound waves pass through the line. Aerodynamic drag analysis is then used to predict resistive and reactive effects from fiber size, mass, and packing density. The concept was later expanded by Leach and applied to closed-box loudspeaker systems [10].

In the 1980s Bullock developed a transmission-line computer simulation based on Bradbury's model, but comparisons with measured performance proved to be ". . . not satisfactory" [11]. The conclusions put forth in Bradbury's paper are also at odds with some of the test results to be described here. One reason may be that his formula for computing the drag coefficient was admittedly tentative. Also, it is not certain that fiber motion is really that important. For example, Hersh and Walker [12] reported excellent predictions of measured behavior, yet their analysis makes the simplifying assumption that fibers are stationary.

For practical loudspeaker system design, our concern is not with the composition of the damping material but rather its actual performance. Moreover, we are only interested in the low-frequency response of a limited range of pipe lengths. On that basis, tests were made with various kinds of lining and stuffing.

Most of the tests used varying densities of four stuffing materials:

1) Ordinary fiberglass thermal blanket. This is readily available with paper backing, which can be removed.

2) Polyester fiber stuffing, "Poly Fluff," a product of Western Synthetic Fiber Inc., Carson, CA.

3) Microfiber stuffing, Celanese "Microfill."

4) "Acousta-Stuf." This is a Nylon polyamide fiber sold for use in loudspeaker enclosures. It is available from Mahogany Sound, Box 9044, Mobile, AL 36691-0044.

These materials are easy to buy, easy to use, and perform well for this application. Numerous other substances were tested, including cotton puffs, steel wool, packing pellets, and plastic foam.

It seems prudent to focus on inert, nonorganic materials. However, long-fiber wool was chosen by Bailey as the ideal stuffing for transmission lines, and his preference was supported by Bradbury. Unfortunately bulk wool is not easy to find in the United States, so fluffy wool yarn was tested instead. It displayed no unusual properties, performing roughly the same as Acousta-Stuf, which is advertised as a superior substitute for wool. A similar comparison was observed between cotton puffs and microfiber.

Microfiber is light and fluffy. Acousta-Stuf is ropy and fairly heavy. For roughly equivalent damping, the packing densities of these two materials must differ by a factor of 2 or more. When this is taken into account, the behavior of all four materials is similar.

Even so, there are some differences in attenuation characteristics. For example, at higher packing densities fiberglass displays a somewhat sharper knee and more rapid high-frequency attenuation than the other materials. On the other hand, it seems to be more prone to unexpected response irregularities at low densities.

Fig. 3 compares the measured pipe output of a test system stuffed with 8 g/L of fiberglass and 16 g/L of Acousta-Stuf. The Acousta-Stuf curve is nicely rounded. In comparison, the fiberglass has a sag around

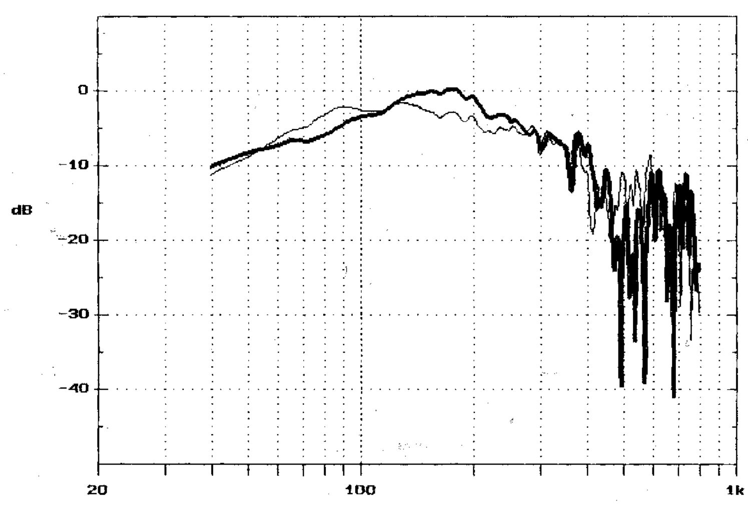


Fig. 3. Pipe output. 8-g fiberglass (bold) and 16-g Acousta-Stuf.

80 Hz, and a broad bump centered at 180 Hz. The remaining peaks and dips are characteristic of the test setup.

This is a typical example. The pipe output is always a little lumpy, and different materials have their own characteristic signatures. In transmission-line systems, if the pipe output is appreciable, then these small differences may be audible.

Most transmission-line literature recommends some optimum stuffing density regardless of pipe length. Common sense suggests that a 100-Hz short pipe should have the same stuffing density as a 50-Hz pipe twice as long. In reality, test results clearly demonstrate that the shorter pipe requires greater packing density for equivalent performance. Moreover, this is apparent from an examination of the analog circuit.

Consider a single transmission-line section. The values of L and C are proportional to the section length x. If the length of the pipe is doubled without changing its cross-sectional area, then L and C must also double. Viewing the section as a low-pass filter, its cutoff frequency has shifted down one octave with no change in impedance. Therefore, since R remains constant but x has doubled, damping per unit length must be halved.

A few tests were run using lining instead of stuffing. However, it became obvious that in pipes of small diameter, even highly absorptive lining cannot provide enough midfrequency attenuation to control passband ripple. Moreover, predicting the performance of lining involves the cross-sectional area and perimeter as well as the length, adding unwanted complications to a basic computer model.

5 MODELING DAMPING MATERIALS

The resistive component of damping is represented by shunt resistance. A frequency-related resistance is required even though fixed losses produce greater attenuation at higher frequencies. Fig. 4 shows analog circuit pipe output with fixed relative damping ranging from 0 to 10. The 1-m pipe is driven by a constant-velocity piston and terminated in its characteristic impedance. Typical absorptive materials exhibit somewhat steeper slopes, and a reactive component is also present.

It is well established that sound wave propagation through tangled fibers is slower than in free air, and is roughly proportional to some power of frequency. In a lightly damped pipe this shows up as a lowering of the nominal quarter-wave resonance frequency plus a smaller shift of the upper harmonics. In a nonresonant transmission-line system, however, damping has a much greater effect on low-frequency performance than propagation velocity. If velocity is set at a fixed value determined by passband ripple frequencies, then any remaining errors mostly affect the response below cutoff.

Four empirical parameters seem to be sufficient to model typical stuffing materials:

1) Fixed losses
2) Variable losses, corner frequency
3) Variable losses, slope
4) Relative sound speed.

The first three are used to calculate the values of shunt resistors at each frequency to be plotted. The last simply sets a scaling factor for capacitance values.

It can be argued that these are unscientific twiddle factors, but they enable the analog circuit described to deliver good approximations of transmission-line behavior. As an example, Fig. 5 shows the system of Fig. 2(b) with the addition of moderate damping. In this case, measured response curves have been omitted because they essentially duplicate the analog response.

6 BASIC SYSTEM BEHAVIOR

With no stuffing, a pipe resonates at odd multiples of its fundamental quarter-wave resonance. The loudspeaker cone is heavily loaded at these frequencies so that loudspeaker output is attenuated and pipe output is accentuated. To complicate the picture, the two are alternately in and out of phase at even multiples of the fundamental, resulting in a highly irregular system response.

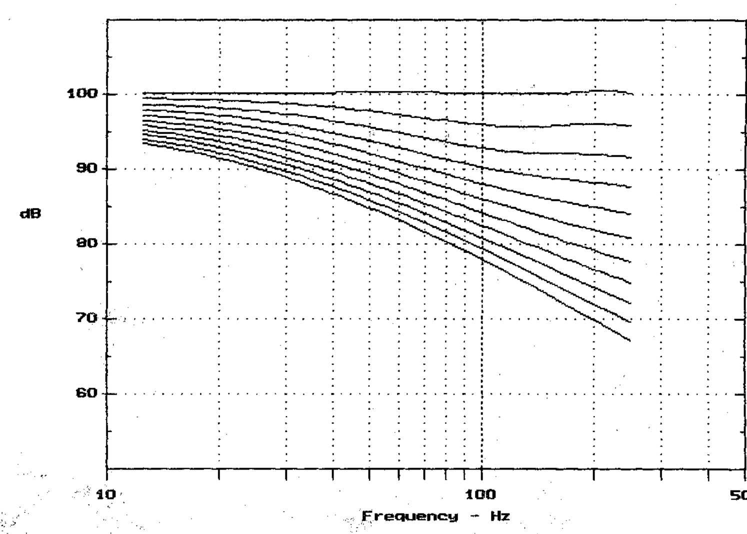


Fig. 4. Terminated pipe output for fixed damping. Relative damping from 0 (top) to 10.

This is clearly shown in Fig. 6, which is the analog response of a small automotive loudspeaker on a 0.78-m pipe. This is nominally a 109-Hz pipe, but it actually resonates at 100 Hz, which is also the loudspeaker's cone resonance. The light solid line represents cone output, the dashed line is pipe output, and the heavy solid line is the combined system response. Note that the upper resonances and antiresonances fall at exact 100-Hz intervals.

The dotted line at the bottom shows voice-coil impedance relative to dc resistance, plotted logarithmically. The impedance curve of this undamped transmission line is obviously similar to that of a vented box. The impedance minimum at 100 Hz is flanked by two peaks at about 64 Hz and 150 Hz. Additional peaks at higher frequencies will disappear as damping is added.

Fig. 7 illustrates what happens when the pipe is stuffed with light, medium, and heavy damping. Like Fig. 6, these are computer analog response curves, but they are all confirmed by actual test data.

When a small amount of damping is introduced, cone and pipe outputs still show resonances, and pipe attenuation is minimal [Fig. 7(a)]. The cone output suggests that quarter-wave loading has moved down to about 80 Hz. However, the fundamental resonance has all but disappeared from the impedance curve. The lower impedance peak no longer exists, and f_H has become a gentle bump. The cone and pipe outputs are additive down to about 85 Hz, and the low-frequency slope is reduced from 24 dB to about 18 dB per octave.

Moderate stuffing density, as shown in Fig. 7(b), results in a well-behaved transmission-line system with a sag of perhaps 2 dB around 300 Hz and gentle rolloff below 150 Hz. Below 100 Hz the slope is about 12 dB per octave. Although pipe output is well below cone output, the two are additive over more than two octaves. The only identifiable resonance in the impedance curve is f_H. The system is starting to behave very much like a closed box.

Still more stuffing results in a purist's transmission line, which effectively swallows up back radiation through the passband, as in Fig. 7(c). Going beyond this point is self-defeating since the output of the loudspeaker cone is progressively reduced by excessive damping.

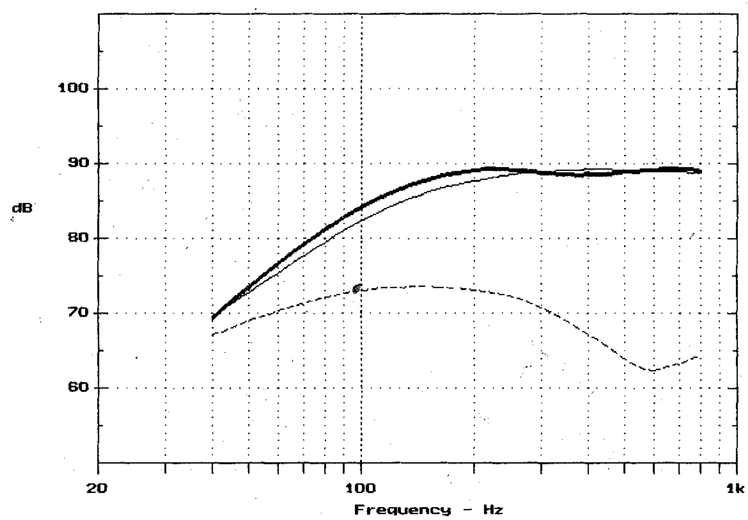


Fig. 5. System of Fig. 2(b) with medium-density stuffing.

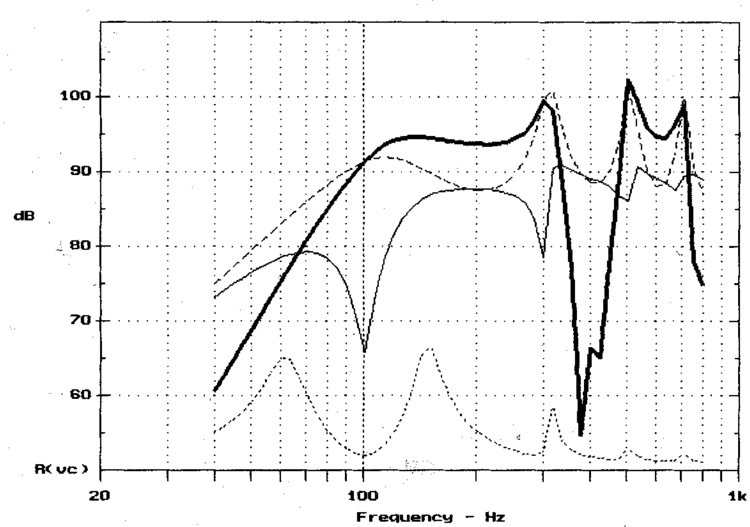
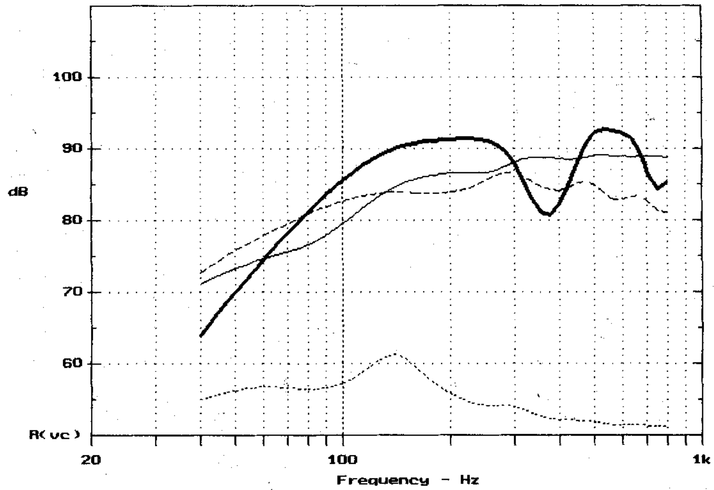


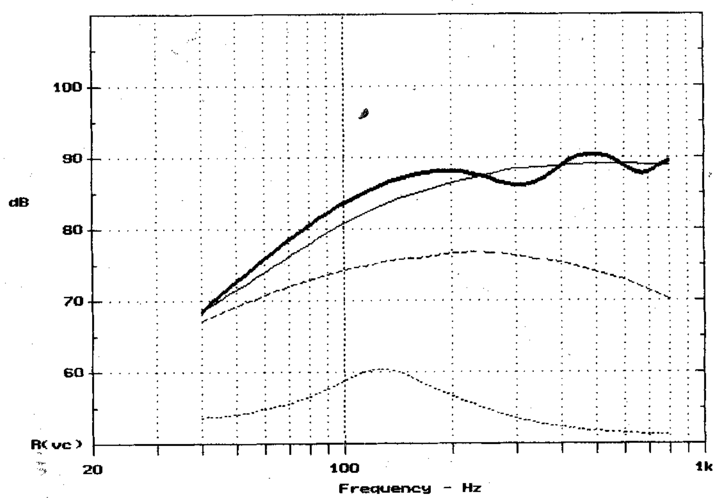
Fig. 6. Response of loudspeaker on undamped straight pipe. Impedance (bottom), cone output, pipe output, and system response (bold).

7 TRANSMISSION-LINE DESIGN FUNDAMENTALS

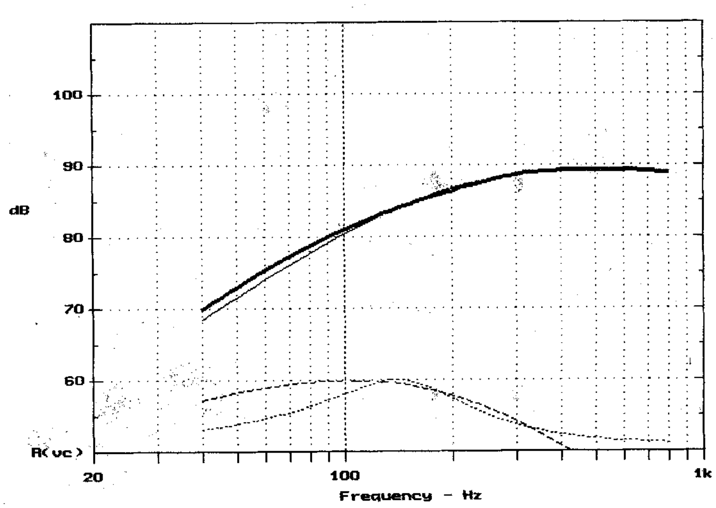
For highest possible efficiency with minimal passband ripple, it is apparent that damping should be negligible up through the second harmonic yet provide more than 20 dB of attenuation at the fourth harmonic. None of the materials tested even comes close. Rather than searching for a new kind of damping material, it seems more reasonable to look at the behavior of Fig. 7(b) and



(a)



(b)



(c)

Fig. 7. Response of loudspeaker on straight pipe. (a) Light damping. (b) Moderate damping. (c) Heavy damping. Impedance, cone output, pipe output, and system response (bold).

see what changes might be made to flatten the response.

First Q_{TS} could be increased to lessen midrange sensitivity and thus level the response above 150 Hz. Also, as in a closed-box system, it appears that f_S should be substantially lower than f_3. Finally, to keep passband ripple within ±1 dB, the stuffing density must be increased slightly.

With some trial-and-error tweaking of the loudspeaker parameters, the response of Fig. 8(a) was achieved. Now f_3 matches f_P while f_S is an octave lower. Q_{TS} is about 0.5 and the pipe volume is one-half V_{AS}. Apart from selecting the right stuffing density, this is sufficient information to duplicate the response curve for any desired low-frequency cutoff. Notice that the pipe diameter is determined by the pipe length and V_P. The cone diameter per se is not a factor in low-frequency enclosure design, as Small proved more than 25 years ago [13].

In Fig. 8(a) the pipe output and cone output add constructively down to 40 Hz or lower. Therefore it might be possible to set f_3 as much as an octave below f_P by specifying the proper loudspeaker parameters, with no change in stuffing density. With the benefit of hindsight, this is a logical assumption. It is confirmed by computer modeling and test results. Fig. 8(b) shows how a nominal 109-Hz pipe can be "tuned" to 65 Hz. Efficiency goes down as well, just as one would expect from Thiele–Small analysis of box-type systems.

Some experimenters have reported a miraculous extension of the low-frequency response by using high-density stuffing in very short pipes. This is wishful thinking. In the real world, as the packing density is increased beyond its optimum value the system behaves more and more like an overdamped infinite pipe. For maximum efficiency it appears that f_3/f_P should be between 0.7 and 1.4.

The following simple alignment table summarizes the Thiele–Small relationships of Fig. 8:

	$\frac{f_3}{f_P}$	$\frac{f_S}{f_P}$	$\frac{V_{AS}}{V_P}$	Q_{TS}
Fig. 8(a)	1.0	0.50	2.0	0.46
Fig. 8(b)	0.6	0.33	1.0	0.36

For a nominal 100-Hz pipe the alignments shown can be realized with 32-g/L polyester stuffing. However,

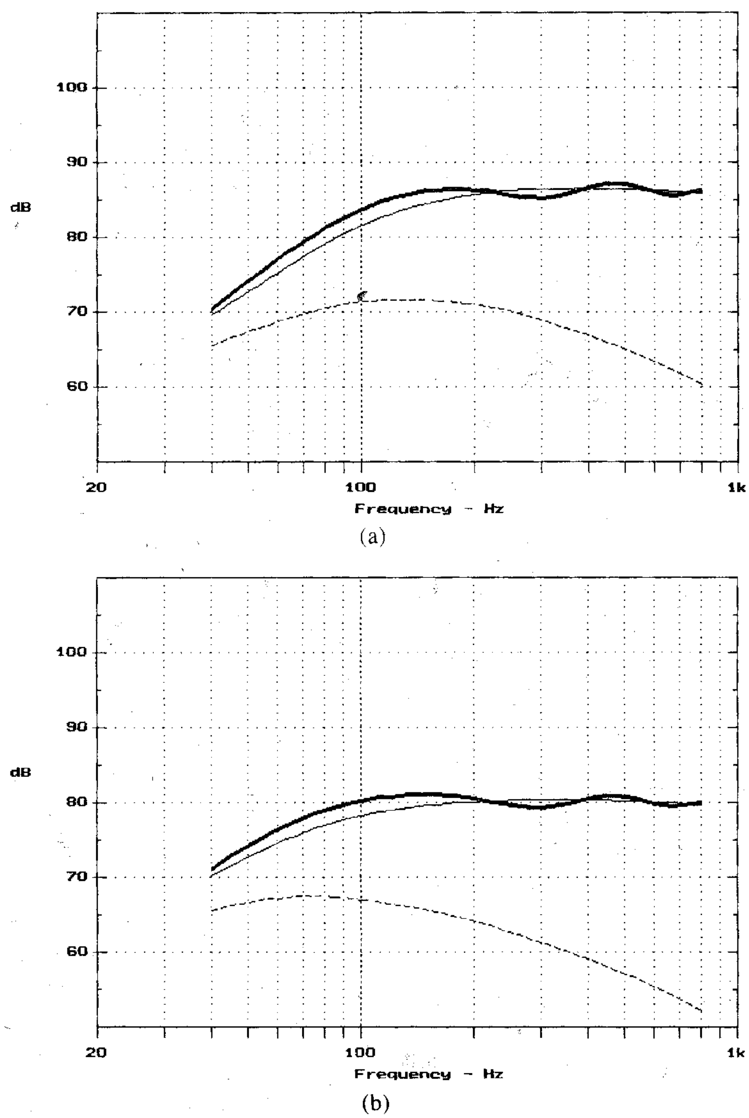


Fig. 8. (a) Response of straight pipe with improved alignment. (b) Response of extended low-frequency alignment. Cone output, pipe output, and system response (bold).

they can also serve as multipurpose alignments. The classic low-frequency rolloff of Fig. 7(b) can be approximated by halving Q_{TS}. To simulate an infinite pipe, the stuffing density should be increased by about 50%.

These examples show what can be done with traditional transmission-line design, but they are lossy. Efficiency is 2–5 dB less than for a comparable closed box. Fortunately the situation can be improved by considering something other than a simple straight pipe.

8 ALTERNATE GEOMETRIES

The computer analog made it easy to experiment with all sorts of geometrical modifications, including tuned stubs, abrupt discontinuities, tapered pipes, coupling chambers, and tricks with damping placement. Promising designs were built and tested. Five of these managed to deliver greater efficiency without sacrificing the traditional transmission-line performance. Fig. 9 shows these variant geometries:

- Tapering the pipe (reverse flare) lowers the fundamental resonance frequency without affecting the upper harmonics. The frequency range of constructive pipe output is broadened [Fig. 9(a)].
- Constricting the pipe exit (a vented pipe) has a similar effect [Fig. 9(b)].
- A coupling chamber lowers the fundamental resonance and increases the high-frequency attenuation of damping materials [Fig. 9(c)].
- An abrupt change in pipe diameter at one-third its length produces a reflection that offsets cancellation in the troublesome fourth-harmonic region [Fig. 9(d)].
- Mounting the loudspeaker at one-fifth the length of the pipe is even more effective in attenuating the pipe output near the fourth harmonic [Fig. 9(e)].

Of these, the tapered pipe, coupling chamber, and offset loudspeaker were chosen for additional analysis.

8.1 Tapered Pipe

Tapered tranmission lines go back to Bailey's design [2]. The reasoning seems to be that since energy decreases along the length of the pipe, space can be saved without making any difference in performance. In fact, tapering makes a big difference.

Tapering an undamped pipe can lower f_0 by more than one-third octave. Upper harmonics are almost unchanged. In a transmission-line loudspeaker system f_3 shifts down, giving a useful extension of the low-frequency bandwidth.

An area reduction between 1:3 and 1:4 seems to work best. If the pipe throat is too large, cross modes can be a problem. If the mouth is too small, excessive air turbulence may result. The taper can be linear or conic, or approximated by cylindrical sections. These variants influence the pipe output, but not enough to appreciably affect the overall system response. Fig. 10 shows the performance of an optimized system, normalized to a low-frequency cutoff of 100 Hz.

8.2 Pipe with Coupling Chamber

Coupling chambers have also been used in many transmission lines. The idea seems to have evolved empirically. Technical explanations range from better impedance matching to suppression of pipe resonances. The latter is close to the truth.

The loudspeaker cone is coupled to the pipe throat by the springiness of the air in the chamber. At mid and high frequencies the throat impedance is largely resistive, and the resulting low-pass action adds another 6 dB per octave of high-frequency rolloff. This can easily be seen by comparing Figs. 10 and 11. Both systems are 9-L nominal 125-Hz pipes stuffed with the same packing density. The coupling chamber is also stuffed. In Fig. 11 the coupling chamber accounts for 3 L and a slimmer pipe contains the remaining 6 L.

The system response of Fig. 11 closely matches that of Fig. 10. Above 200 Hz, however, the pipe output rolls off more rapidly and passband ripple is reduced even though the ripple frequencies have moved down. Also, in the 100-Hz region the cone excursion is slightly less.

For the system to function as modeled there must be a clear demarcation between pipe and coupling chamber. On the other hand, if the chamber is too large, then we have restored the cavity that the transmission line was supposed to eliminate. A good compromise is to make the chamber volume one-third of the total volume.

8.3 Offset Loudspeaker

Quarter-wave stubs are sometimes used in duct silencers to suppress specific frequencies. In a damped transmission line the effect is more akin to a shelving filter. Fig. 12 shows how this geometry can be used to

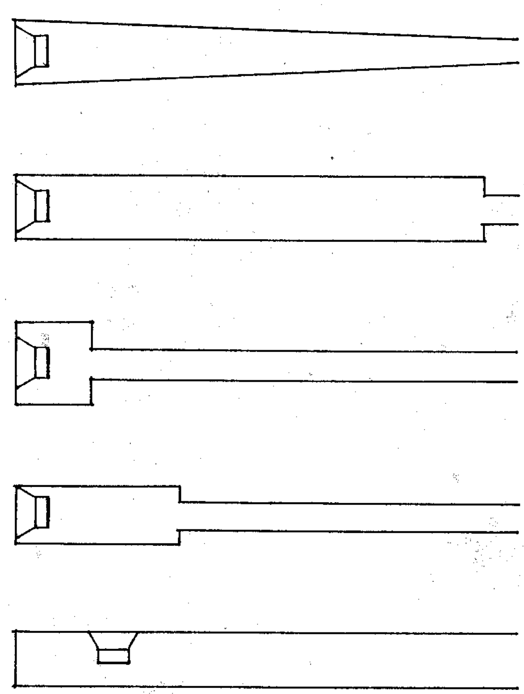


Fig. 9. Alternate pipe geometries. (a) Tapered. (b) Vented. (c) Chamber. (d) Stepped. (e) Offset loudspeaker.

achieve performance at least as good as in the previous examples. A straight pipe is used with the loudspeaker located at one-fifth the length of the pipe.

This system has the same volume, the same stuffing density, and the same low-frequency cutoff as the previous two examples. However, f_3 is now higher than f_P. A longer, thinner pipe is required for comparable performance.

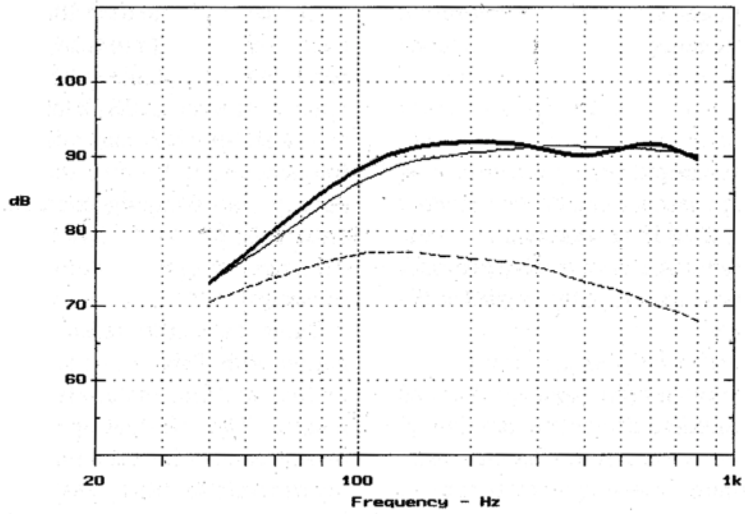


Fig. 10. Tapered pipe transmission-line response. Cone output, pipe output, and system response (bold).

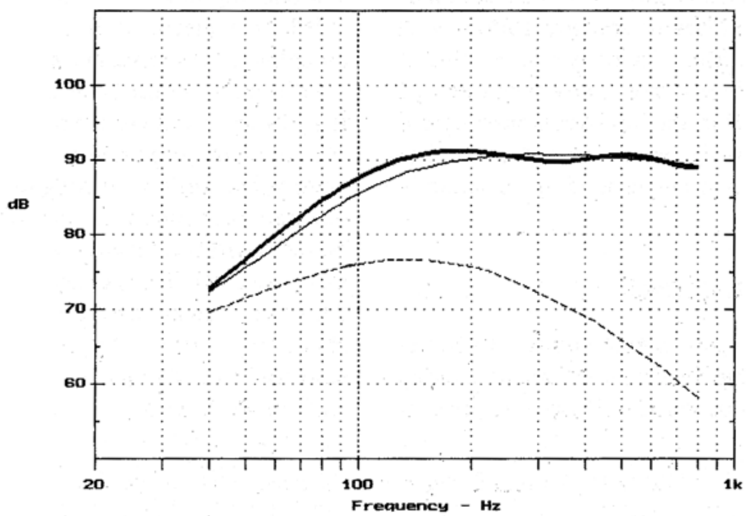


Fig. 11. Coupling chamber transmission-line response. Cone output, pipe output, and system response (bold).

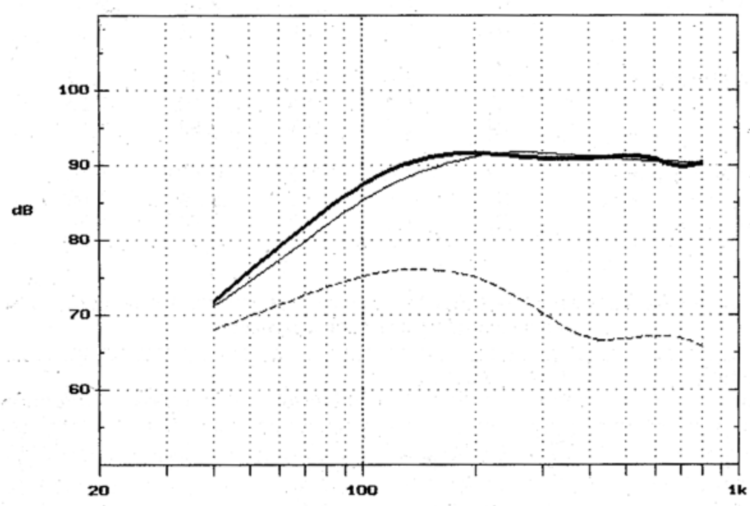


Fig. 12. Offset loudspeaker transmission-line response. Cone output, pipe output, and system response (bold).

8.4 Alignment Table

Alignments based on the Thiele–Small parameters are listed in Table 1 for all three systems. Three sets of values for each design offer a reasonable spread of loudspeaker choices. In reality, all three alternates represent the same loudspeaker mechanism with different cone suspension compliances.

8.5 Combinations

It is possible to combine various pipe geometries. As a case in point, a coupling chamber can drive a tapered pipe. Although it has been used in at least two commercial transmission-line designs, this combination provides no reduction in pipe volume and slightly degrades the low-frequency performance.

Other combinations are similarly disappointing. For example, a tapered pipe with an offset loudspeaker can be made to squeeze out another decibel of efficiency, but at the cost of greater cone excursion. The net result is a decrease in the maximum low-frequency output.

8.6 General Comments

The optimized transmission lines described are characterized by second-order low-frequency rolloff with minimal passband ripple. The efficiency can match that of an equivalent closed-box system, however the pipe output contributes 2–3 dB in the low-frequency region. Since loudspeakers are displacement limited at low frequencies, the net result is a corresponding increase in maximum output.

9 STUFFING SPECIFICATIONS

In theory, separate system alignments would be required for different stuffing materials, different packing densities, and different pipe lengths. However, the general trend for all materials is an increase in sound attenuation with an increase in frequency. As previously noted, if appropriate packing densities are chosen for the four materials studied, then their damping characteristics are similar over a moderate range of frequencies.

There is little concern about the pipe output at frequencies well below f_3 because this region is out of the passband. For acceptable passband ripple, the pipe output must be at least 15 dB below the cone output at frequencies well above f_3 and will continue to drop at higher frequencies. It follows that for a specified cutoff frequency, the damping characteristics must be matched only over a range of two octaves or less. However, densities for pipes of various lengths must be specified separately.

Table 2 is a cross reference chart to be used in combination with Table 1. It shows the equivalent densities of four stuffing materials over a range of useful pipe lengths. The information is derived from several sets of measurements for each material and should be reasonably accurate for fiberglass, Acousta-Stuf, and polyester. Fewer tests were made with microfiber and a fair amount of interpolation is included. Also, tests made with very low packing densities show appreciable variations.

Which material is best? Each material has its own damping characteristics, and even with close matching the differences may be audible. On this basis the choice is arbitrary, but there are other factors to consider. Probably the most important is consistency.

Polyester pillow stuffing seems to be fairly generic, but there is no guarantee that a batch from another manufacturer will be the same as Poly Fluff. Fiberglass thermal blanket delivers consistent performance at packing densities greater than 15 g/L. Its unpacked density is about 10 g/L (0.6 lb/ft^3), and its physical properties are held to close tolerances. On the debit side, it is nasty stuff to work with.

Table 1. Optimized alignments for three practical systems.

Design		f_3/f_S	f_3/f_P	f_S/f_P	V_{AS}/V_P	Q_{TS}
Tapered	I	2.0	0.8	0.40	3.10	0.36
(nom. 4:1)	II	1.6	0.8	0.50	2.00	0.46
	III	1.3	0.8	0.63	1.20	0.58
Coupling chamber	I	2.0	0.8	0.40	2.14	0.31
	II	1.6	0.8	0.50	1.35	0.39
	III	1.3	0.8	0.63	0.84	0.50
Offset loudspeaker	I	2.0	1.2	0.60	3.10	0.36
	II	1.6	1.2	0.74	2.00	0.46
	III	1.3	1.2	0.94	1.20	0.58

Table 2. Packing densities (g/L) for various pipe lengths for tapered, offset, and coupling chamber alignments.

Length (m)	f_P (Hz)	Acousta-Stuf	Polyester	Fiberglass	Microfiber
0.61	140	27.0	29.0	14.5	10.5
0.91	94	21.0	22.5	11.0	9.0
1.22	71	16.0	17.5	9.5	7.5
1.83	48	12.0	13.5	—	5.5
2.44	36	8.0	10.5	—	4.3

Acousta-Stuf is more expensive than fiberglass or polyester but its characteristics are closely specified. As delivered, it is lumpy and must be thoroughly teased, especially for low packing densities. Otherwise, it is easy to use and delivers predictable results.

Microfiber is very light and fluffy. Once packed to the desired density it seems to stay in place, but loose wisps will drift around for days. If the brand name Celanese Microfill is used, then its acoustical properties should be as predictable as those of fiberglass or Acousta-Stuf.

All of these materials can be tricky to use in long, large pipes requiring low packing densities. Partitioning a fat pipe into two or more thin pipes will help keep the stuffing in place and make the structure more rigid. Using thick lining instead of stuffing is another alternative, but is outside the scope of this study.

10 DIRECTIONAL EFFECTS

Letts [8] seems to be the only researcher to have noticed the unusual directional properties of transmission lines.

If the dimensions of a sound radiator are very small in comparison with the wavelength, then it is assumed to behave like a point source. Its coverage pattern is omnidirectional, constrained only by large adjacent surfaces. A small sealed or vented loudspeaker system is essentially omnidirectional at frequencies below 200 Hz or so.

In contrast, a loudspeaker on a lightly damped straight pipe is a unidirectional gradient source at low frequencies. Its coverage pattern is the same as that of a cardioid microphone. If the pipe output is less than the cone output, the directional effects are less pronounced but still in evidence. The pipe output must be at least 15 dB below the cone output for the directivity to be determined by the loudspeaker alone.

All of the system response curves in this paper are on-axis curves, that is, they represent a response at some point equidistant from the loudspeaker and the pipe mouth. If the pipe output is appreciable, then the off-axis response and the total power response may both be quite different from the on-axis curve. Such differences are minimized if the loudspeaker and the pipe mouth are very close together.

Since the low-frequency tonal balance heard in a typical listening room is dominated by generally reflected sound, it follows that a loudspeaker on a straight pipe may indeed sound different than one on an otherwise identical folded pipe.

11 CONCLUSION

This study has attempted to demystify the nonresonant transmission-line design pioneered by Bailey: a loudspeaker is mounted on a pipe stuffed with tangled fibrous material of uniform density, providing sufficient damping to control the passband ripple yet allow useful reinforcement of the cone output at low frequencies.

With a few small modifications, Locanthi's horn analog was shown to be an excellent tool for modeling transmission-line loudspeaker systems. However, to derive usable parameters for real-world damping material it was necessary to test a number of pipes with different materials of varying densities.

Based on test results, four empirical parameters were found sufficient to approximate the performance of damped transmission lines. Three of these define a frequency-dependent resistive component. Surprisingly, the relative propagation velocity can then be set to a constant value even though, in reality, it is also frequency dependent.

For a pipe of given length, different materials require different packing densities to achieve desired damping. Once this is done, the passband performance is essentially the same for any of the materials tested.

The pipe length establishes a usable range of cutoff frequencies, typically a one-octave band centered at f_P. Within that range, f_3 is controlled by the loudspeaker parameters in relation to pipe length and volume. Damping remains unchanged.

In contrast to a basic cylindrical pipe, at least four other geometries allow lighter damping, which results in higher efficiency. Systems can be scaled to any cutoff frequency and any practical efficiency by using simple alignment tables. Optimized alignments were developed for three alternate geometries. Allowing for $\pm$1-dB passband ripple, these new alignments approximate the response of an equal-volume closed box, but with reduced cone excursion and correspondingly greater maximum low-frequency output.

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Passive-Radiator Loudspeaker Systems Part I: Analysis*

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The passive-radiator loudspeaker system is a close relative of the vented-box system and is capable of similar low-frequency performance. The passive radiator may be of any area but should preferably have a suspension with high compliance and low mechanical losses. It should also possess a linear volume displacement limit at least twice that of the system driver.

1. INTRODUCTION

Historical Background

The use of passive radiators in direct-radiator loudspeaker systems was described by Olson in a U.S. patent of 1935 [1]. Apparently, commercial exploitation of the principle was not immediate. The first description of the physical performance of such a loudspeaker system was published by Olson in 1954 [2]. Olson made direct comparisons between the use of a vent and a passive radiator (or drone cone) with the same driver and enclosure and claimed several advantages in favor of the passive radiator [2], [3].

Despite the very favorable results reported by Olson, only a few manufacturers have attempted to produce passive-radiator loudspeaker systems commercially. Perhaps an important reason for the limited interest in these systems has been the lack of any comprehensive published quantitative analysis or guide to their design.[1]

* Abridged version of this paper was presented September 10, 1973, at the 46th Convention of the Audio Engineering Society, New York

[1] This was written before the publication of the small-signal analysis by Nomura and Kitamura [9]. The present paper uses a slightly different approach, contains a somewhat wider range of useful alignments, and also deals with large-signal performance and design.

Technical Background

The passive-radiator loudspeaker system is a direct-radiator system using an enclosure which has two apertures. One aperture accommodates a driver, the other contains a suspended diaphragm which may resemble a driver but which has no voice coil or magnet assembly. The second undriven diaphragm is variously called a drone cone, passive radiator, or auxiliary bass radiator.

At low frequencies, the passive-radiator diaphragm moves in response to pressure variations within the enclosure [1]. It thus contributes to the total volume velocity crossing the enclosure boundaries and therefore to the system acoustic output [4].

The operation of the passive-radiator system is very similar to that of the vented-box system [5], the principal difference being the presence of a compliant suspension in the passive radiator which is not present with a simple vent. Because of this similarity, the passive-radiator system can be expected to perform in a manner similar to the vented-box system if passive-radiator compliance is made large enough.

In Part I of this paper, the passive-radiator system is analyzed by the general method described in [4]. Important objectives of this analysis are to determine the effects of limited passive-radiator compliance and to discover any advantages or disadvantages of this system compared to the vented-box system. The basic analytical results reveal the important physical relationships governing the small-signal and large-signal performance of passive-radiator systems and provide a quantitative basis for the measurement, assessment, and design of these systems.

Part II will provide a discussion of these results and present methods of synthesis (system design) which facilitate the design of an enclosure and passive radiator for a given driver or the specification of all system components required to meet a complete and realizable set of system performance specifications.

2. BASIC ANALYSIS

The impedance-type acoustical analogous circuit of a passive-radiator loudspeaker system is presented in Fig. 1.

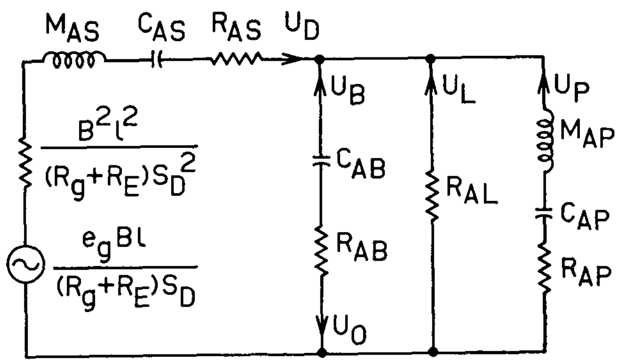

Fig. 1. Acoustical analogous circuit of passive-radiator loudspeaker system.

The symbols in this circuit are defined as follows.

- e_g open-circuit (Thevenin) output voltage of source or amplifier
- B magnetic flux density in driver air gap
- l length of voice-coil conductor in magnetic field of air gap
- S_D effective projected surface area of driver diaphragm
- R_g output (Thevenin) resistance of source or amplifier
- R_E dc resistance of driver voice coil
- C_{AS} acoustic compliance of driver suspension
- M_{AS} acoustic mass of driver diaphragm assembly including voice coil and air load
- R_{AS} acoustic resistance of driver suspension losses
- C_{AB} acoustic compliance of air in enclosure
- R_{AB} acoustic resistance of enclosure losses contributed by internal energy absorption
- R_{AL} acoustic resistance of enclosure losses contributed by leakage
- C_{AP} acoustic compliance of passive-radiator suspension
- M_{AP} acoustic mass of passive-radiator diaphragm including air load
- R_{AP} acoustic resistance of passive-radiator suspension losses
- U_D volume velocity of driver diaphragm
- U_P volume velocity of passive-radiator diaphragm
- U_L volume velocity of enclosure leakage
- U_B volume velocity entering enclosure
- U_0 total volume velocity leaving enclosure boundaries.

This circuit may be simplified by combining the series resistances in the driver branch to form a single acoustic resistance R_{AT} where

$$R_{AT} = R_{AS} + \frac{B^2l^2}{(R_g + R_E)S_D^2} \quad (1)$$

by defining

$$p_g = \frac{e_gBl}{(R_g + R_E)S_D} \quad (2)$$

as the value of the pressure generator at the left of the circuit, and by ignoring losses in the enclosure and passive radiator. The effects of these losses are examined indirectly later in the paper. The simplified circuit is presented in Fig. 2.

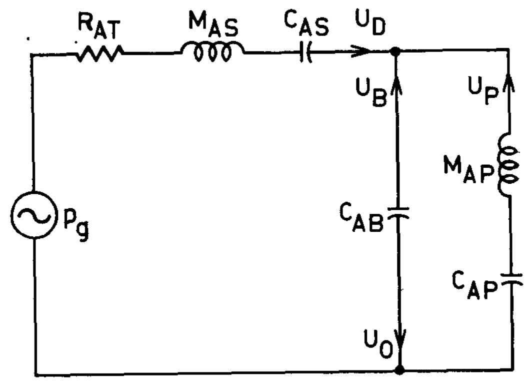

Fig. 2. Simplified acoustical analogous circuit of passive-radiator loudspeaker system with no enclosure or passive-radiator losses.

The complete electrical equivalent circuit of the passive-radiator system is the dual of Fig. 1. The electrical circuit elements are related to the acoustical circuit elements by the relationship

$$Z_E = \frac{B^2l^2}{S_D^2\ Z_A} \quad (3)$$

where Z_E is the impedance of an element in the electrical equivalent circuit and Z_A is the impedance of the corresponding element in the acoustical analogous circuit.

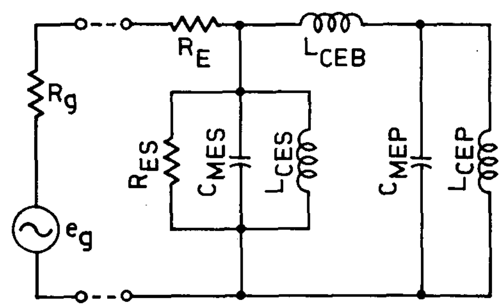

Fig. 3. Simplified electrical equivalent circuit of passive-radiator loudspeaker system.

A simplified electrical equivalent circuit corresponding to Fig. 2 is presented in Fig. 3. The symbols in this circuit are defined as follows.

- C_{MES} electrical capacitance representing driver mass, $= M_{AS}S_D^2/(Bl)^2$
- L_{CES} electrical inductance representing driver suspension compliance $= C_{AS}\ B^2l^2/S_D^2$
- R_{ES} electrical resistance representing driver suspension losses $= B^2l^2/(S_D^2R_{AS})$

L_{CEB} electrical inductance representing enclosure compliance, $= C_{AB}B^2l^2/S_D^2$
C_{MEP} electrical capacitance representing passive-radiator mass, $= M_{AP}S_D^2/(Bl)^2$
L_{CEP} electrical inductance representing passive-radiator suspension compliance, $= C_{AP}B^2l^2/S_D^2$.

The circuit of Fig. 3 has been arranged so that the actual system voice-coil terminals are accessible. This facilitates the study of the system voice-coil impedance and its relationship to the system element values.

The circuits presented above are valid only for frequencies within the piston range of the system driver. The element values are assumed to be independent of frequency within this range.

As discussed in [4], both voice-coil inductance and radiation load resistance are neglected in the construction of these circuits. Also neglected is the effect of external acoustic interaction between driver and passive radiator; this approximation is justified later in the paper.

The analysis of the system and the interpretation of its describing functions are simplified by defining a number of component and system parameters. For the driver, these are [4]

$$T_S^2 = 1/\omega_S^2 = C_{AS}M_{AS} = C_{MES}L_{CES} \tag{4}$$

$$Q_{MS} = \omega_S C_{MES}R_{ES} = 1/(\omega_S C_{AS}R_{AS}) \tag{5}$$

$$Q_{ES} = \omega_S C_{MES}R_E = \omega_S R_E M_{AS} S_D^2/(Bl)^2 \tag{6}$$

$$V_{AS} = \rho_0 c^2 C_{AS} \tag{7}$$

Eq. (4) defines the resonance frequency of the driver ($\omega_S = 2\pi f_S$). In Eq. (7) ρ_0 is the density of air (1.18 kg/m³) and c is the velocity of sound in air (345 m/s). Eq. (7) expresses the acoustic compliance of the driver suspension in terms of a volume of air (under standard conditions of temperature and pressure) which has the same acoustic compliance. In this paper it is assumed that M_{AS} and hence the values of f_S, Q_{MS}, and Q_{ES} apply to the driver when the diaphragm air-load mass has the value normally imposed by the system enclosure; where appropriate, this is indicated explicitly by using the symbol f_{SB} for f_S [4], [5].

Similar parameters are defined for the passive radiator, except that there is no equivalent to Q_{ES}. There is only one Q, related to suspension losses. Thus,

$$T_P^2 = 1/\omega_P^2 = C_{AP}M_{AP} = C_{MEP}L_{CEP} \tag{8}$$

$$Q_{MP} = \omega_P C_{MEP}R_{EP} = 1/(\omega_P C_{AP}R_{AP}) \tag{9}$$

$$V_{AP} = \rho_0 c^2 C_{AP}. \tag{10}$$

It is assumed in this paper that the values of ω_P (or the corresponding f_P) and Q_{MP} apply to the passive radiator when the diaphragm air-load mass has the value normally imposed by the system enclosure.

The enclosure, with the passive radiator installed, exhibits a resonance frequency $\omega_B = 2\pi f_B$ in the same manner as does a vented enclosure. This frequency is given by

$$T_B^2 = \omega_B^2 l = \frac{C_{AB}M_{AP}}{1+\dfrac{C_{AB}}{C_{AP}}} = \frac{C_{MEP}L_{CEB}}{1+\dfrac{L_{CEB}}{L_{CEP}}}. \tag{11}$$

The losses in the enclosure and passive radiator are conveniently defined as Q at the enclosure resonance frequency in the same manner as for the vented-box system [5, sec. 3]. Thus for absorption, leakage, and passive-radiator suspension losses respectively,

$$Q_A = 1/(\omega_B C_{AB}R_{AB}) \tag{12}$$

$$Q_L = \omega_B C_{AB}R_{AL} \tag{13}$$

$$Q_P = 1/(\omega_B C_{AB}R_{AP}). \tag{14}$$

The total enclosure loss Q_B at f_B is then given by

$$1/Q_B = 1/Q_A + 1/Q_L + 1/Q_P. \tag{15}$$

The interaction of the source, driver, enclosure, and passive radiator give rise to further system parameters. These are the system compliance ratio

$$\alpha = C_{AS}/C_{AB} = L_{CES}/L_{CEB} \tag{16}$$

the passive-radiator compliance ratio

$$\delta = C_{AP}/C_{AB} = L_{CEP}/L_{CEB} \tag{17}$$

the system tuning ratio

$$h = f_B/f_S = \omega_B/\omega_S = T_S/T_B \tag{18}$$

the passive-radiator tuning ratio

$$y = f_P/f_S = \omega_P/\omega_S = T_S/T_P \tag{19}$$

and the total Q of the driver connected to the source

$$Q_T = 1/(\omega_S C_{AS}R_{AT}). \tag{20}$$

In dealing with the system-describing functions it is useful to recognize that from Eqs. (8), (11), and (17)–(19)

$$T_P/T_B = f_B/f_P = h/y = (\delta+1)^{1/2}. \tag{21}$$

Following the method of [4], analysis of Figs. 2 and 3, and substitution of the parameters defined above yields the system-describing functions. The response function is

$$G(s) = \frac{s^2T_S^2(s^2T_P^2+1)}{D(s)} \tag{22a}$$

where

$$D(s) = s^4T_P^2T_S^2 + s^3T_P^2T_S/Q_T + s^2[(\alpha+1)T_P^2 + (\delta+1)T_S^2] + s(\delta+1)T_S/Q_T + (\alpha+\delta+1) \tag{22b}$$

and $s = \sigma + j\omega$ is the complex frequency variable.

The displacement function for the driver diaphragm, normalized to unity at zero frequency, is

$$X(s) = \frac{(\alpha+\delta+1)(s^2T_B^2+1)}{D(s)} \tag{23}$$

and the displacement constant is

$$k_x = \frac{\delta+1}{\alpha+\delta+1}. \tag{24}$$

Because the displacement capability of a passive-radiator diaphragm is limited by the suspension design, it is important to assess the required displacement as a function of frequency and power level. It is easily shown that at zero frequency the volume displacement of the passive radiator is equal to that of the driver multiplied by the factor $\delta/(\delta+1)$. The displacement function for the passive-radiator diaphragm $X_P(s)$, normalized to unity at zero frequency, is then given by

$$X_P(s) = \frac{(\alpha+\delta+1)}{D(s)}. \quad (25)$$

Analysis of the electrical equivalent circuit of Fig. 3 for the impedance of the circuit to the right of the voice-coil terminals gives the system voice-coil impedance function

$$Z_{VC}(s) = R_E + R_{ES}\frac{(\delta+1)\,(sT_S/Q_{MS})\,(s^2T_B{}^2+1)}{D'(s)} \quad (26)$$

where $D'(s)$ is the function $D(s)$ of Eq. (22) but with Q_T wherever it appears replaced by Q_{MS}.

3. RESPONSE

Response Function

The response function of the passive-radiator system given by Eq. (22) may be rearranged into the general form

$$G(s) = \frac{s^4T_0{}^4 + b_2s^2T_0{}^2}{s^4T_0{}^4 + a_1s^3T_0{}^3 + a_2s^2T_0{}^2 + a_3sT_0 + 1}. \quad (27)$$

This response function has a fourth-order denominator polynomial which is similar to that of the vented-box system. But unlike the vented-box system, two of the zeros of the numerator are located away from the origin of the s plane. It is the relocation of these zeros, caused by the passive-radiator suspension compliance, which makes the response of a passive-radiator system different from that of a comparable vented-box system.

Frequency Response

The frequency response $|G(j\omega)|$ of Eq. (27) is examined in the Appendix; coefficient values are given for a variety of system alignments which have useful response characteristics.

The distinguishing feature of the frequency response of the passive-radiator system is the presence of a notch or dip which appears at the resonance frequency f_P of the passive radiator as indicated by Eq. (22a). This frequency is normally located below the system cutoff frequency. The effect of the notch generally is to sharpen the "corner" of the frequency response characteristic and to give a steeper initial cutoff slope compared to the equivalent vented-box system.

In this respect the passive-radiator system response may be loosely compared to that of the "m-derived" high-pass filter of classical image-parameter theory and the vented-box system response to that of the "constant-k" high-pass filter [6, pp. 181-183, 652]. In the terminology of the modern insertion-loss filter theory on which the Appendix is based, the passive-radiator system response is that of an elliptic-function filter [7, pp. 489, 532].

Alignment

The response notch of the passive-radiator system may be eliminated by adjusting the system parameters so that two of the denominator poles exactly cancel the numerator zeros contributing the notch. Considerable damping must be introduced into the passive radiator to achieve this. The result is a system with pure second-order response (a nominal 12-dB per octave cutoff slope), but unfortunately one which is demonstrably inferior to a normal closed-box system in terms of the efficiency constant and power rating constant obtained [8].

Allowing the notch to remain, the high-pass behavior of the system above the notch frequency can be made to have equal-ripple, maximally flat, or quasi maximally flat properties as discussed in the Appendix. The response characteristic below the notch frequency is not of particular interest because it is very far down in the stop band.

Comparison of Eqs. (22) and (27) reveals that the five mathematical variables required to specify a given alignment (T_0, a_1, a_2, a_3, and b_2) are related to the five independent system parameters (T_S, T_P, Q_T, α, and δ). This means that every specification of a particular alignment corresponds to a unique set of system parameters. However, unlike the simpler case of the vented-box system, specified *conditions* such as "maximally flat" do not define a unique set of coefficients for Eq. (27). There are now an infinite variety of maximally flat (passband) responses having notches at various frequencies below cutoff. Thus one system parameter may be specified arbitrarily if desired without necessarily restricting the range of types of passband alignments available; only the specific shape of each alignment type is fixed.

Fig. 4 illustrates some of the maximally flat responses[2] which may be obtained for various chosen values of the passive-radiator compliance ratio δ. As the value of δ approaches infinity (infinite passive-radiator compliance, and hence f_P or notch frequency of zero), the response characteristic approaches that of the pure fourth-order Butterworth alignment obtainable from the vented-box system [5].

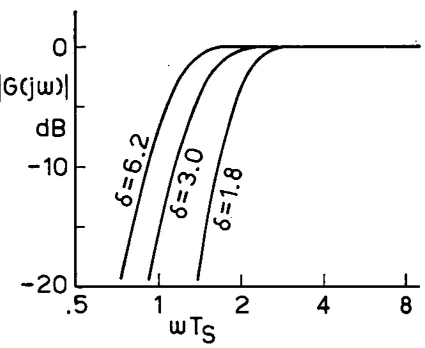

Fig. 4. Maximally flat passband responses obtainable from the passive-radiator loudspeaker system.

Fig. 5 is an alignment chart based on the range of maximally flat alignments obtainable from the lossless passive-radiator system, including those illustrated in Fig. 4. The system compliance ratio α is chosen as the primary independent variable and plotted as the abscissa. The curves then give the values of h (or y), δ, and Q_T required to obtain a maximally flat alignment as well as the normalized half-power cutoff frequency f_3/f_S at which the response is 3 dB below the passband reference level. Note that for the lossless passive-radiator system, maximally flat responses can be obtained only for values of α

[2] The maximally flat alignments of this paper are identical with the general Butterworth alignments of [9].

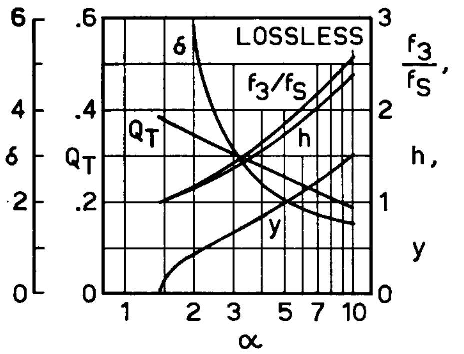

Fig. 5. Alignment chart for lossless passive-radiator system providing maximally flat passband responses.

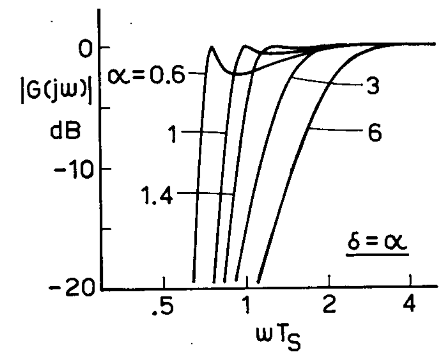

Fig. 6. Responses obtainable from passive-radiator system for the condition $\delta=\alpha$ (equal passive-radiator and driver compliances).

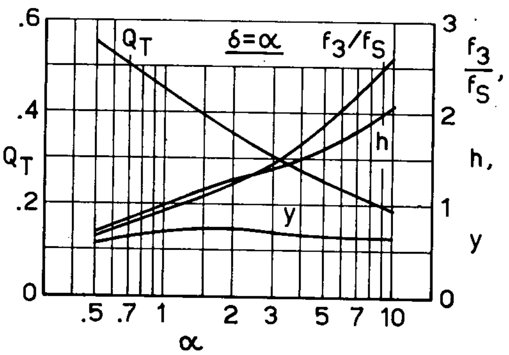

Fig. 7. Alignment chart for lossless passive-radiator systems with $\delta=\alpha$.

that are equal to or larger than the value required ($\sqrt{2}$) for a lossless vented-box system.

It may be shown from Eq. (22) that if the passive-radiator compliance is made infinite, the response is the same as for the vented-box system i.e., Eq. (22) reduces to [5, eq. (13)]. However, a common practical condition in a passive-radiator system is $\delta=\alpha$. This is because the passive radiator is often made from the same frame and suspension as the driver; the diaphragm is simply made heavier and the magnet and voice coil omitted. For the condition $\delta=\alpha$, Fig. 6 illustrates some of the response characteristics obtainable from the passive-radiator system. These include equal-ripple, maximally flat, and quasi maximally flat alignments.[3]

[3] The equal-ripple alignments used in this paper have negative ripple and are not the same kind used in [9]; those alignments have positive ripple and are obtained for somewhat different conditions. Both kinds are useful but possess slightly different values of the efficiency factor $k_{\eta(G)}$.

Fig. 7 is an alignment chart for lossless passive-radiator systems with $\delta=\alpha$. The range of alignments include those illustrated in Fig. 6. For a value of α very close to 3, the response is maximally flat. For lower values of α, the response is equal-ripple; for higher values of α, the response is quasi maximally flat.

Misalignment

The effect of an incorrectly adjusted parameter on the frequency response of a passive-radiator system is illustrated in Figs. 8 and 9. These curves were obtained with the use of an analog simulator. Fig. 8 shows the variation produced in the response of the lossless $\delta=\alpha$ maximally flat alignment by changes in the value of Q_T of $\pm 20\%$, -50%, and $+100\%$ from the nominally correct value. Fig. 9 shows the variations produced in the response of the same alignment by mistuning (a change in value of h or f_B) of $\pm 20\%$ and $\pm 50\%$. The effects are very similar to those for the vented-box system [5, Figs. 7 and 8], as might be expected.

System Losses

It can be expected in practice that Q_A and Q_L will have about the same values for a passive-radiator system as for a comparable vented-box system, provided that no additional leakage is introduced by such sources as faulty passive-radiator sealing gaskets. However, Q_P may be ex-

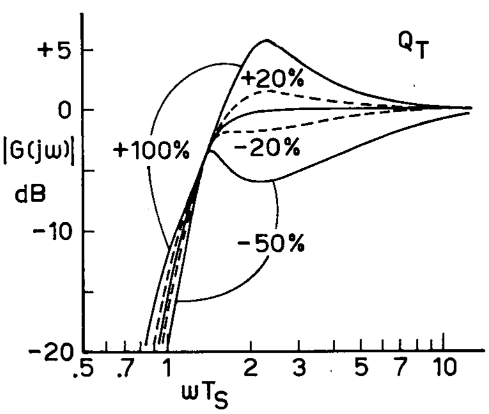

Fig. 8. Variations in frequency response of lossless maximally flat $\delta=\alpha$ passive-radiator system for misalignment of Q_T (from simulator).

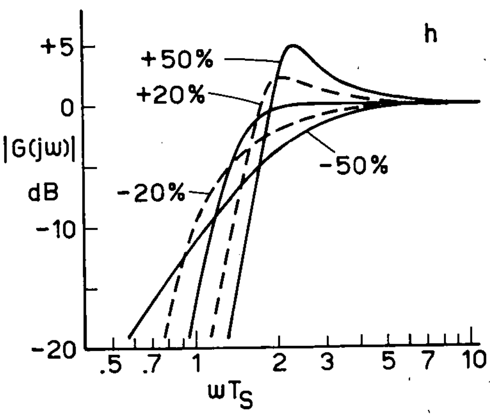

Fig. 9. Variations in frequency response of lossless maximally flat $\delta=\alpha$ passive-radiator system for misalignment of h (from simulator).

pected to be lower for the passive-radiator system, because R_{AP} in this system is commonly of the same order of magnitude as R_{AS}.

The effects of enclosure losses in the passive-radiator system can be evaluated by introducing finite values of Q_A, Q_L, and Q_P into a correctly aligned lossless system. Fig. 10 shows the effect of Q values of 5 on the lossless $\delta=\alpha$ maximally flat alignment, obtained by analog simulation. Fortunately, passive-radiator losses have the least effect on the system response.

All this suggests that the passive-radiator system will

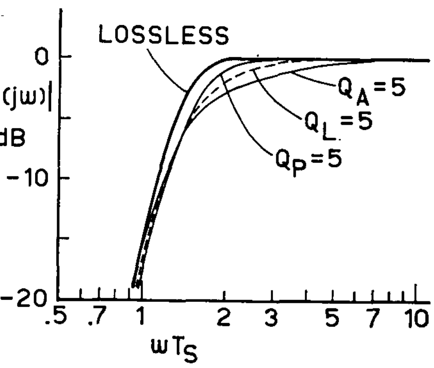

Fig. 10. Effects of enclosure and passive-radiator losses on response of a lossless maximally flat $\delta=\alpha$ passive-radiator system (from simulator).

exhibit a lower measured value of Q_B than its vented-box counterpart, but that the total effect of this loss on response will be only slightly greater. The lower value of Q_B has been confirmed by measurement on a number of passive-radiator systems for which the passive radiator could be replaced by an adaptor plate and a vent giving the same value of f_B.

Alignment with Enclosure Losses

The exact alignment parameters for lossy passive-radiator systems are extremely difficult to calculate from the relevant expanded form of Eq. (22). For this investigation, a shortcut was taken by observing the effects of losses on the vented-box system alignment and modifying the lossless passive-radiator system alignments similarly. The resulting alignments were tested by analog simulation and corrected as necessary to produce the desired response shapes. The final alignment data were then used

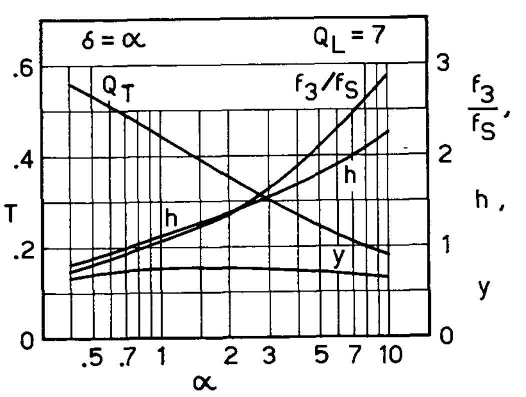

Fig. 11. Alignment chart for $\delta=\alpha$ passive-radiator systems with $Q_B = Q_L = 7$.

to construct the alignment chart of Fig. 11. This covers the same $\delta=\alpha$ alignments as Fig. 7, but for the condition $Q_B=Q_L=7$. This condition is so typical of the total-loss structure of a wide variety of passive-radiator systems that have been tested (actual measured Q_B of 5) that no alignment charts for other values would appear to be useful. As a representation of typical conditions, Fig. 11 may be compared directly with [5, Fig. 11] for vented-box systems with $Q_B=Q_L=7$.

Transient Response

The step responses of a selection of $\delta=\alpha$ lossless passive-radiator alignments are presented in Fig. 12. If these

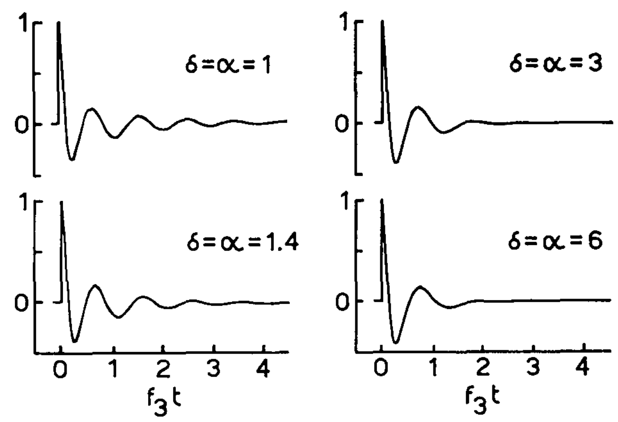

Fig. 12. Normalized step response of passive-radiator loudspeaker system (from simulator).

are compared to the corresponding step responses of equivalent vented-box alignments [5, Fig. 14], it is clear that the steeper cutoff slopes of the passive-radiator system contribute greater overshoot and transient ringing, particularly for systems with low compliance ratios. However, as pointed out earlier, it is the value of δ which is of greatest importance. If δ is made high, then even the low-α alignments for the passive-radiator system become very much like their vented-box system counterparts.

4. EFFICIENCY

Reference Efficiency

The piston-range reference efficiency η_0 of the passive-radiator system is the reference efficiency of the system driver when the total air-load mass on the driver diaphragm is that imposed by the enclosure. Thus [4, eq. (32)],

$$\eta_0 = \frac{4\pi^2}{c^3} \cdot \frac{f_S{}^3 V_{AS}}{Q_{ES}} . \tag{28}$$

Efficiency Factors

Following the method of [5, sec. 5], Eq. (28) may be put into the form

$$\eta_0 = k_\eta \, f_3{}^3 \, V_B \tag{29}$$

where f_3 is the half-power or −3-dB cutoff frequency of the system, V_B is the net internal volume of the system enclosure, and k_η is an efficiency constant consisting of two factors; namely,

$$k_\eta = k_{\eta(Q)}\, k_{\eta(G)} \tag{30}$$

where

$$k_{\eta(Q)} = Q_T/Q_{ES} \tag{31}$$

$$k_{\eta(G)} = \frac{4\pi^2}{c^3} \cdot \frac{V_{AS}}{V_B} \cdot \frac{f_S{}^3}{f_3{}^3} \cdot \frac{1}{Q_T} . \tag{32}$$

Driver Loss Factor

If $R_g = 0$, then $Q_T = Q_{TS}$, where

$$Q_{TS} = \frac{Q_{ES}Q_{MS}}{Q_{ES}+Q_{MS}} . \tag{33}$$

Thus

$$k_{\eta(Q)} = Q_{TS}/Q_{ES} = 1 - \frac{Q_{TS}}{Q_{MS}} . \tag{34}$$

This efficiency factor reflects the effects of mechanical losses in the system driver. For typical drivers used in passive-radiator systems, $k_{\eta(Q)}$ has a value in the range of 0.8 to 0.95.

System Response Factor

For normal passive-radiator system enclosures containing only a small amount of damping material used as a lining,

$$C_{AB} = V_B/(\rho_0 c^2) \tag{35}$$

and Eq. (32) can be written as

$$k_{\eta(G)} = \frac{4\pi^2}{c^3} \cdot \frac{\alpha}{Q_T(f_3/f_S)^3} . \tag{36}$$

For any passive-radiator system alignment contained in Figs. 7 or 11, the values of α, Q_T, and f_3/f_S are known

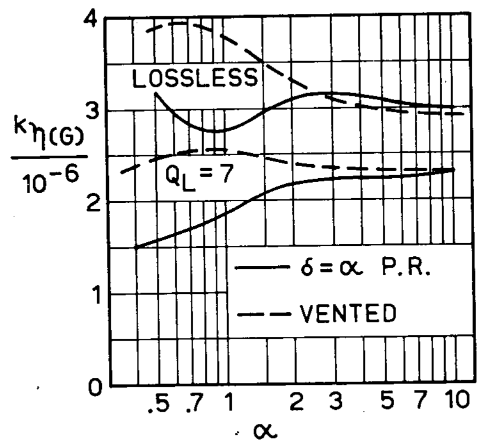

Fig. 13. Response factor $k_{\eta(G)}$ of efficiency constant for $\delta=\alpha$ passive-radiator systems (solid lines) and vented-box systems (broken lines) with lossless enclosures and with $Q_B = Q_L = 7$.

and the value of $k_{\eta(G)}$ may be calculated. Fig. 13 is a plot of the value of $k_{\eta(G)}$ as a function of α for Q_L equal to 7 and infinity. For comparison, the corresponding curves for vented-box systems [5, Fig. 15] are shown by broken lines. Note that the pairs of curves differ only in the value of δ; this is infinite for the vented-box system but equal to α for the passive-radiator system. Thus for the alignment types included here, there is little difference in $k_{\eta(G)}$ for δ values above about 2; lower values, however, place the passive-radiator system at a definite disadvantage.

5. DISPLACEMENT-LIMITED POWER RATINGS

Driver Diaphragm Displacement

The passive-radiator system displacement function given by Eq. (23) has essentially the same form as that for the vented-box system [5, eq. (14)]. However, k_x for the passive-radiator system, as given by Eq. (24), is less than unity. This indicates that for very low frequencies at least, the driver diaphragm displacement for the passive-radiator system is less than that for the vented-box system. Fig. 14 is a plot of $k_x|X(j\omega)|$ for several of the lossless $\delta=\alpha$ passive-radiator system alignments. The frequency scale is normalized to f_B. As expected, this plot

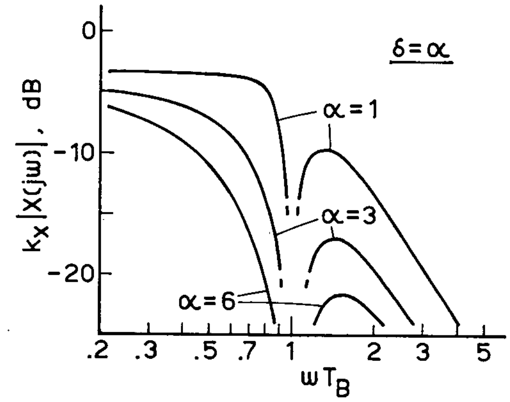

Fig. 14. Normalized diaphragm displacement of passive-radiator system driver as a function of normalized frequency for several typical $\delta=\alpha$ lossless alignments (from simulator).

is very similar to the corresponding vented-box data [5, Fig. 17], except at very low frequencies. But the low-frequency displacement decrease is not large.

From Eq. (24) the displacement at very low frequencies can be reduced by up to 6 dB if $\delta=\alpha>>1$. Significantly greater reduction is possible only if α is large and δ is small. Because small values of δ lead to rather poor performance in terms of transient response and the value of $k_{\eta(G)}$, it is clear that no dramatic reduction of very-low-frequency diaphragm displacement sensitivity over that of the vented-box system can be achieved with the passive-radiator system, unless a considerable sacrifice of performance can be tolerated.

Acoustic Power Rating

Assuming linear large-signal diaphragm displacement, the steady-state displacement-limited acoustic power rating P_{AR} of a loudspeaker system, from [4, eq. (42)], is

$$P_{AR} = \frac{4\pi^3\rho_0}{c} \cdot \frac{f_S{}^4 V_D{}^2}{k_x{}^2|X(j\omega)|_{max}{}^2} \tag{37}$$

where $|X(j\omega)|_{max}$ is the maximum magnitude attained by the displacement function and V_D is the peak displacement volume of the driver diaphragm. The latter is given by

$$V_D = S_D\, x_{max} \tag{38}$$

where x_{max} is the peak linear displacement of the driver diaphragm.

Eq. (37) may be written in the form

$$P_{AR} = k_P f_3^4 V_D^2 \qquad (39)$$

where k_P is a power rating constant given by

$$k_P = \frac{4\pi^3\rho_0}{c} \cdot \frac{1}{(f_3/f_S)^4 \, (k_x|X(j\omega)|_{max}{}^2} \, . \qquad (40)$$

Values of (f_3/f_S) may be calculated for any alignment. From Fig. 14 the quantity $k_x|X(j\omega)|$ has two maxima, one within and one below the passband, just as for the vented-box system. For the passband maxima, the magnitudes are very little different from those of comparable vented-box alignments. The alignment data are also similar, particularly for large δ. Thus for average program material having most of its energy within the system passband, the power ratings must be about the same as for vented-box systems, i.e. [5, eq. (41)],

$$P_{AR} = 3.0 \, f_3^4 \, V_D^2 \, . \qquad (41)$$

For a graphical illustration of this relationship between acoustic power rating, cutoff frequency, and driver displacement volume, see [5, Fig. 19].

Note that this rating is not affected by the displacement reduction that occurs at *very* low frequencies for the passive-radiator system, because this reduction does not extend to the frequency range near cutoff. However, it is reasonable to expect that the passive-radiator system should be somewhat less vulnerable to *very*-low-frequency signals such as amplifier turn-on and turn-off transients and the too hastily lowered pickup stylus.

Electrical Power Rating

The displacement-limited electrical input power rating P_{ER} of the passive-radiator system may be obtained by dividing the acoustic power rating by the system reference efficiency. The dependence of this rating on the important system parameters is observed by dividing Eq. (39) by Eq. (29):

$$P_{ER} = \frac{P_{AR}}{\eta_0} = \frac{k_P}{k_\eta} f_3 \frac{V_D^2}{V_B} \, . \qquad (42)$$

6. PASSIVE-RADIATOR REQUIREMENTS

The effective surface area of the passive radiator is usually made equal to that of the driver. This condition is not necessary for successful operation, but several factors encourage it. It was stated earlier that the passive radiator is often made from the same frame and suspension as the driver; the economic advantages of this approach are readily apparent, and it results in equal areas.

The use of a passive radiator which is substantially larger than the driver is seldom feasible because of the required baffle area. In most cases the size of both driver and passive radiator are limited by the enclosure dimensions, and it is impractical to make the passive radiator area more than about twice that of the driver.

The alternative of making the passive radiator smaller than the driver is almost never encountered. The principal reason for this is that the volume displacement required of the passive radiator is quite substantial. A small area therefore requires a very large linear displacement capability which can be difficult to achieve in practice.

In Section 5 the power capacity of the passive-radiator system is determined on the basis that the limiting factor is the displacement volume of the driver. If this power capacity is to be realized in practice, the passive radiator must be designed so that it is capable of displacing the maximum volume required of it by the system at rated power output. This volume displacement requirement is normally larger than that of the driver and is the physical reason for the relatively high power rating constant of the system.

The relative maximum volume displacement requirements for the driver and passive radiator may be found from Eqs. (23) and (25), recognizing that at zero frequency the passive-radiator volume displacement must be $\delta/(\delta+1)$ of that of the driver as noted in Section 2. Fig. 15 illustrates the relative displacements as a function of frequency for the lossless maximally flat $\delta=\alpha$ alignment. The maxima occur at different frequencies, but, most importantly, high passive-radiator displacement is required within the system passband.

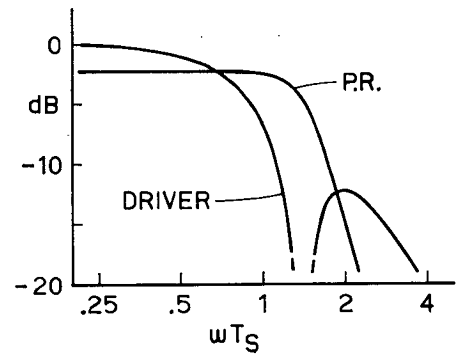

Fig. 15. Normalized displacements of driver and passive-radiator as a function of normalized frequency for lossless maximally flat $\delta=\alpha$ passive-radiator system alignment.

For program-rated systems, the passive radiator displacement volume V_{PR} must typically be about twice the rated driver displacement volume V_D. Fig. 16 is a plot of the required ratio of V_{PR} to V_D as a function of α for all of the $\delta=\alpha$ alignments. If driver and passive radiator have the same effective surface areas, the maximum linear displacements must be in this ratio.

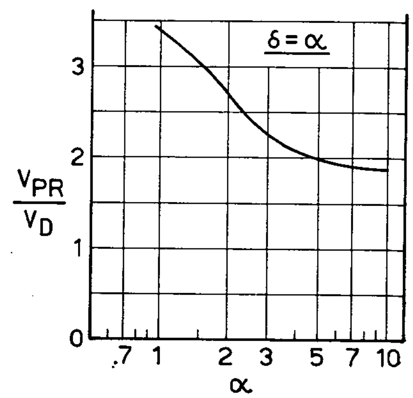

Fig. 16. Required ratio of passive-radiator displacement volume V_{PR} to driver displacement volume V_D as a function of α for program-rated $\delta=\alpha$ passive-radiator systems (from simulator).

Not all high-quality drivers have a suspension capable of more than twice the linear displacement that the magnet/voice-coil structure can provide with good linearity. For this reason, optimum design of passive-radiator systems may require that the passive-radiator suspension be somewhat different from that of the driver. The "convenience" of using the same suspension may in fact result in limited power capacity compared to that which could be achieved with a specially designed passive radiator.

An interesting feature of the $\delta = \alpha$ alignments is the small variation of the required value of $y = (f_P/f_S)$. For the most common alignments, a passive radiator made from the same frame and suspension as the driver (assuming adequate displacement capability) consistently requires a diaphragm mass almost twice that of the driver for correct system alignment.

The general requirements for a passive radiator may be summarized as acoustic mass and displacement volume roughly twice those of the driver, acoustic compliance equal to or greater than that of the driver, and suspension losses as low as possible.

7. MUTUAL COUPLING IN PASSIVE-RADIATOR SYSTEMS

Mutual coupling in passive-radiator systems takes the same form as for vented-box systems [5, sec. 8]. However, the effects are generally even smaller than for the vented-box system.

If the diameter of the passive radiator is equal to that of the driver, as is usual, the minimum center-to-center aperture spacing is greater than for the vented-box system, and the mutual coupling mass is therefore smaller. Furthermore, passive radiators are most often used in smaller loudspeaker systems which require relatively heavy driver cones to obtain extended low-frequency response. The mutual-coupling mass under these conditions represents only a tiny fraction of the total driver moving mass, giving quite negligible effects on both performance and measurement.

8. PARAMETER MEASUREMENT

Voice-Coil Impedance

The voice-coil impedance function of the passive-radiator system is given by Eq. (26). The steady-state magnitude $|Z_{VC}(j\omega)|$ of this function has the shape plot-

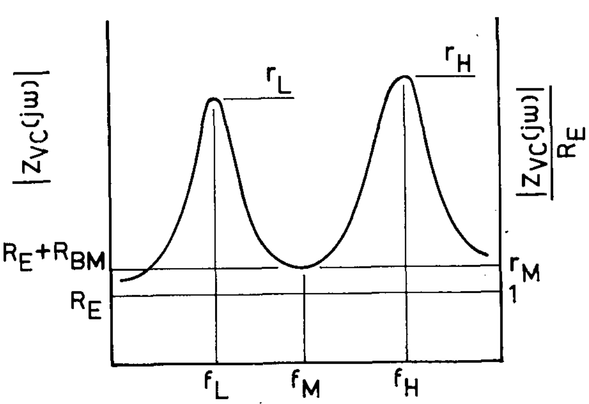

Fig. 17. Voice-coil impedance magnitude of passive-radiator loudspeaker system as a function of frequency.

ted in Fig. 17. This shape is exactly the same as that for the vented-box system [5, Fig. 20]. The plot has two maxima, at the frequencies labeled f_L and f_H. Between these maxima, there is a minimum at a frequency near f_B which is labeled f_M. At f_M the minimum impedance is slightly greater than R_E; the additional resistance is contributed by enclosure and passive-radiator losses and designated R_{BM}.

Small-Signal Parameter Measurement

The measured impedance curve of a passive-radiator system conforms closely to the shape of Fig. 17. The impedance maximum at f_L is usually lower than that at f_H because of passive-radiator losses. As in the case of the vented-box system, the basic system parameters may be evaluated with satisfactory accuracy by ignoring enclosure and passive-radiator losses for initial calculations and then calculating the system losses using the approximate system data.

Ignoring enclosure and passive-radiator losses, and assuming that $f_M \doteq f_B$, Eq. (26) may be used to derive the following parameter-impedance-plot relationships:

$$\frac{\delta+1}{\alpha+\delta+1} = \frac{f_B^2\, f_{SB}^2}{f_L^2\, f_H^2} \tag{43}$$

$$\frac{\alpha\,\delta}{\alpha+\delta+1} = \frac{(f_H+f_B)\;(f_H-f_B)\;(f_B+f_L)\;(f_B-f_L)}{f_L^2\, f_H^2}. \tag{44}$$

These relationships do not give an immediate solution for any of the passive-radiator system parameters as do their counterparts for the vented-box system [5, eqs. (44) and (45)]. This is because only the same amount of information is available from the impedance curve while the system has the additional parameter δ to be evaluated.

However, it is relatively easy to evaluate α. If the passive radiator can be removed from the enclosure, it can be replaced temporarily by a vent. Then f_{SB} and α can be calculated as for a vented-box system from [5, eqs. (44) and (45)]. The passive-radiator aperture can also be blocked off and α evaluated as for a closed-box system from [8, eq. (48)]. Alternatively, the driver resonance frequency f_S may be measured and adjusted to correspond to the air-load mass applicable in the enclosure; then, using the passive-radiator system impedance-plot data,

$$\alpha = \frac{f_H^2 + f_L^2 - f_B^2}{f_{SB}^2} - 1 \tag{45}$$

where Eq. (45) is derived directly from Eqs. (43) and (44).

With α and f_{SB} determined, δ may be found from either Eq. (43) or Eq. (44). A useful check for errors of measurement, calculation, or approximation is the computation of δ from both equations and comparison of the values obtained. Using the measured values of δ and f_B, f_P may be calculated from Eq. (21).

The remaining system parameters are measured in the manner described in [4, Appendix] and [5, sec. 6]. The value of Q_B computed from [5, eq. (49)] includes the effect of passive-radiator losses; assigning a value about 30–40% greater than this to Q_L gives a very satisfactory picture of the system response for evaluation purposes.

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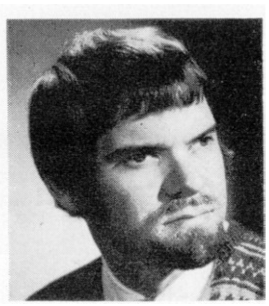
THE AUTHOR



Richard H. Small received the degrees of Bachelor of Science (1956) from the California Institute of Technology and Master of Science in Electrical Engineering (1958) from the Massachusetts Institute of Technology.

He was employed in electronic circuit design for high-performance analytical instruments at the Bell & Howell Research Center from 1958 to 1964, except for a one-year visiting fellowship to the Norwegian Technical University in 1962. After a working visit to Japan in 1964, he moved to Australia. In 1972, following the completion of a program of research into direct-radiator electrodynamic loudspeaker systems, he was awarded the degree of Doctor of Philosophy by The University of Sydney. He is now a Lecturer in Electrical Engineering at The University of Sydney.

Dr. Small is a member of the Institute of Electrical and Electronics Engineers, a member of the Institution of Radio and Electronics Engineers Australia, and a Fellow of the Audio Engineering Society.

Passive-Radiator Loudspeaker Systems Part II: Synthesis*

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Passive-radiator loudspeaker systems can be designed to specification as easily as vented-box systems. Driver requirements are generally about the same as for comparable vented-box systems, and the requirements of the passive radiator are directly related to those of the driver. The passive-radiator principle is particularly useful in compact systems where vent realization is difficult or impossible, but it can also be applied satisfactorily to larger systems.

INTRODUCTION: The analysis presented in Part I shows that the passive-radiator system is a very close relative of the vented-box system. The principal difference in performance is the presence of a notch in the frequency response below cutoff. While this notch can noticeably degrade performance, it can through the provision of high passive-radiator suspension compliance be placed so low in frequency that the system performance is virtually indistinguishable from that of a vented-box system in most fundamental respects.

However, the passive-radiator system has the distinct advantage that it is physically realizable in many cases where the vented-box system is not. This is particularly true of very compact designs which are required to have a low cutoff frequency. Fortunately it is just this requirement which is easiest to realize with the notch frequency well below cutoff. In this regard, the passive-radiator system may be considered as a most natural and logical extension of the vented-box system [10].

9. DISCUSSION

Comparison of Passive-Radiator and Vented-Box Systems

Many of the major differences between vented-box and passive-radiator systems have already been presented in Part I. However, some of the particular similarities and differences merit further discussion.

Driver Requirements

For a given specification of enclosure size, system response, and power capacity, the required driver parameters are virtually the same for both vented-box and passive-radiator systems. Expressed in another way, a particular driver will give substantially the same performance in a given enclosure, regardless of whether the enclosure has a vent or a passive radiator, so long as the passive-radiator compliance ratio δ is high, the passive-radiator losses are not excessively large, and the enclosure is tuned to the correct frequency in each case.

Design Complexity

The additional design complexity of the passive-radiator system is entirely associated with the passive-radiator suspension compliance. Fortunately, this compliance is not critical in the sense that it must always be adjusted to a precise value. The general requirements are easily summed up: allow for the required displacement, and provide maximum compliance (at least equal to that of the driver) with minimum losses. If these requirements are observed, the design of passive-radiator systems is no more complex than that of vented-box systems. The only practical difference is that the required value of f_B is obtained by adjusting the passive-radiator diaphragm mass instead of the acoustic mass of a vent.

* An abridged version of this paper was presented September 10, 1973, at the 46th Convention of the Audio Engineering Society.

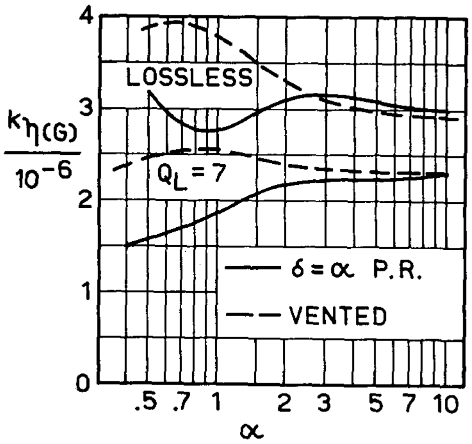

Fig. 13. Response factor $k_{\eta(G)}$ of efficiency constant for $\delta=\alpha$ passive-radiator systems (solid lines) and vented-box systems (broken lines) with lossless enclosures and with $Q_B=Q_L=7$.

Small-Signal Performance

Fig. 13 (repeated from Part I) shows that the two systems have comparable small-signal performance limits when δ is large. For small values of δ, however, passive-radiator systems have significantly lower values of $k_{\eta(G)}$ than do their vented-box counterparts. This is why passive-radiator suspension compliance should always be made as high as practicable.

For a range of alignments near and above $\alpha = 3$, Fig. 13 shows that the lossless $\delta = \alpha$ passive-radiator system has a value of $k_{\eta(G)}$ slightly greater than that for the lossless vented-box system. Fig. 18 compares the responses for $\alpha = 3$; the driver parameters are virtually identical for both systems. The value of f_3 for the passive-radiator system is indeed about 1% lower, while the cutoff slope is visibly steeper.

For systems with realistic losses, the passive-radiator system appears to be at a disadvantage compared to the vented-box system, although the difference is very small when δ is large. Fig. 19 shows the frequency response measured by the method of [11] for a laboratory driver and test enclosure, first with a vent and again with a passive radiator. The compliance ratios (δ and α) of about unity for this particular system theoretically should favor the use of a vent. The penalty for low passive-radiator compliance is readily apparent in the higher cutoff frequency and steeper initial cutoff slope for the passive radiator.

It is emphasized that the condition $\delta = \alpha$, though common in practice, is used in this paper only as a matter of convenience to simplify the vast range of possible alignments. For best performance it is clearly advisable to use the highest practicable value of passive-radiator compliance.

Large-Signal Performance

Given adequate passive-radiator displacement volume, only small differences are likely to exist in the power capacities of the two systems. These would depend upon the specific relationship between the power spectrum of the driving signal and the exact alignment of the systems.

Popular Beliefs about Passive Radiators

Two particular advantages which are widely claimed for passive-radiator systems, either in popular magazine articles or in advertisements, deserve specific comment in the light of the preceding analysis and discussion.

The first claimed advantage is that the uniform air-particle velocity in the region of the passive radiator is an improvement over the comparatively nonuniform amplitude and phase conditions existing over the aperture of a vent.

This observation first appeared [2, p. 225] in support of a claim that the nonuniform particle velocity in a vent gives rise to vent losses which are eliminated by the use of a passive radiator. This is of course nominally true, but if a vent is properly designed and unobstructed, then the amount of energy dissipated as a result of nonuniform air velocity is relatively small compared to other enclosure losses [5, sec. 3] and easily may be exceeded by that dissipated in the suspension of typical contemporary passive radiators.

Other authors have sometimes misinterpreted the text of [2] and have claimed or suggested that nonuniform particle velocity in a vent is by its very nature inefficient or even nonlinear. But from [4], the relative amplitudes and phases of individual particles are not important. It is their total integrated effect, i.e., the total (phasor sum) volume velocity crossing the enclosure boundaries, that determines the system output. So long as the average particle velocity in the vent is held within the limit discussed in [5, sec 8], all air movement can remain substantially linear and no loss of output or significant nonlinear distortion will occur.

The second claimed advantage of passive-radiator systems (which is particularly popular with advertising copy-

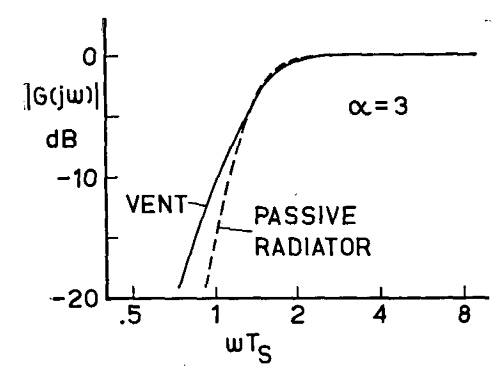

Fig. 18. Response of lossless vented-box and $\delta=\alpha$ passive-radiator systems for $\alpha=3$ (from simulator).

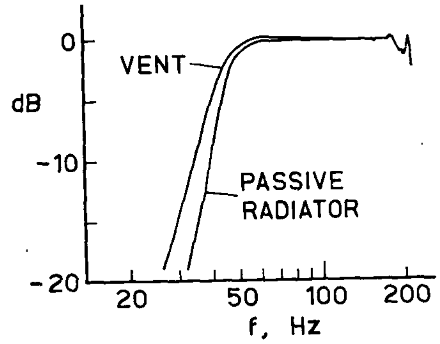

Fig. 19. Response of experimental loudspeaker system with interchangeable vent and passive radiator. Parameters with vent: $\alpha = 1.0$, $h=1.1$, $Q_T=0.37$, $Q_B=9$; passive-radiator compliance ratio $\delta = 1.0$.

writers) is that the use of a passive radiator "doubles the radiating area at low frequencies." It is naturally implied that this is somehow beneficial to performance.

The passive-radiator system does indeed possess the same advantages over the single-diaphragm closed-box system as does the vented-box system [5, Part II]. These advantages, however, depend simply on the *presence* of the secondary aperture, not on its *area*. The passive radiator aids the driver only to the same degree as does a vent. In fact, over the frequency range near f_B where the passive radiator (or vent) contributes most usefully to the system output, it does so through reducing and replacing, rather than supplementing (as so often implied) the motion of the driver.

Additional Features of Passive-Radiator Systems

It might appear from the discussion so far that there is no advantage to using a passive radiator in larger systems for which a satisfactory vent could be realized. Certainly the passive radiator represents a moderate additional cost. Measurements made on systems of this type using interchangeable vents and passive radiators indicate consistently that a passive radiator has greater losses and gives a slightly higher f_3 compared with a vent. But there are at least two features of the passive radiator which do not appear in the basic analysis of the system that are worth taking note of.

First, a passive radiator is entirely free of the windage and resonant-tube noises which are often generated by a vent operated at high volume velocity. So long as the passive radiator is designed to accommodate large linear volume displacements, the total spurious distortion of the passive radiator may then be less.

Second, the passive radiator acts as a physical barrier to the propagation of sound at high frequencies from within the enclosure. Some of the sound coloration which results from the coupling of internal standing-wave modes of the enclosure to the room via natural propagation through the air of a vent is thus substantially reduced or eliminated by the use of a passive radiator.

These two features of the passive-radiator system are perhaps secondary in nature, but they could be important in particular applications.

Typical Passive-Radiator System Performance

During 1969 and 1970 a sample of commercially produced passive-radiator systems was tested by measuring the basic system parameters and obtaining the system response from an analog simulator adjusted to duplicate the system parameters. Only five such systems could be obtained at the time, ranging in enclosure volume from 12 to 56 dm^3 (0.4 to 2 ft^3). They were produced by one manufacturer in the United States and one in Great Britain. Three used 8-in (20-cm) drivers and passive radiators, one used 10-in (25-cm) units, and the last used 12-in (30-cm) units.

Four of the systems had cutoff frequencies f_3 below 50 Hz (the lowest was 39 Hz) and response peaks less than 1 dB. The fifth (and smallest) system had a cutoff frequency of 60 Hz and a response peak of 3 dB; this performance was expected because the enclosure volume was only 12 dm^3 (0.4 ft^3) and the driver and passive radiator were identical to those used in one of the larger systems for which they were more ideally suited.

All systems had values of α and δ equal to or greater than 3, and for the most part these were equal. Three systems had measured Q_B values of 5; the others had values of 4 and 6. Reference efficiencies were all between 0.4 and 0.6%.

All the systems tested were extremely well made and appeared to be the result of very careful testing, as would be expected from these particular manufacturers. It appears that the lack of generally available design information for passive-radiator systems has limited their application to only the most competent manufacturers who have the skill and facilities to carry out careful design and evaluation.

10. SYSTEM SYNTHESIS

System-Component Relationships

The design of passive-radiator systems is exactly analogous to that of vented-box systems [5, sec. 10]. The basic small-signal alignment data are obtained from Fig. 11 (repeated from Part I) for the vast majority of systems having $\delta=\alpha$ and a typical (effective) Q_B value of 7. The alignment chart for vented-box systems with $Q_B = 7$ [5, Fig. 11] is also valid for passive-radiator systems with infinite δ and is reproduced here as Fig. 20. This chart may be used in conjunction with Fig. 11 to interpolate

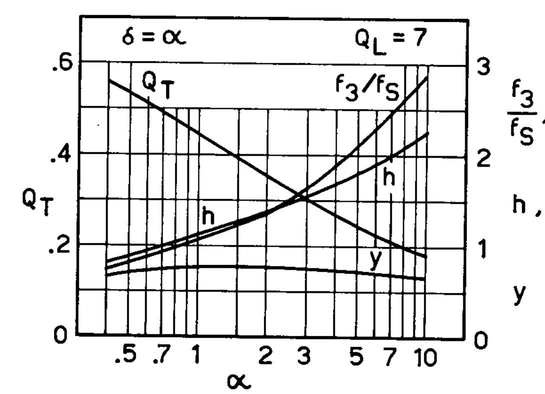

Fig. 11. Alignment chart for $\delta=\alpha$ passive-radiator system with $Q_B=Q_L=7$.

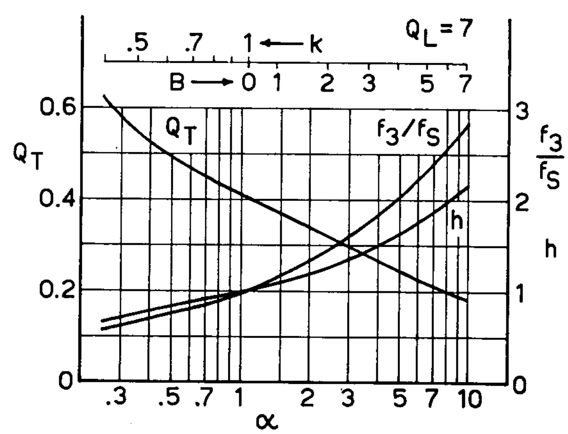

Fig. 20. Alignment chart for vented-box systems with $Q_B=Q_L=7$. Also valid for passive-radiator systems with infinite δ ($y = 0$).

alignments for systems with values of δ greater than α. Comparison of the two figures shows that there is little difference in Q_T or f_3/f_S for large values of α; only h varies noticeably with δ, but not very much.

For unusual design conditions wherein Q_B is quite high or low, but provided that α is large and δ is equal to or greater than α, any of the alignment charts of [5] may be used in place of Fig 11. It is the rarity of either extreme-loss condition, the usefulness of these alternate charts, and the relative unimportance of the actual value of δ (so long as it is large) that make it unnecessary for any charts other than Figs. 11 and 20 to be provided here. For extremely unusual design cases, alignment data may be calculated from the relationships given in the Appendix.

System design procedures are summarized below for both the optimum use of a given driver and the design of a complete system from specifications. Each summary is followed by a specific design example.

Design with a Given Driver

The design of an enclosure and passive radiator to suit a given driver begins with knowledge of the basic small-signal parameters of the driver: f_S, Q_{TS} and V_{AS}. If these are not already known, they may be measured by the method given in [4]. The measurements should be made with the driver on a standard test baffle or the results otherwise adjusted to correspond to the air-mass loading conditions of an enclosure; i.e., it is f_{SB} (and the corresponding value of Q_{TS}), not f_{SA} (the value for free-air loading) that is needed.

The value of Q_{TS} must be no larger than about 0.5 for use in a passive-radiator system. Larger values lead to alignments with excessive passband ripple. It is assumed here that the system will be used with an amplifier having negligible output (Thevenin) resistance so that $Q_T = Q_{TS}$. Thus if the value of Q_{TS} is reasonable, find this value on the Q_T curve in Fig. 11. The value of α on the abscissa corresponding to this value of Q_T is the system compliance ratio required for an optimum "flat" alignment. Using this value of α, the other curves of the figure give the required values of h or y (and therefore f_B or f_P) and the resulting value of f_3 for the system. The required enclosure volume is $V_B = V_{AS}/\alpha$.

The system reference efficiency η_0 is calculated from the driver parameters using Eq. (28). The approximate displacement-limited acoustic power capacity P_{AR} is calculated from Eq. (41) if V_D is known; V_D can be evaluated as described in [8, sec. 6]. The approximate displacement-limited input power capacity P_{ER} is found by dividing P_{AR} by η_0 as indicated by Eq. (42).

If the passive radiator is made from the same frame and suspension as the driver (assuming adequate displacement capability), the diaphragm mass is adjusted to obtain the required value of f_B as indicated by the system impedance curve (see Section 8, Part I).

Example of Design with a Given Driver

It is instructive to repeat here the design example carried out in [5, sec. 10] for two reasons. First, in that example the required vent dimensions of 65-mm (2.6-in) diameter and 175-mm (7-in) length are not wholly desirable. The length is somewhat excessive for a compact enclosure, and the ratio of length to diameter is great enough to promote resonant-pipe amplification of vent windage noises. This suggests that a passive radiator would probably give better overall system performance.

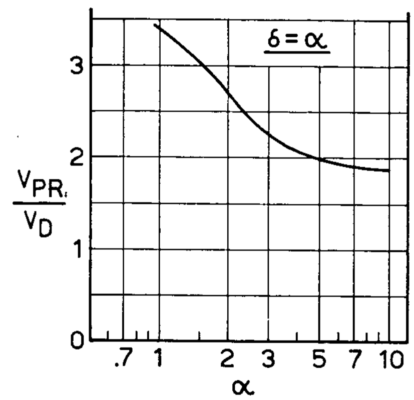

Fig. 16. Required ratio of passive-radiator displacement volume V_{PR} to driver displacement volume V_D as a function of α for program-rated, $\delta=\alpha$ passive-radiator systems (from simulator data).

Second, the driver parameters used in this example are in fact those of a driver of the same type as that contained in one of the commercial passive-radiator systems described in the previous section. The calculated enclosure design may thus be compared to that found desirable by the manufacturer.

The driver parameters are

$$f_S = 33 \text{ Hz}$$
$$Q_{MS} = 2.0$$
$$Q_{ES} = 0.45$$
$$V_{AS} = 57 \text{ dm}^3 \text{ (2 ft}^3)$$
$$V_D = 120 \text{ cm}^3$$
$$P_{ER} = \text{(adequate for use with 25-W amplifier)}$$

and by calculation using Eq. (33) and (28),

$$Q_{TS} = 0.37$$
$$\eta_0 = 0.44\%.$$

For the vented-box design example, the modest enclosure size led to the assumption of $Q_B = 10$. Clearly, the enclosure loss must be higher with the passive radiator, especially if the latter is constructed from the same suspension that produced $Q_{MS} = 2$ for the driver. Hence, using the alignment data from Fig. 11, and assuming negligible driving source impedance so that $Q_T = Q_{TS} = 0.37$, the appropriate system small-signal parameters are

$$\alpha = 1.72$$
$$h = 1.30 \ (y = 0.79)$$
$$f_3/f_S = 1.28$$

and the system design is thus

$$V_B = 33 \text{ dm}^3 \text{ (1.2 ft}^3)$$
$$f_B = 43 \text{ Hz } (f_P = 26 \text{ Hz})$$
$$f_3 = 42 \text{ Hz.}$$

From Eqs. (41) and (42),

$$P_{AR} = 3f_3^4 V_D^2 = 130 \text{ mW}$$
$$P_{ER} = P_{AR}/\eta_0 = 30 \text{ W.}$$

For the 25-W input limit recommend by the manufacturer for this driver, the useful value of P_{AR} is 110 mW.

For $\delta=\alpha$, Fig. 16 suggests that V_{PR} must be about 2.9 times V_D. Because the input power is restricted to 25 W, not quite all of the available V_D is used; the required value of V_{PR} is therefore about 320 cm³. If the passive radiator has the same diaphragm area as the driver, its total "throw" must be a substantial 32 mm (1.3 in).

The vented-box system designed around this driver in [5, sec. 10] has a 37-dm³ (1.3-ft³) enclosure, a cutoff frequency of 38 Hz, but a power capacity of only 90 mW acoustical and 20 W electrical. The passive-radiator design, as a result of its higher cutoff frequency, makes better use of the maximum thermal power capacity of the driver. But because the values of α and especially δ are not particularly high, the value of k_η for this system is noticeably poorer. A higher value of δ (greater passive-radiator suspension compliance), if physically realizable, would be an advantage to this system.

For comparison, the commercial system which uses this driver has the measured properties

$$V_B = 21\ \text{dm}^3\ (0.74\ \text{ft}^3)$$
$$f_B = 44\ \text{Hz}\ (f_P = 23\ \text{Hz})$$
$$f_3 = 46\ \text{Hz}$$
$$Q_B = 5.1.$$

This represents a higher α (and δ) alignment which has a slight (1-dB) response peak and quite satisfactory cutoff frequency. And significantly, the displacement requirements for both driver and passive radiator are considerably reduced for this system if the input power is still restricted to 25 W.

Design from Specifications

The procedure for designing a passive-radiator system from specifications essentially follows that of [5, sec. 10] for vented-box systems. For passive-radiator systems, however, the range of alignments specified should be limited to system compliance ratios (or at least δ values) of 3 or more. For $\delta = \alpha$ designs, Fig. 11 of the present paper can be used for determination of the driver and passive-radiator small-signal parameters. As with the vented-box system, an alignment with passband peaking may be obtained by allowing a modest increase in Q_T and/or h over the values required for flat response.

The mechanical properties of both driver and passive radiator are calculated from the acoustical requirements by the method of [8, sec. 10] or [5, sec. 11]. The required value of V_{PR} is found from Fig. 16 after the required value of V_D has been calculated.

Example of Design from Specifications

One of the ideal applications of the passive-radiator principle is in compact systems where a low cutoff frequency is required together with a relatively high value of the efficiency constant k_η. Such loudspeaker systems can be expected to provide satisfactory acoustical performance when driven from amplifiers of moderate power and indeed would typically be sold in pairs for use in small rooms together with a stereo amplifier having a continuous power rating of about 15 W per channel. Accordingly, let the system specifications start with the following:

$$V_B = 25\ \text{dm}^3\ (0.9\ \text{ft}^3)$$
$$f_3 = 40\ \text{Hz}$$
$$P_{ER} = 15\ \text{W}$$

Use: normal program material with 10-dB peak-average power ratio.

The actual alignment has not yet been specified.

For the specified enclosure size it is assumed that both driver and passive radiator must be 8-in (20-cm) units. With such a configuration, it should readily be possible to obtain α and δ values of 3. From Figs. 6, 12, 13, and 16 this alignment provides satisfactory response with a reasonable value of $k_{\eta(G)}$ and a moderate passive-radiator–driver displacement ratio. This completes the system specifications. It is assumed that amplifier driving impedance will be negligible and that system losses will be of normal magnitude.

Design then begins with Fig. 11. For $\delta = \alpha = 3$, the required alignment parameters are

$$Q_T = 0.30$$
$$h = 1.52\ (y = 0.76)$$
$$f_3/f_S = 1.63.$$

Thus the required driver parameters are

$$f_S = 24.5\ \text{Hz}$$
$$V_{AS} = 75\ \text{dm}^3$$
$$Q_{TS} = 0.30$$

and the passive radiator mass must be adjusted so that

$$f_B = 37.3\ \text{Hz}$$

or, from Eq. (21),

$$f_P = 18.6\ \text{Hz}.$$

If it is assumed that the driver Q_{MS} will be about 3, a typical value for such a driver, then the required electrical damping is

$$Q_{ES} = 0.33.$$

Then from Eq. (28),

$$\eta_0 = 0.32\%.$$

From the large-signal specification, Eqs. (42) and (41) give

$$P_{AR} = 15(0.0032) = 48\ \text{mW}$$

and

$$V_D = 80\ \text{cm}^3.$$

From Fig. 16, $V_{PR}/V_D = 2.25$, so

$$V_{PR} = 180\ \text{cm}^3.$$

For 8-in (20-cm) units with a typical diaphragm area of 2.0×10^{-2} m², the total "throw" must then be 8 mm (0.31 in) for the driver and 18 mm (0.7 in) for the passive radiator.

Finally, the driver voice coil must be able to dissipate as heat an average nominal input power of at least 1.5 W.

The remaining physical properties of the driver and passive radiator are calculated as outlined in [8, sec. 10]. For the driver, these are

$$C_{MS} = V_{AS}/(\rho_0 c^2 S_D^2) = 1.34 \times 10^{-3}\ \text{m/N}$$
$$M_{MS} = (\omega_S^2 C_{MS})^{-1} = 31.5\ \text{g}$$
$$M_{MD} = M_{MS} - (\text{air load}) = 28.7\ \text{g (a heavy cone)}$$
$$B^2l^2/R_E = \omega_S M_{MS}/Q_{ES} = 14.7\ \text{N·s/m}$$

or, for $R_E = 6.5\ \Omega$ (typical for 8-Ω rating),

$$Bl = 9.8\ \text{T·m}.$$

Similarly, for the passive radiator,

$$C_{MP} = 1.34 \times 10^{-3}\ \mathrm{m/N}$$
$$M_{MP} = 54.3\ \mathrm{g\ (including\ air\ load)}$$
$$M_{MD} = 51.5\ \mathrm{g.}$$

11. CONCLUSION

The passive-radiator loudspeaker system is a nearly equivalent alternative to the vented-box system. It is particularly adaptable to compact enclosures for which a vented-box design cannot be satisfactorily realized.

It is important that the passive-radiator suspension compliance be made as high as conveniently possible and that the displacement limit be large enough to complement the full output capability of the driver. Beyond this, the design requirements are no more difficult than for the vented-box system; maximum performance generally results from the intelligent selection of alignment type and the avoidance of unnecessary losses.

APPENDIX

ELLIPTIC FILTER FUNCTIONS AND ALIGNMENT OF THE LOSSLESS PASSIVE-RADIATOR SYSTEM

General Expressions

The general form of filter function given in Eq. (27) is expressed in magnitude-squared form as

$$|G_H(j\omega)|^2 = \frac{\omega^8 T_0^8 + B_1\omega^6 T_0^6 + B_2\omega^4 T_0^4}{\omega^8 T_0^8 + A_1\omega^6 T_0^6 + A_2\omega^4 T_0^4 + A_3\omega^2 T_0^2 + 1} \qquad \text{(A-1)}$$

where

$$\begin{aligned} A_1 &= a_1^2 - 2a_2 \\ A_2 &= a_2^2 + 2 - 2a_1 a_3 \\ A_3 &= a_3^2 - 2a_2 \\ B_1 &= -2b_2 \\ B_2 &= b_2^2. \end{aligned} \qquad \text{(A-2)}$$

It is convenient to use a restricted form of Eq. (A-1) in which the polynomial coefficients are replaced by constants which relate directly to the types of responses found to be useful. This is

$$|G_H(j\omega)|^2 = \frac{\omega^4 T_0^4 (k_1^2 - \omega^2 T_0^2)^2}{\omega^4 T_0^4 (k_1^2 - \omega^2 T_0^2)^2 + (1 - k_2^2\omega^2 T_0^2)^2 + k_3^2\omega^2 T_0^2}. \qquad \text{(A-3)}$$

It is obvious from Eq. (A-3) that the system response null occurs when $\omega T_0 = k_1$; i.e., k_1 is the normalized frequency of the response null and is equal to $(b_2)^{1/2}$.

For equivalence of Eqs. (A-1) and (A-3),

$$\begin{aligned} B_1 &= -2k_1^2 \\ B_2 &= k_1^4 \\ A_1 &= -2k_1^2 \\ A_2 &= k_1^4 + k_2^4 \\ A_3 &= k_3^2 - 2k_2^2. \end{aligned} \qquad \text{(A-4)}$$

This imposes the constraint $A_1 = B_1$, but this constraint is common to all of the responses found useful.

The half-power (− 3 dB) frequency f_3 of any alignment is given by

$$f_3/f_0 = d^{1/2} \qquad \text{(A-5)}$$

where

$$f_0 = 1/(2\pi T_0) \qquad \text{(A-6)}$$

and d is the largest positive real root of the equation

$$d^4 - (A_1 - 2B_1)d^3 - (A_2 - 2B_2)d^2 - A_3 d - 1 = 0. \qquad \text{(A-7)}$$

For a response function specified in terms of the values of k_1, k_2, and k_3, the A_i and B_i coefficients can be calculated from Eq. (A-4). Then, using Eq. (A-2), the a_i and b_i coefficients may be found as follows:

$$b_2 = B_2^{1/2} \qquad \text{(A-8)}$$

a_2 is found as a positive real root of

$$a_2^4 - 2(A_2 + 6)a_2^2 - 8(A_1 + A_3)a_2 + (A_2 - 2)^2 - 4A_1 A_3 = 0 \qquad \text{(A-9)}$$

then

$$\begin{aligned} a_1 &= (A_1 + 2a_2)^{1/2} \\ a_3 &= (A_3 + 2a_2)^{1/2}. \end{aligned} \qquad \text{(A-10)}$$

Types of Response

Elliptical Responses [7]

This family of responses is characterized by $k_3 = 0$. The amplitude response has equal-ripple characteristics in both passband and stopband.

Symmetrical Elliptical Responses

This family of responses is characterized by $k_3 = 0$ and $k_2 = k_1$. It has the same properties as the general elliptical family with the addition of the symmetry characteristic

$$G(sT_0) = 1 - G(1/sT_0). \qquad \text{(A-11)}$$

Maximally-Flat-Passband-Amplitude Responses [12]

The general maximally flat passband requirement [7] is satisfied by Eq. (A-3) for $k_2 = k_3 = 0$. This requires that, for any suitable value of a_1,

$$\begin{aligned} a_3 &= 2/a_1 + a_1/\sqrt{2} \\ a_2 &= a_3^2/2 \\ b_2 &= a_2 - a_1^2/2. \end{aligned} \qquad \text{(A-12)}$$

"Quasi-Maximally-Flat" Responses

The condition of Thiele's "quasi-Butterworth" responses [13] is met by Eq. (A-3) for $k_2 = 0$ and $k_3 > 0$.

Alignment of Lossless Passive-Radiator System

For the lossless passive-radiator system, the response function given in Eq. (22) is equivalent to Eq. (27) for

$$\begin{aligned} T_0 &= (T_P T_S)^{1/2}/\gamma^{1/4} \\ b_2 &= y/\gamma^{1/2} \\ a_1 &= 1/(Q_T y^{1/2}\gamma^{1/4}) \\ a_2 &= (1/\gamma^{1/2})[y(\delta + 1) + (\alpha + 1)/y] \\ a_3 &= y^{1/2}(\delta + 1)/(Q_T\gamma^{3/4}) \end{aligned} \qquad \text{(A-13)}$$

where y is given by Eq. (19) and

$$\gamma = \alpha + \delta + 1. \qquad \text{(A-14)}$$

For any given response, the parameters of a lossless system which will produce this response are thus

$$\alpha = \frac{a_2 a_3/a_1 - (a_3/a_1)^2 - 1}{1 - b_2(a_2 - a_3/a_1)}$$
$$\delta = (1/b_2)(a_3/a_1) - 1$$
$$Q_T = 1/[a_1(\gamma b_2)^{1/2}] \quad \text{(A-15)}$$
$$f_0/f_S = (\gamma b_2)^{1/2}$$
$$h = (\gamma b_2)^{1/2}(a_3/a_1)^{1/2}$$
$$(\text{or } y = \gamma^{1/2} b_2).$$

The normalized cutoff frequency is found from

$$f_3/f_S = (f_0/f_S)(f_3/f_0) = (\gamma b_2)^{1/2}(f_3/f_0). \quad \text{(A-16)}$$

For the elliptical and quasi-maximally-flat alignments there is an extra degree of freedom, and it is useful to fix an additional parameter relationship so that only a single family of parameter adjustments remains. The practical (and common) restriction $\delta = \alpha$ is used in this paper. This constrains the polynomial coefficients so that

$$\frac{a_3}{a_1} = \frac{2a_2 + b_2 - 1/b_2 \pm \sqrt{(2a_2 + b_2 - 1/b_2)^2 - 8a_2 b_2}}{4}. \quad \text{(A-17)}$$

For this constraint, the elliptical alignment parameters can be obtained as follows. For a given value of α, find the positive real root r of the equation

$$\frac{(\alpha+1)^4(2\alpha+1)^2}{2\alpha^2} r^3 + \frac{(\alpha+1)^3(2\alpha+1)}{\alpha} r^2 + \frac{(\alpha^2-\alpha-1)^2}{2\alpha^2} r - 1 = 0. \quad \text{(A-18)}$$

Then

$$b_2 = r^{1/2}$$
$$a_3/a_1 = b_2(\alpha+1)$$
$$a_2 = b_2(\alpha+1) + (\alpha+1)/[b_2(2\alpha+1)]$$
$$a_1 = [2(a_2 - b_2)]^{1/2}$$
$$a_3 = a_1 b_2(\alpha+1). \quad \text{(A-19)}$$

The remaining alignment parameters are then found from Eqs. (A-15) and (A-16).

From the calculated alignment data used to construct Fig. 5, if was found that a maximally flat $\delta = \alpha$ alignment occurs for $\delta = \alpha \approx 3.01$. By analogy with the vented-box system, only smaller values of $\delta = \alpha$ should be investigated for elliptical responses, and larger values for quasi-maximally-flat responses. The alignment for which $\delta = \alpha = 1 + \sqrt{2}$ is a symmetrical alignment.

The alignment parameters for quasi-maximally-flat responses are obtained as above except that Eq. (A-18) simplifies to

$$r = \frac{1}{2\alpha+1} \frac{\sqrt{b^2 + 4ac} - b}{2a} \quad \text{(A-20)}$$

where

$$a = \alpha(3\alpha+2)$$
$$b = 2\alpha^2$$
$$c = (\alpha+1)^2.$$

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